Lecture 11: Game Theory // Preliminaries and dominance

Mauricio Romero

Introduction - Continued

Static games with complete information

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Static games with complete information

- We will represent games in two different ways

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- This is just a way to schematizing the game and in general it makes the analysis simpler

Normal form

The normal form consists of:

- The list of players
- The strategy space
- The pay-off functions

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There is no mention of rules or available information. Where is this hidden?

When there a few players (2 or 3) a matrix is used to represent the game in the normal form.

	s_1	s_1'	s_1''
s_2	$(u_1(s_1, s_2), u_2(s_1, s_2))$	$(u_1(s_1', s_2), u_2(s_1', s_2))$	$(u_1(s_1'', s_2), u_2(s_1'', s_2))$
s_2'	$(u_1(s_1, s_2'), u_2(s_1, s_2'))$	$(u_1(s_1', s_2'), u_2(s_1', s_2'))$	$(u_1(s_1'', s_2''), u_2(s_1'', s_2''))$

Matching-Pennies (Pares y Nones) I

Both players play at the same time

	1_B	2_B
1_A	(1000,-1000)	(-1000,1000)
2_A	(-1000,1000)	(1000,-1000)

Matching-Pennies (Pares y Nones) II

A plays first, then B

	$(1,1)$	$(1,2)$	$(2,1)$	$(2,2)$
1_A	(1000,-1000)	(1000,-1000)	(-1000,1000)	(-1000,1000)
2_A	(-1000,1000)	(-1000,1000)	(1000,-1000)	(1000,-1000)

Prisoner's Dilemma

There are two players $I = \{1, 2\}$ that are members of a drug cartel who are both arrested an imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack enough evidence to convict the pair on the principal charge so they must settle for a lesser charge. Simultaneously, the prosecutor offers each prisoner a deal. Each prisoner is given the opportunity to either 1) betray the other by testifying the other committed the crime or to 2) cooperate with the other prisoner and stay silent.

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$$S_1 = \{\text{betray}_1, \text{silent}_1\}.$$

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Prisoner's Dilemma

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The strategies of player 2:

$$S_2 = \{\text{betray}_2, \text{silent}_2\}.$$

The utility function of the players is given by:

$$u_1(b_1, b_1) = -2, u_2(b_1, b_1) = -2$$
$$u_1(b_1, s_2) = 0, u_2(b_1, s_2) = -3$$
$$u_1(s_1, b_2) = -3, u_2(s_1, b_2) = 0$$
$$u_1(s_1, s_2) = -1, u_2(s_1, s_2) = -1.$$

Prisoner's Dilemma

Prisoner's Dilemma

	s_2	b_2
s_1	$-1, -1$	$-3, 0$
b_1	$0, -3$	$-2, -2$

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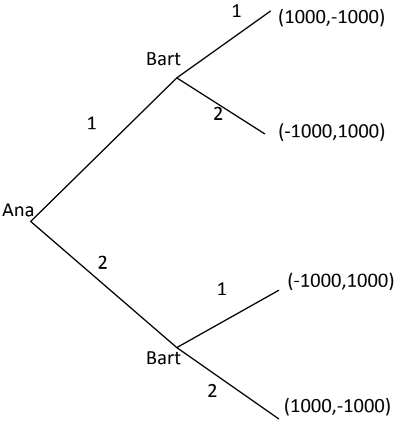
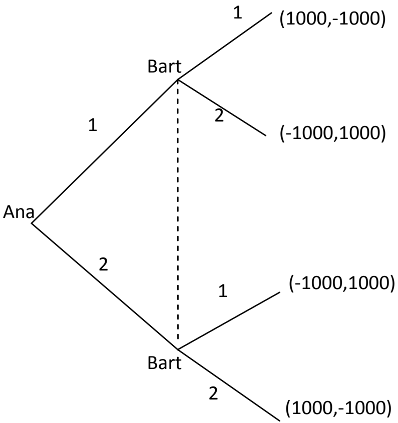
- If two nodes are connected by a dotted line, it means they are in the same information set (i.e., the player is not sure in which node she is in)

