

# Lecture 1: General Equilibrium

Mauricio Romero

# Lecture 1: General Equilibrium

Introduction

Pure Exchange Economies

Pareto efficiency

Edgeworth Box

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## Previous classes

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- ▶ But markets are often intertwined

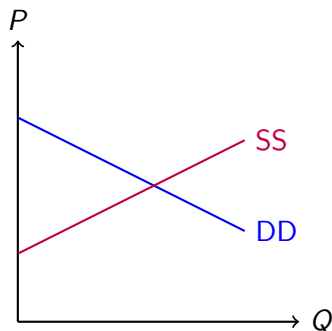
## Previous classes

- ▶ Consumers behavior (decision theory) was often analyze separately from firm behavior (producer theory)
- ▶ When analyzed together, each market was viewed in isolation
- ▶ But markets are often intertwined
  - ▶ Transportation: Uber/metro/ecobici/car
  - ▶ Wages across sectors
  - ▶ Fruits
  - ▶ Beer and tacos

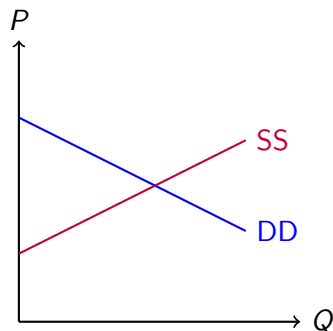
## Example - Fruits

- ▶ Suppose that apple and bananas are substitutes

Apples



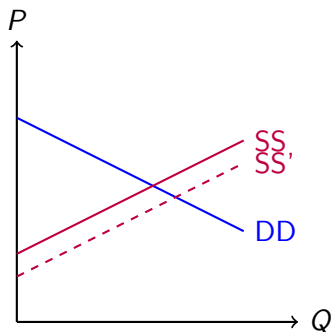
Bananas



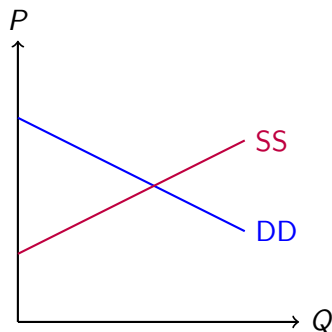
## Example - Fruits

- ▶ Suppose that apple and bananas are substitutes
- ▶ Supply curve for apples shifts out

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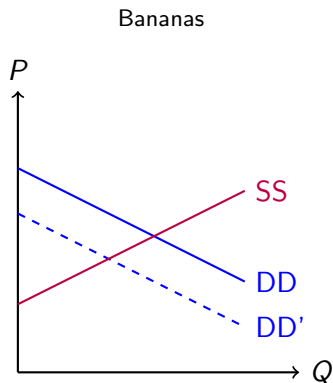
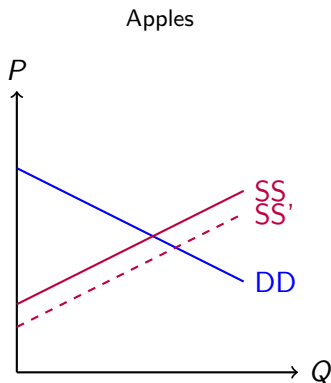
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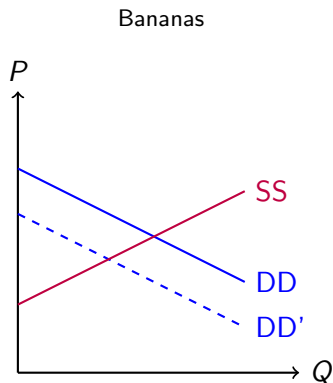
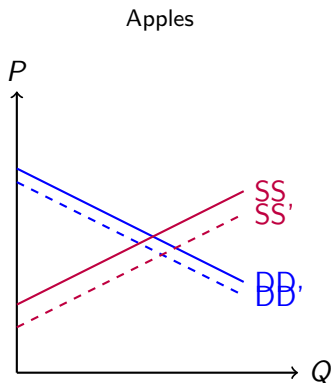
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- ▶ DD for bananas decreases (exogenous)



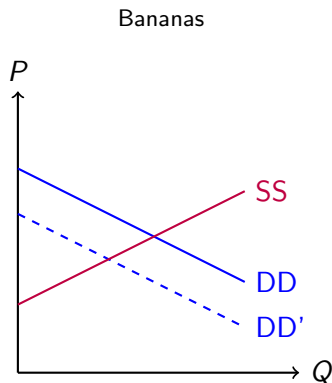
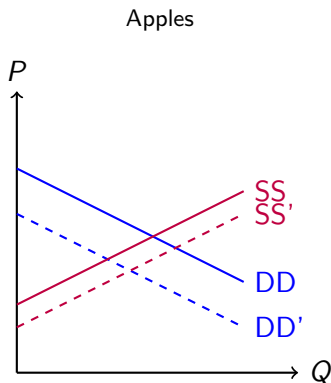
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## Example - Fruits

- ▶ What happens if apple and bananas are complements?

