Lecture 9: Price Discrimination

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Lecture 9: Price Discrimination

Third Degree Price Discrimination

Monopsony

Double Marginalization Problem

Profit Sharing and Double Marginalization
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Profit Sharing and Double Marginalization
Market is segmented (no re-selling across markets)

Firm knows the characteristics of each market (demand curve)

Consider the following example: Two kinds of consumers:

\[
q_A(p_A) = 24 - p_A \\
q_B(p_B) = 24 - 2p_B.
\]

constant marginal cost of production of 6
If the firm were allowed to set different prices in the different markets, then he would choose:

\[
\max_{p_A} (24 - p_A)(p_A - 6) \Rightarrow p_A^* = 15
\]

\[
\max_{p_B} (24 - 2p_B)(p_B - 6) \Rightarrow p_B^* = 9.
\]
Total consumer surplus (CS) and profits of the firm in each market:

\[ \pi^*_A = 81, \pi^*_B = 18, CS_A = 40.5, CS_B = 9. \]
Firm chose to set the same price in each market. Then he would maximize the following:

\[
\max \left\{ \max_{p \geq 12} (24 - p)(p - 6), \max_{p < 12} (24 - p)(p - 6) + (24 - 2p)(p - 6) \right\} \\
= \max\{81, 75\} = 81
\]
Price of $p^* = 15$ in both markets, which leads to only consumers in market $A$ buying.

To summarize, the consumer surplus and profits in each market are:

$$\pi_A^* = 81, \pi_B^* = 0, CS_A = 40.5, CS_B = 0.$$  

Prohibiting third degree price discrimination can exclude a whole market altogether.

Highly inefficient compared to the social welfare outcome given third degree price discrimination.
Suppose that the constant marginal cost of production is now 4 instead of 6.

With third degree price discrimination, the firm sets the following prices:

\[
\max_{p_A} (24 - p_A)(p_A - 4) \implies p_A^* = 14,
\]

\[
\max_{p_B} (24 - 2p_B)(p_B - 4) \implies p_B^* = 8.
\]

In this case, the profits and consumer surplus in each market is given by:

\[
\pi_A^* = 100, \pi_B^* = 32, CS_A = 50, CS_B = 16, TS = 198.
\]
If the firm were prohibited from using third degree price discrimination, then:

\[
\max \left\{ \max_{p \geq 12} (24 - p)(p - 4), \max_{p < 12} (48 - 3p)(p - 4) \right\} = \max\{100, 108\} = 108.
\]

\( p = 10 \)

profits in both markets and the consumer surplus in both markets:

\( \pi_A^* = 84, \pi_B^* = 24, CS_A = 98, CS_B = 4, TS = 210. \)
- Consumers in region $B$ are hurt but consumers in region $A$ gain significantly leading to an increase in consumer surplus

- The firm’s joint profits are hurt but the total surplus actually increases

- Total surplus decreases
Third degree price discrimination is considered illegal in many countries and the European union.

It is possible to get around such allegations by claiming that the differential pricing comes from cost reasons.
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Profit Sharing and Double Marginalization
When someone or some firm is the sole buyer (monopoly is the sole seller)

Often arises in the context of firms being the sole buyers of labor
Let us study the profit maximization problem of a firm:

$$\max_{K,L} pf(K, L) - rK - w(L)L.$$ 

$w$ is now a function of the amount of labor demanded (reflecting the power of the firm in the labor market).
The first order condition yields:

\[ p \frac{\partial f}{\partial L}(K^*, L^*) = w'(L^*)L^* + w(L^*) \Rightarrow pMPL = L^*w' + w. \]

In a competitive market \( w' = 0 \) and so \( pMPL = w \)

Wages and labor below the competitive level (an argument for minimum wages and union)
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Profit Sharing and Double Marginalization
What happens when there are multiple monopolies involved in the market?
What happens when there are multiple monopolies involved in the market?

Firm A produces factor a at no cost

Firm B produces according to a cost function:

\[ C(q_b) = (p_a + c)q_b \]

Demand equation for good b is linear:

\[ q_b(p_b) = 100 - p_b \]
What happens when there are multiple monopolies involved in the market?

Firm A produces factor $a$ at no cost

Firm $b$ in order to supply $q_b$ units of $b$ must buy $q_a$ units of $a$
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Demand equation for good $b$ is linear:

$$q_b(p_b) = 100 - p_b.$$
Firm B’s optimization problem becomes:

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\max_{q_b} (100 - q_b)q_b - p_a q_b - c q_b.
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The first order condition tells us:

$$100 - 2q_b = p_a + c \implies p_a = 100 - 2q_b - c.$$
Firm $B$’s optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - p_a q_b - cq_b.$$ 

The first order condition tells us:

$$100 - 2q_b = p_a + c \implies p_a = 100 - 2q_b - c.$$ 

Since firm $b$ is the only demander of commodity $a$, we have:

$$p_a = 100 - 2q_b - c = 100 - 2q_a - c.$$
If the price is $p_a$ then the $q_a$ that solves the above equation would be the amount demanded of good $a$. 

Thus firm B's maximization problem has given us an inverse demand function for commodity $a$. 