Matching-Pennies (Pares y Nones) I



Pares o Nones I

Matching-Pennies (Pares y Nones) II



Pares o Nones II

Theorem

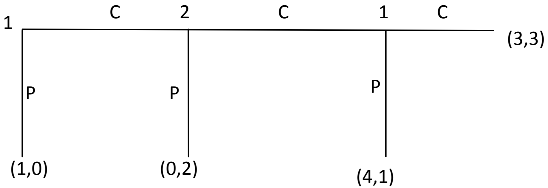
Every game can be represented in both forms (extensive and normal). The representation you choose will not alter the analysis, but it may be simpler to do the analysis with one form or another. A normal form game may have several extensive representations (but every extensive form has a single normal form equivalent to it); however, all of the results we will see/use are robust to the representation used.

Centipede Game

Suppose there are two individuals Ana and Bernardo. Ana is given a chocolate. She can stop the game and keep the chocolate or she can continue. If she continues, Ana's chocolate is taken away and Bernardo is given two. Bernardo can then stop the game and keep two chocolates (and Ana will get zero) or can continue. If he continues, a chocolate is taken away from him and Ana is given four. Ana can stop the game and keep 4 chocolates (and Bernardo will keep one), or she can continue, in which case the game ends with three chocolates for each one.

Centipede Game

The extensive form is

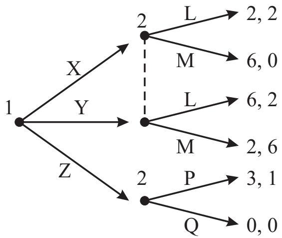


Centipede Game

The normal form is

	C	P
C,C	3,3	0,2
C,P	4,1	0,2
P,C	1,0	1,0
P,P	1,0	1,0

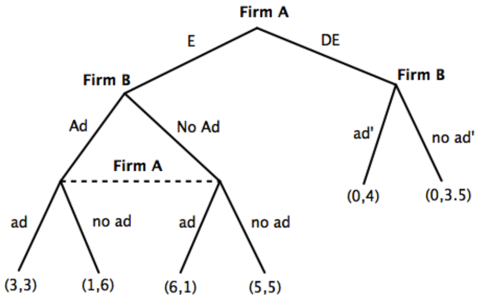
Consider the following game in extensive form:



The normal form is:

1 \ 2	LP	LQ	MP	MQ
X	2, 2	2, 2	6, 0	6, 0
Y	6, 2	6, 2	2, 6	2, 6
Z	3, 1	0, 0	3, 1	0, 0

Consider the following game in extensive form



The normal form is:

	Ad, ad'	$Ad, no\ ad'$	$No\ Ad, ad'$	$No\ Ad, no\ ad'$
(E, ad)	3,3	3,3	6,1	6,1
$(E, no\ ad)$	1,6	1,6	5,5	5,5
(DE, ad)	0, 4	0,3.5	0,4	0,3.5
$(DE, no\ ad)$	0, 4	0,3.5	0,4	0,3.5

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- That is, strategies where no individual has incentives to deviate or to do something different, given what others do.
- This is a concept equivalent to general equilibrium, where given market prices, everyone is optimizing, markets empty, and therefore no one has incentives to deviate, but nobody told us how we got there . . . pause (the Walrasian auctioneer?)

Introduction - Continued

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- If players move sequentially, but can not observe what other people played, it's equivalent to a static game
- Only consider games of complete information (all players know the objective functions of their opponents)
- These are very restrictive conditions but they will allow us to present very important concepts that will be easy to extend to more complex games
- As each player faces one contingency, the strategies are identical to the actions.