## A tour down memory lane

- ▶ Léon Walras started it all (1834-1910)
  - ▶ First to use mathematical tools in economics
  - ▶ Supply and demand curves as solutions to a maximization problem
  - ▶ Started the “marginal revolution”
- ▶ Walras was ultimately after normative questions (is the market economy good?)
- ▶ But first, he tackled positive questions (is there an equilibrium? is it unique?)
- ▶ Made a lot of progress. In particular came up with “Walras Law”: Sum of the values of excess demands across all markets must equal zero always

## A tour down memory lane

- ▶ Vilfredo Pareto was Walras student (1848-1923)
  - ▶ Abandoned utilitarianism (i.e., utility functions)
  - ▶ Embraced “preferences”
    - ▶ Utility functions only have cardinal content
    - ▶ Comparing “utilis” across individuals is meaningless
  - ▶ (Pareto) optimum/efficiency: Achieved if we can't make someone better-off without making someone worse-off

## A tour down memory lane

- ▶ Francis Edgeworth (1845 – 1926)
  - ▶ Introduced indifference curves
  - ▶ Was the first to ask: Where will voluntary exchange lead to?
  - ▶ He conjecture his result was aligned with Walras' result

## A tour down memory lane

- ▶ No more advances for a while (until 1950's) then
  - ▶ Kenneth Arrow
  - ▶ Gerard Debreu
  - ▶ Lionel McKenzie
- ▶ Existence
- ▶ Showed it was Pareto efficient
- ▶ Two Nobel prizes (Arrow — 1972 and Debreu — 1974)



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## Pure Exchange Economies

- ▶ How are goods distributed among consumers?
- ▶ What incentives are there to exchange goods? What institutions mediate the exchange?
- ▶ Is there a distribution of goods that leaves everyone satisfied and there aren't any incentives to deviate?

# Pure Exchange Economies

- ▶ What are the properties of such an equilibrium?
  - ▶ Is it unique?
  - ▶ Is it stable?
  - ▶ Is it efficient?

# Pure Exchange Economies

- ▶ Assume there are
  - ▶  $I$  consumers,  $\mathcal{I} = \{1, \dots, I\}$
  - ▶  $L$  goods,  $\mathcal{L} = \{1, \dots, L\}$
  - ▶ Each consumer  $i$  is characterized by a utility function  $u^i$ .
  - ▶ Each consumer can consume goods in  $X_i = \mathbb{R}_+^L$
  - ▶ Each consumer has an initial endowment of  $w^i \in \mathbb{R}_+^L$ .
  - ▶ Each consumer is characterized by the pair:  $(u^i, w^i)$ .
  - ▶ Assume the utility functions represent neoclassic preferences

# Utility functions and neoclassic preferences

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- ▶ They are used to represent preferences
  - ▶ If  $x \succ_i y$  then  $u^i(x) > u^i(y)$
  - ▶ If  $f$  is any increasing function then  $f(u^i(x)) > f(u^i(y))$
  - ▶ Hence  $f(u^i(\cdot))$  also represents  $\succ_i$
  - ▶  $u^i(x) > u^i(y)$  means something, but  $u^i(x) - u^i(y)$  does not
- ▶ Neoclassic preferences are well behaved

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  - ▶  $u^i(x) > u^i(y)$  means something, but  $u^i(x) - u^i(y)$  does not
- ▶ Neoclassic preferences are well behaved
  - ▶ They can be represented by a utility function
  - ▶ They are weakly monotonic
  - ▶ They are quasi-concave

## Pure Exchange Economies

### Definition (Exchange economy)

A pure exchange economy is  $\mathcal{E} = \langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \rangle$  where  $\mathcal{I}$  is the set of agents,  $u^i$  is a representation of consumer  $i$ 's preferences and  $w^i$  is consumer  $i$ 's initial endowment.

- ▶ Let  $w = \sum_{i=1}^I w^i$  be the total endowment of the economy.
- ▶ An *allocation* of resources is denoted by  $x = (x^1, x^2, \dots, x^I)$  where  $x^i \in \mathbb{R}_+^L$ .