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Thus firm $B$’s maximization problem has given us an inverse demand function for commodity $a$. 
Since firm A is also a monopolist in producing good a, we can solve firm A’s maximization problem in the following way:

$$\max_{q_a} q_a (100 - 2q_a - c).$$
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As a result, we get:

$$100 - 4q_a - c = 0 \Rightarrow q^*_a = \frac{100 - c}{4}, p^*_a = 50 - \frac{c}{2}.$$
Since firm A is also a monopolist in producing good a, we can solve firm A’s maximization problem in the following way:

$$\max_{q_a} q_a (100 - 2q_a - c).$$

As a result, we get:

$$100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, \quad p_a^* = 50 - \frac{c}{2}.$$  

Firm a decides to supply the above units of a at a price $50 - c/2$. 

Firm $B$ will produce $q_b^* = q_a^* = \frac{100-c}{4}$
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Then the price is given by:

$$p_b^* = 100 - \frac{100 - c}{4} = 75 + \frac{c}{4}.$$
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Then the price is given by:

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To summarize, we have:

$$p_a^* = 50 - \frac{c}{2} \quad (1)$$
$$q_a^* = \frac{100 - c}{4} \quad (2)$$
$$p_b^* = 75 + \frac{c}{4} \quad (3)$$
$$q_b^* = \frac{100 - c}{4} \quad (4)$$
Case 1: $c = 0$

\[ p_a^* = 50, \, q_a^* = 25, \, p_b^* = 75, \, q_b^* = 25. \]

If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?

The monopolists problem becomes:

\[
\max_q q(100 - q).
\]

The first order condition states that:

\[ 100 - 2q^* = 0 \implies q^* = 50, \, p^* = 50. \]

Price of good $b$ comes down from 75 to 50

Production of good $b$ goes up from 25 to 50

This increases both the profits of the firm and the consumer surplus!
Case 1: $c = 10$

$p_a^* = 45, q_a^* = 22.5, p_b^* = 77.5, q_b^* = 22.5.$

If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?

The monopolists problem becomes:

$$\max_q q(100 - q) - 10q$$

The first order condition states that:

$$100 - 2q = 10 \implies p^* = 55, q^* = 45.$$  

This increases both the profits of the firm and the consumer surplus!
What is going on in the above examples?

because the first firm is a monopolist, it charges a mark up above marginal cost for its intermediate good

This then distorts the marginal cost of firm B up additionally

This then leads an even larger mark up on top of this additional marginal cost that affects the price of good b

Essentially a markup on product a indirectly leads to an even larger markup on the final product b

This is called the **double marginalization problem**
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Profit Sharing and Double Marginalization
Double marginalization can lead to inefficiently high prices and inefficiently low levels of production.

By merging, both profits of the firm and consumer surplus may simultaneously go up.

Difficult to tell if two firms are merging to solve a double marginalization problem or if they are simply merging to create a monopoly.

What are some potential ways to solve this problem without mergers?

One possible way might be to engage in profit sharing.
Firms agree to share profits according to the following rule

Prices charged for good $a$ are zero

In exchange, the profits of firm $B$ are shared via a split of $\alpha$ going to firm $A$ and $(1 - \alpha)$ going to firm $B$
Firms agree to share profits according to the following rule

Prices charged for good \( a \) are zero

In exchange, the profits of firm \( B \) are shared via a split of \( \alpha \) going to firm \( A \) and \((1 - \alpha)\) going to firm \( B \)

Firm \( A \)'s decision is trivial. He simply produces \( q_a = q_b \)

Firm \( B \) chooses to maximize:

\[
\max_q (1 - \alpha) ((100 - q)q - cq) = (1 - \alpha) \left( \max_q (100 - q)q - cq \right).
\]

Term inside the parentheses is just the monopoly profits if the two firms merged:

\[
(1 - \alpha) \max_q \Pi^m(q).
\]
The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement.
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Really, for any $\alpha$?
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Really, for any $\alpha$?

Such arrangements can break down easily. Profits are hard to verify.
Profits are usually difficult to verify. However, revenues are much easier to check.

\[ \alpha \] firms enter into an arrangement where the revenues are shared according to \( \alpha \) (firm A) and \( (1 - \alpha) \) (firm B) split.

Suppose that \( \alpha = \frac{1}{2} \) and \( c = 10 \). Then firm 2 maximizes:

\[
\max_{q_1, q_2} q_1 (100 - q_1) - 10 q_2.
\]

The first order condition gives:

\[
\frac{1}{2} \frac{d}{dq}(100 - q_1) = 10 = \Rightarrow \frac{d}{dq}(100 - q_1) = 20.
\]

Firm will produce below monopoly profits since it will produce at a point where \( \frac{d}{dq}(100 - q_1) = 20 \) instead of \( \frac{d}{dq}(100 - q_1) = 10 \).
Profits are usually difficult to verify. However, revenues are much easier to check.

Firms enter into an arrangement where the revenues are shared according to $\alpha$ (firm A) and $(1 - \alpha)$ (firm B) split.

Suppose that $\alpha = \frac{1}{2}$ and $c = 10$. Then firm 2 maximizes:

$$\max q_1 q_2 (100 - q_2) - 10 q_2.$$ 

The first order condition gives:

$$\frac{1}{2} MR(q_2) = MC = 10 = \Rightarrow MR(q_2) = 2 MC = 20.$$ 

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Solving, we get:

\[
100 - 2q = 20 \implies p^* = 60 > p^m = 55, \quad q^* = 40 < q^m = 45.
\]
Solving, we get:

\[100 - 2q = 20 \implies p^* = 60 > p^m = 55, q^* = 40 < q^m = 45.\]

This does solve the double marginalization problem slightly:

\[p_b^* = 77.5 > p^* = 60, q_b^* = 22.5 < q^* = 40.\]