# Lecture 11: Game Theory // Preliminaries and dominance

Mauricio Romero

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Introduction - Continued

Static games with complete information

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Introduction - Continued

Static games with complete information

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- This is just a way to schematizing the game and in general it makes the analysis simpler

# Normal form

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- The list of players
- The strategy space
- The pay-off functions

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There is no mention of rules or available information. Where is this hidden?

When there a few players (2 or 3) a matrix is used to represent the game in the normal form.

| | $s_2$ | $s_2'$ |
|---|---|---|
| $s_1$ | $(u_1(s_1, s_2), u_2(s_1, s_2))$ | $(u_1(s_1, s_2'))$ |
| $s_1'$ | $(u_1(s_1', s_2), u_2(s_1', s_2))$ | $(u_1(s_1', s_2'))$ |
| $s_1''$ | $(u_1(s_1'', s_2), u_2(s_1'', s_2))$ | $(u_1(s_1'', s_2'))$ |

# Matching-Pennies (Pares y Nones) – Simultaneous

Both players play at the same time

| | $1_B$ | $2_B$ |
|---|---|---|
| $1_A$ | (1000,-1000) | (-1000,1000) |
| $2_A$ | (-1000,1000) | (1000,-1000) |

# Matching-Pennies (Pares y Nones) – Sequential

A plays first, then B

| | $(1,1)$ | $(1,2)$ | $(2,1)$ | $(2,2)$ |
|---|---|---|---|---|
| $1_A$ | (1000,-1000) | (1000,-1000) | (-1000,1000) | (-1000,1000) |
| $2_A$ | (-1000,1000) | (1000,-1000) | (-1000,1000) | (1000,-1000) |

# Prisoner's Dilemma

There are two players $I = \{1, 2\}$ that are members of a drug cartel who are both arrested an imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack enough evidence to convict the pair on the principal charge so they must settle for a lesser charge. Simultaneously, the prosecutor offers each prisoner a deal. Each prisoner is given the opportunity to either 1) betray the other by testifying the other committed the crime or to 2) cooperate with the other prisoner and stay silent.

## Prisoner's Dilemma

The strategies of player 1:

$$S_1 = \{\mathsf{betray}_1, \mathsf{silent}_1\}.$$

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# Prisoner's Dilemma

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The strategies of player 2:

$$S_2 = \{\text{betray}_2, \text{silent}_2\}.$$

The utility function of the players is given by:

$$u_1(b_1, b_1) = -2, u_2(b_1, b_1) = -2$$
$$u_1(b_1, s_2) = 0, u_2(b_1, s_2) = -3$$
$$u_1(s_1, b_2) = -3, u_2(s_1, b_2) = 0$$
$$u_1(s_1, s_2) = -1, u_2(s_1, s_2) = -1.$$

# Prisoner's Dilemma

Prisoner's Dilemma

| | $s_2$ | $b_2$ |
|---|---|---|
| $s_1$ | $-1, -1$ | $-3, 0$ |
| $b_1$ | $0, -3$ | $-2, -2$ |

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  - The actions available to each player in each point in time
  - The pay-off functions

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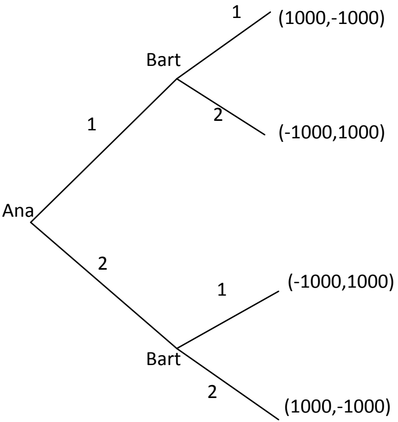
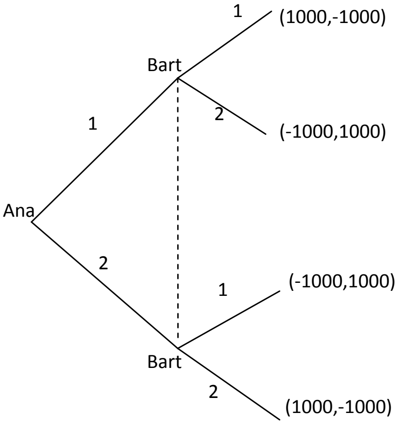
- Each node where a branch begins is a decision node, where a player needs to choose an action
- If two nodes are connected by a dotted line, it means they are in the same information set (i.e., the player is not sure in which node she is in)