# Pure Exchange Economies

## Definition (Feasible allocation)

The set of *feasible* allocation  $F$  of an economy

$\mathcal{E} = \langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \rangle$  is defined by:

$$F = \left\{ x = (x^1, x^2, \dots, x^I) : x^i \in \mathbb{R}_+^L, \sum_{i=1}^I x^i = \sum_{i=1}^I w^i \right\}$$

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## Pareto efficiency

### Definition (Pareto efficiency)

Let  $\mathcal{E}$  be an economy. A feasible allocation of resources  $x = (x^1, x^2, \dots, x^I)$  is Pareto efficient if there isn't another feasible allocation  $\hat{x} = (\hat{x}^1, \hat{x}^2, \dots, \hat{x}^I)$  such that for every agent  $i$ ,  $u^i(\hat{x}^i) \geq u^i(x^i)$  and for at least one agent  $i^*$ ,  $u^{i^*}(\hat{x}^{i^*}) > u^{i^*}(x^{i^*})$ .

### Definition (Pareto domination)

Take two feasible allocations  $x$  and  $\hat{x}$ . We say that  $\hat{x}$  Pareto dominates  $x$  if for all  $i = 1, \dots, I$ ,

$$u_i(\hat{x}_1^i, \dots, \hat{x}_L^i) \geq u_i(x_1^i, \dots, x_L^i)$$

and there is at least one consumer  $j$  for which

$$u_j(\hat{x}_1^j, \dots, \hat{x}_L^j) > u_j(x_1^j, \dots, x_L^j).$$



## Thinking about Pareto efficiency

- ▶ If  $x$  is a Pareto efficient feasible allocation, does it mean that  $x$  Pareto dominates all other feasible allocations?
- ▶ If there are two allocations ( $x$  and  $y$ ) is it always the case that one Pareto dominates the other?
- ▶ For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy
- ▶ Social planner should strive to achieve Pareto efficiency at the very least!

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- ▶ For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy
- ▶ Social planner should strive to achieve Pareto efficiency at the very least! However, she may have other concerns such as fairness

## Thinking about Pareto efficiency

- ▶ If utility is strictly increasing, then can a Pareto efficient allocation be such that  $\sum_{i=1}^I x_j^i < \sum_{i=1}^I w_j^i$ ?
- ▶ The set of all Pareto allocations is known as the **contract curve**

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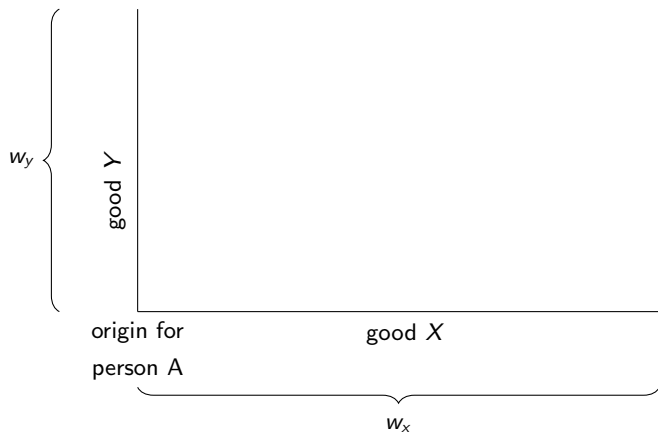
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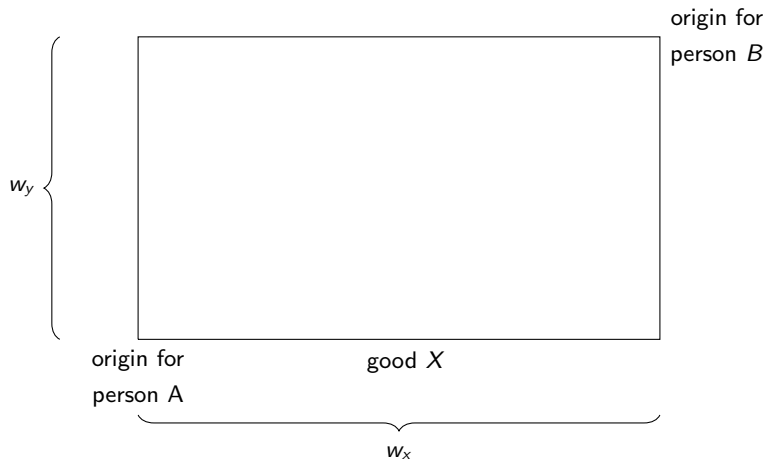
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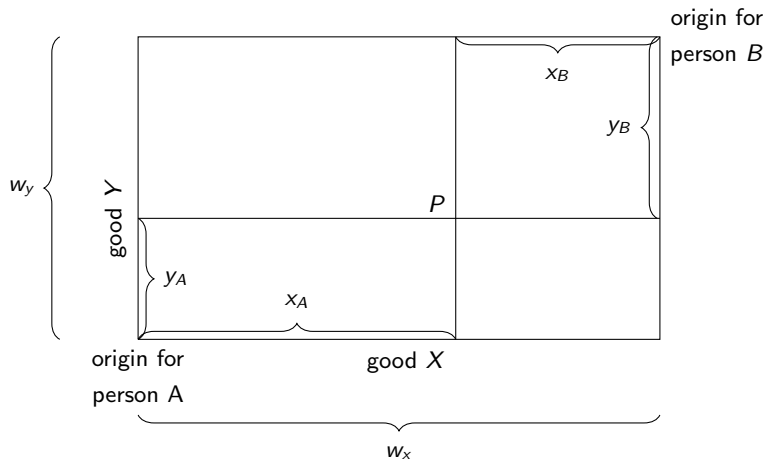
The Edgeworth box

