Lecture 1: General Equilibrium

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Lecture 1: General Equilibrium

Introduction

Pure Exchange Economies

Pareto efficiency

Edgeworth Box
Introduction

Pure Exchange Economies

Pareto efficiency

Edgeworth Box
Previous classes

- Consumers behavior (decision theory) was often analyze separately from firm behavior (producer theory)

- When analyzed together, each market was viewed in isolation
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- But markets are often intertwined
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- Consumers behavior (decision theory) was often analyze separately from firm behavior (producer theory)

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- But markets are often intertwined
  - Transportation: Uber/metro/ecobici/car
  - Wages across sectors
  - Fruits
  - Beer and tacos
Example - Fruits

Suppose that apple and bananas are substitutes.

Diagram of supply and demand for apples and bananas.

- Apples:
  - Supply (SS)
  - Demand (DD)

- Bananas:
  - Supply (SS)
  - Demand (DD)
Example - Fruits

- Suppose that apple and bananas are substitutes
- Supply curve for apples shifts out
Example - Fruits

▶ Suppose that apple and bananas are substitutes
▶ Supply curve for apples shifts out
▶ DD for bananas decreases (exogenous)
Example - Fruits

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![Graph showing supply and demand curves for apples and bananas.](image-url)
Example - Fruits

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- DD for apples decreases (exogenous) - maybe a lot
Example - Fruits

What happens if apple and bananas are complements?
A tour down memory lane

- Léon Walras started it all (1834-1910)
  - First to use mathematical tools in economics
  - Supply and demand curves as solutions to a maximization problem
  - Started the “marginal revolution”

- Walras was ultimately after normative questions (is the market economy good?)

- But first, he tackled positive questions (is there an equilibrium? is it unique?)

- Made a lot of progress. In particular came up with “Walras Law”: Sum of the values of excess demands across all markets must equal zero always
A tour down memory lane

- Vilfredo Pareto was Walras student (1848-1923)

- Abandoned utilitarianism (i.e., utility functions)

- Embraced “preferences”

  - Utility functions only have cardinal content

  - Comparing “utilis” across individuals is meaningless

- (Pareto) optimum/efficiency: Achieved if we can’t make someone better-off without making someone worst-off
A tour down memory lane

▸ Francis Edgeworth (1845 – 1926)

▸ Introduced indifference curves

▸ Was the first to ask: Where will voluntary exchange lead to?

▸ He conjecture his result was aligned with Walras’ result
A tour down memory lane

No more advances for a while (until 1950’s) then

- Kenneth Arrow
- Gerard Debreu
- Lionel McKenzie

Existence

Showed it was Pareto efficient

Two Nobel prizes (Arrow — 1972 and Debreu — 1974)
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Pure Exchange Economies

- How are goods distributed among consumers?

- What incentives are there to exchange goods? What institutions mediate the exchange?

- Is there a distribution of goods that leaves everyone satisfied and there aren’t any incentives to deviate?
Pure Exchange Economies

What are the properties of such an equilibrium?

- Is it unique?

- Is it stable?

- Is it efficient?
Pure Exchange Economies

▶ Assume there are

▶ \( I \) consumers, \( \mathcal{I} = \{1, ..., I\} \)

▶ \( L \) goods, \( \mathcal{L} = \{1, ..., L\} \)

▶ Each consumer \( i \) is characterized by a utility function \( u^i \).

▶ Each consumer can consume goods in \( X_i = \mathbb{R}^L_+ \)

▶ Each consumer has an initial endowment of \( w^i \in \mathbb{R}^L_+ \).

▶ Each consumer is characterized by the pair: \( (u^i, w^i) \).

▶ Assume the utility functions represent neoclassic preferences.
A brief reminder

- Utility functions are cardinal not ordinal.
- They are used to represent preferences.
- If $x \succ_i y$ then $u_i(x) > u_i(y)$.
- If $f$ is any increasing function then $f(u_i(x)) > f(u_i(y))$.
- Hence $f(u_i(\cdot))$ also represents $\succ_i$.
- $u_i(x) > u_i(y)$ means something, but $u_i(x) - u_i(y)$ does not.

Neoclassic preferences are well behaved.
- They can be represented by a utility function.
- They are weakly monotonic.
- They are quasi-concave.
Utility functions and neoclassic preferences

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Definition (Exchange economy)

A pure exchange economy is \( \mathcal{E} = \left\langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \right\rangle \) where \( \mathcal{I} \) is the set of agents, \( u^i \) is a representation of consumer \( i \)'s preferences and \( w^i \) is consumer \( i \)'s initial endowment.

- Let \( w = \sum_{i=1}^{I} w^i \) be the total endowment of the economy.

- An allocation of resources is denoted by \( x = (x^1, x^2, \ldots, x^I) \) where \( x^i \in \mathbb{R}_+^L \).
Definition (Feasible allocation)

The set of feasible allocation $F$ of an economy $\mathcal{E} = \langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \rangle$ is defined by:

$$F = \left\{ x = (x^1, x^2, ..., x^l) : x^i \in \mathbb{R}^L_+, \sum_{i=1}^{l} x^i = \sum_{i=1}^{l} w^i \right\}$$
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Definition (Pareto efficiency)

Let $E$ be an economy. A feasible allocation of resources $x = (x^1, x^2, \ldots, x^I)$ is Pareto efficient if there isn’t another feasible allocation $\hat{x} = (\hat{x}^1, \hat{x}^2, \ldots, \hat{x}^I)$ such that for every agent $i$, $u^i(\hat{x}^i) \geq u^i(x^i)$ and for at least one agent $i^*$, $u^{i^*}(\hat{x}^{i^*}) > u^{i^*}(x^{i^*})$. 
Pareto efficiency

Definition (Pareto domination)
Take two feasible allocations \( x \) and \( \hat{x} \). We say that \( \hat{x} \) Pareto dominates \( x \) if for all \( i = 1, \ldots, I \),

\[
u_i(\hat{x}_1^i, \ldots, \hat{x}_L^i) \geq u_i(x_1^i, \ldots, x_L^i)
\]

and there is at least one consumer \( j \) for which

\[
u_j(\hat{x}_1^j, \ldots, \hat{x}_L^j) > u_j(x_1^j, \ldots, x_L^j).
\]
Thinking about Pareto efficiency

- If $x$ is a Pareto efficient feasible allocation, does it mean that $x$ Pareto dominates all other feasible allocations?

- If there are two allocations ($x$ and $y$) is it always the case that one Pareto dominates the other?

- For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy

- Social planner should strive to achieve Pareto efficiency at the very least!
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- For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy.

- Social planner should strive to achieve Pareto efficiency at the very least! However, she may have other concerns such as fairness.
Thinking about Pareto efficiency

- If utility is strictly increasing, then can a Pareto efficient allocation be such that $\sum_{i=1}^{l} x_j < \sum_{i=1}^{l} w_j$?

- The set of all Pareto allocations is known as the contract curve
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Pareto efficiency

Edgeworth Box
The Edgeworth Box

origin for person A

$w_y$

$w_x$

good $Y$

good $X$

The Edgeworth box
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