# Matching-Pennies (Pares y Nones) – Sequential




Pares o Nones I

# Matching-Pennies (Pares y Nones) – Simultaneous




Pares o Nones II

Theorem

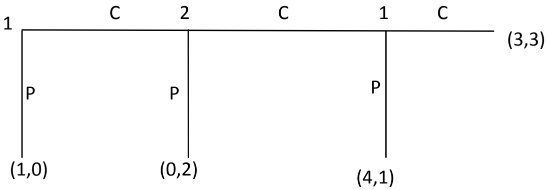
*Every game can be represented in both forms (extensive and normal). The representation you choose will not alter the analysis, but it may be simpler to do the analysis with one form or another. A normal form game may have several extensive representations (but every extensive form has a single normal form equivalent to it); however, all of the results we will see/use are robust to the representation used.*

# Centipede Game

Suppose there are two individuals Ana and Bernardo. Ana is given a chocolate. She can stop the game and keep the chocolate or she can continue. If she continues, Ana's chocolate is taken away and Bernardo is given two. Bernardo can then stop the game and keep two chocolates (and Ana will get zero) or can continue. If he continues, a chocolate is taken away from him and Ana is given four. Ana can stop the game and keep 4 chocolates (and Bernardo will keep one), or she can continue, in which case the game ends with three chocolates for each one.

# Centipede Game

The extensive form is

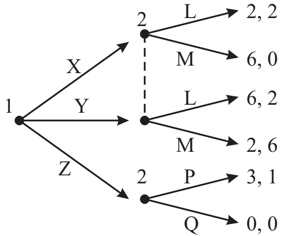


# Centipede Game

The normal form is

| | C | P |
|---|---|---|
| C,C | 3,3 | 0,2 |
| C,P | 4,1 | 0,2 |
| P,C | 1,0 | 1,0 |
| P,P | 1,0 | 1,0 |

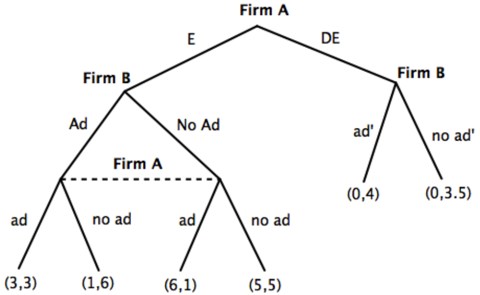
Consider the following game in extensive form:



The normal form is:

| 1 \ 2 | LP | LQ | MP | MQ |
|---|---|---|---|---|
| X | 2, 2 | 2, 2 | 6, 0 | 6, 0 |
| Y | 6, 2 | 6, 2 | 2, 6 | 2, 6 |
| Z | 3, 1 | 0, 0 | 3, 1 | 0, 0 |

Consider the following game in extensive form



The normal form is:

| | $Ad, ad'$ | $Ad, no\ ad'$ | $No\ Ad, ad'$ | $No\ Ad, no\ ad'$ |
|---|---|---|---|---|
| $(E, ad)$ | 3,3 | 3,3 | 6,1 | 6,1 |
| $(E, no\ ad)$ | 1,6 | 1,6 | 5,5 | 5,5 |
| $(DE, ad)$ | 0, 4 | 0,3.5 | 0,4 | 0,3.5 |
| $(DE, no\ ad)$ | 0, 4 | 0,3.5 | 0,4 | 0,3.5 |

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- That is, strategies where no individual has incentives to deviate or to do something different, given what others do.
- This is a concept equivalent to general equilibrium, where given market prices, everyone is optimizing, markets empty, and therefore no one has incentives to deviate, but nobody told us how we got there .. . pause (the Walrasian auctioneer?)