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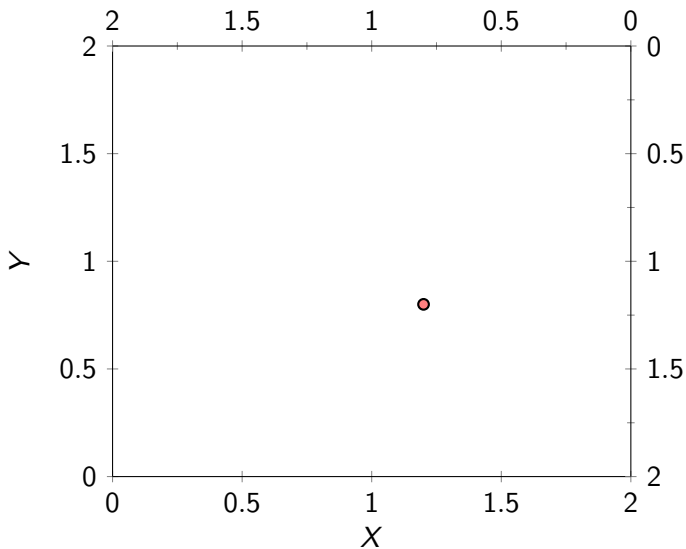
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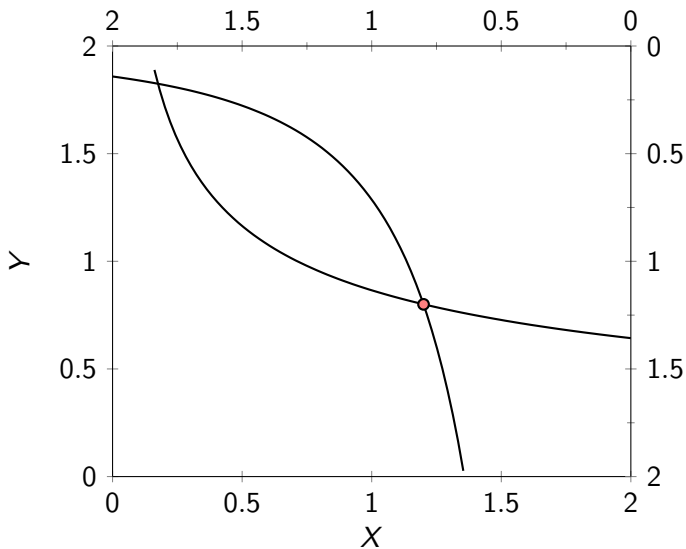
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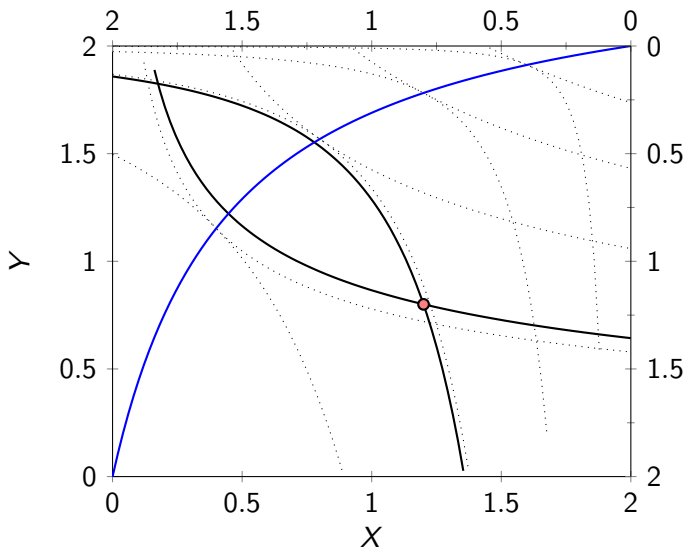
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