Dominance

- Intuitively if a strategy s_i always results in a greater utility than s_i', regardless of the strategy followed by the other players then the strategy s_i' should never be chosen by individual i

Dominance

s_i **strictly dominates** s_i' if no matter what the opponent does, s_i gives a better payoff to i than s_i'

Definition

Let s_i, s_i' be two pure strategies. Then we say that s_i strictly dominates s_i' if for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

Dominance

A pure strategy s_i is **strictly dominant** if s_i strictly dominates every other strategy s_i'

Definition

Let s_i be a pure strategy of player i. Then s_i is strictly dominant if for all $s_i' \neq s_i$, s_i strictly dominates s_i'.

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- Intuitively if a strategy s_i always results in a greater utility than s_i', regardless of the strategy followed by the other players then the strategy s_i' should never be chosen by individual i

- We can eliminate any strategy that is strictly dominated

Dominance in the prisoners dilemma

	C	NC
C	5,5	0,10
NC	10,0	2,2

- NC dominates C for both individuals

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- *NC* dominates *C* for both individuals
- NC, NC is not a Pareto Optimum.

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- NC, NC is not a Pareto Optimum.
- What happened to the first welfare theorem? Is it incorrect?

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
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- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*

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A	5, 5	0, 10	3, 4
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- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*
- Player 1 would foresee this...

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- A now dominates B for player 1

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- *A* now dominates *B* for player 1
- Player 2 would foresee this (that player 1 foresees that 2 will not play a, and thus he will not play B)

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B

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- We have reached a solution (B, c)

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Dominance (iterated)

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- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c)
- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)
- The equilibrium is the set of strategies, not the payoff!

IDSDS

Definition (Solvable by IDSDS)

A game is solvable by **Iterated Deletion of Strictly Dominated Strategies** if the result of the iteration is a single strategy profile (one strategy for each player)

IDSDS

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- Is the order of elimination of the strategies important? **No**
- Not all games are solvable by IDSDS

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

- No strategy is dominated for either player

Beauty contest

- Consider the next game among 100 people. Each individual selects a number, s_i, between 20 and 60.
- Let a_{-i} be the average of the number selected by the other 99 people. i.e. $a_{-i} = \sum_{j\ neqi} \frac{s_j}{99}$.
- The utility function of the individual i is $u_i(s_i, s_{-i}) = 100 - (s_i - \frac{3}{2}a_{-i})^2$

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

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- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others
- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$
- but $a_{-i} \in [20, 60]$
- Therefore $s_i = 20$ is dominated by $s_i = 30$

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- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45

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- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)

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- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.

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- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.
- The solution by means of iterated elimination of dominated strategies is $\underbrace{(60, 60, ..., 60)}_{100 \text{ times}}$

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- here is no strictly dominated strategy

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- here is no strictly dominated strategy
- However, C always gives at least the same utility to player 1 as B
- It's tempting to think player 1 would never play C
- However, if player 1 is sure that player two is going to play a he would be completely indifferent between playing B or C

Definition

s_i weakly dominates s_i' if for all opponent pure strategy profiles, $s_{-i} \in S_{-i}$,

$$u_i(s_i, s_{-i}) \geq u_i(\_i', s_{-i})$$

and there is at least one opponent strategy profile $s_{-i} \in S_{-i}$ for which

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}).$$

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- Rationality is not enough
- Even so, it sounds “logical” to do so and has the potential to greatly simplify a game
- There is a problem, and that is that the order in which we eliminate the strategies matters

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- If we eliminate B (C dominates weakly), then a weakly dominates b and we can eliminate b and therefore player 1 would never play A. This leads to the result (C, a).

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- If we eliminate B (C dominates weakly), then a weakly dominates b and we can eliminate b and therefore player 1 would never play A. This leads to the result (C, a).
- If on the other hand, we notice that A is also weakly dominated by C then we can eliminate it in the first round, and this would eliminate a in the second round and therefore B would be eliminated. This would result in (C, b).