Introduction - Continued

Static games with complete information

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## Static games with complete information

- Games where all players move simultaneously and only once
- If players move sequentially, but can not observe what other people played, it's equivalent to a static game
- Only consider games of complete information (all players know the objective functions of their opponents)
- These are very restrictive conditions but they will allow us to present very important concepts that will be easy to extend to more complex games
- As each player faces one contingency, the strategies are identical to the actions.

# Dominance

- Intuitively if a strategy $s_i$ always results in a greater utility than $s_i'$, regardless of the strategy followed by the other players then the strategy $s_i'$ should never be chosen by individual $i$

# Dominance

$s_i$ **strictly dominates** $s_i'$ if no matter what the opponent does, $s_i$ gives a better payoff to $i$ than $s_i'$

## Definition

Let $s_i, s_i'$ be two pure strategies. Then we say that $s_i$ strictly dominates $s_i'$ if for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

# Dominance

A pure strategy $s_i$ is **strictly dominant** if $s_i$ strictly dominates every other strategy $s_i'$

## Definition

Let $s_i$ be a pure strategy of player $i$. Then $s_i$ is strictly dominant if for all $s_i' \neq s_i$, $s_i$ strictly dominates $s_i'$.

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- Intuitively if a strategy $s_i$ always results in a greater utility than $s_i'$, regardless of the strategy followed by the other players then the strategy $s_i'$ should never be chosen by individual $i$

- We can eliminate any strategy that is strictly dominated

# Dominance in the prisoners dilemma

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

- $NC$ dominates $C$ for both individuals

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# Dominance in the prisoners dilemma

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- $NC$ dominates $C$ for both individuals
- $(NC, NC)$ is not a Pareto Optimum.
- What happened to the first welfare theorem? Is it incorrect?

# Dominance (iterated)

Consider this game

| | a | b | c |
|---|---|---|---|
| A | 5, 5 | 0, 10 | 3, 4 |
| B | 3, 0 | 2, 2 | 4, 5 |

- Player 1 has no strategy that is strictly dominated

# Dominance (iterated)

Consider this game

| | a | b | c |
|---|---|---|---|
| A | 5, 5 | 0, 10 | 3, 4 |
| B | 3, 0 | 2, 2 | 4, 5 |

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*

# Dominance (iterated)

Consider this game

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|---|---|---|---|
| A | 5, 5 | 0, 10 | 3, 4 |
| B | 3, 0 | 2, 2 | 4, 5 |

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*
- Player 1 would foresee this...

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| | b | c |
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- $B$ now dominates $A$ for player 1

# Dominance (iterated)

| | b | c |
|---|---|---|
| A | 0, 10 | 3, 4 |
| B | 2, 2 | 4, 5 |

- $B$ now dominates $A$ for player 1
- Player 2 would foresee this (that player 1 foresees that 2 will not play a, and thus he will not play B)

# Dominance (iterated)

| | b | c |
|---|---|---|
| B | 2, 2 | 4, 5 |

- Player 2 would play $c$ and player 1 would play $B$

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- Player 2 would play $c$ and player 1 would play $B$
- We have reached a solution $(B, c)$
- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)
- The equilibrium is the set of strategies, not the payoff!

### Definition (Solvable by IDSDS)

A game is solvable by **Iterated Deletion of Strictly Dominated Strategies** if the result of the iteration is a single strategy profile (one strategy for each player)

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- Two key assumptions:
- 1) Nobody plays a strictly dominated strategy (that is, the agents are rational)
- 2) Everyone trusts others are rational (i.e., they do not play strictly dominated strategies). That is, agents' rationality is *common information*
- Is the order of elimination of the strategies important? **No**
- Not all games are solvable by IDSDS

# Battle of the sexes

| | G | P |
|---|---|---|
| G | 2,1 | 0,0 |
| P | 0,0 | 1,2 |

- No strategy is dominated for either player