Lecture 16: Applications of Subgame Perfect Nash Equilibrium

Mauricio Romero

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Ultimatum Game

Alternating offers

Stackelberg Competition

Entry Deterrence in Quantity Competition

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1. Player 1 makes a proposal $(x, 1000 - x)$ of how to split 100 pesos among $(100, 900), \ldots, (800, 200), (900, 100)$

2. Player 2 accepts or rejects the proposal

3. If player 2 rejects both obtain 0. If 2 accepts, then the payoffs or the two players are determined by $(x, 1000 - x)$

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- In any SPNE, player 1 makes the proposal $(900, 100)$

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- When extreme offers like $(900, 100)$ are made, player 2 rejects in many cases
- Player 2 may care about inequality or positive utility associated with "punishment" aversion

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- Two players are deciding how to split a pie of size 1

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- The players would rather get an agreement today than tomorrow (i.e., discount factor)

- Player 1 makes an offer θ_1

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- ... and on and on for T periods

- Player 1 makes an offer θ_1
- Player 2 accepts or rejects the proposal
- If player 2 rejects, player 2 makes an offer θ_2
- If player 1 accepts or rejects the proposal
- If player 1 rejects, player 1 makes an offer θ_3
- ... and on and on for T periods
- If no offer is ever accepted, both payoffs equal zero

The discount factor is $\delta \leq 1$.
If Player 1 offer is accepted by Player 2 in round m,

$$\pi_1 = \delta^m \theta_m,$$

$$\pi_2 = \delta^m (1 - \theta_m).$$

If Player 2 offer is accepted, reverse the subscripts

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- There is a unique SPNE: The player that makes the last offer gets the whole pie
- Last-mover advantage

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- In period $(T - 1)$, Player 2 could offer Smith δ, keeping $(1 - \delta)$ for himself
- Player 1 would accept (indifferent between accepting and rejecting) since the **whole pie** in the next period is worth δ

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- Player 1 would accept...
- ...
- In equilibrium, the very first offer would be accepted, since it is chosen precisely so that the other player can do no better by waiting

Table 1 shows the progression of Player 1's shares when $\delta = 0.9$.

Table 1: Alternating Offers over Finite Time

Round	**1's share**	**2's share**	**Total value**	**Who offers?**
$T-3$	$\delta(1-\delta(1-\delta))$	$1-\delta(1-\delta(1-\delta))$	δ^{T-4}	2
$T-2$	$1-\delta(1-\delta)$	$\delta(1-\delta)$	δ^{T-3}	1
$T-1$	δ	$1-\delta$	δ^{T-2}	2
T	1	0	δ^{T-1}	1

- If $T = 3$ (i.e, 1 offers, 2 offers, 1 offers)

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- One offers $\delta(1-\delta)$, 2 accepts in period 1

- Player 1 always does a little better when he makes the offer than when Player 2 does

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- If we consider just the class of periods in which Player 1 makes the offer, Player 1's share falls

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- Firms have the cost functions $c_i(q_i)$.

Te timing of the game is given by:

1. First Firm 1 chooses $q_1 \geq 0$
2. Second Firm 2 observes the chosen q_1 and then chooses q_2

- The game tree in this game is then depicted by an infinite tree

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- A pure strategy for firm 1 is just a choice of $q_1 \geq 0$

- A strategy for firm 2 specifies what it does after every choice of q_1

- Firm 2's strategy is a function $q_2(q_1)$ which specifies exactly what firm 2 does if q_1 is the chosen strategy of player 1

The utility functions for firm i when firm 1 chooses q_1 and firm 2 chooses the strategy (or function) $q_2(\cdot)$ is given by:

$$\pi_1(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_1 - c_1(q_1)$$
$$\pi_2(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_2(q_1) - c_2(q_2(q_1))$$

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- Cconsider the following specific game with demand function given by:

$$P(q_1 + q_2) = A - q_1 - q_2.$$

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- Then the normal form simplifies:

$$u_1(q_1, q_2(\cdot)) = (A - q_1 - q_2(q_1))q_1,$$
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$$q_1^* = \alpha, q_2^*(q_1) = \begin{cases} A & \text{if } q_1 \neq \alpha, \\ \frac{A-\alpha}{2} & \text{if } q_1 = \alpha. \end{cases}$$

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- Let us check that indeed this constitutes a Nash equilibrium

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$$u_1(q_1, q_2^*(\cdot)) = \begin{cases} \left(A - \alpha - \left(\frac{A-\alpha}{2}\right)\right)\alpha > 0 & \text{if } q_1 = \alpha \\ -q_1^2 \leq 0 & \text{if } q_1 \neq \alpha. \end{cases}$$

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- In particular, in the Nash equilibrium corresponding to $\alpha = 0$, the equilibrium outcome is for firm 1 to choose a quantity of 0 and firm 2 setting a price of $A/2$
- This would be the same outcome if firm 2 were the monopolist in this market

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- As a result, the best that firm 1 can do is to produce nothing
- If firm 1 were to hypothetically choose $q_1 > 0$, then firm 2 would obtain negative profits if it indeed follows through with $q_2^*(q_1)$.

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- To eliminate such counterintuitive equilibria, we focus instead on SPNE instead of NE
- Lets continue with the setting in which marginal costs are zero and the demand function is given by $A - q_1 - q_2$

- We always start with the smallest/last subgames which correspond to the decisions of firm 2 after firm 1's choice of q_1 has been made

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- So, player 2 solves:

$$\max_{q_2(\cdot)}(A - q_1 - q_2(q_1))q_2(q_1).$$

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- In this case, the first order condition implies:

$$q_2^*(q_1) = \frac{A - q_1}{2}.$$

- Thus, in any SPNE, player 2 must play the following strategy:

$$q_2^*(q_1) = \begin{cases} \frac{A-q_1}{2} & \text{if } q_1 \leq A \\ 0 & \text{if } q_1 > A. \end{cases}$$

- Then player 1's utility function given that player 2 plays q_2^* is given by:

$$u_1(q_1, q_2^*(\cdot)) = q_1(A - q_1 - q_2^*(q_1)) = \begin{cases} q_1(A - q_1) & \text{if } q_1 > A, \\ q_1 \frac{A - q_1}{2} & \text{if } q_1 \leq A. \end{cases}$$

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- Firm 1 will never choose $q_1 > A$ since then it obtains negative profits
- Thus, firm 1 maximizes:

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- The **equilibrium outcome** is for firm 1 to choose $A/2$ and firm 2 to choose $A/4$

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- In that game, since there is only one subgame, SPNE was the same as the set of NE
- Lets solve for the set of SPNE (which is the same as NE) in the Cournot game with the same demand function and same costs
- In this case, (q_1^*, q_2^*) is a NE if and only if

$$q_1^* \in BR_1(q_2^*), q_2^* \in BR_2(q_1^*).$$

- For $q_1^* \in BR_1(q_2^*)$, we need q_1^* to solve the following maximization problem:

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- Similarly for $q_2^* \in BR_2(q_1^*)$,

$$q_2^* = \frac{A - q_1^*}{2}.$$

- As a result, solving these two equations, we get:

$$q_1^* = q_2^* = \frac{A}{3}.$$

In the Cournot game, note that firms' payoffs are:

$$\pi_1^c = \frac{A^2}{9}, \pi_2^c = \frac{A^2}{9}.$$

As we already saw, this was not Pareto efficient since each firm is getting a payoff that is strictly less than $1/2$ of the monopoly profits.

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- Thus, the firms' payoffs in the SPNE is:

$$\pi_1^s = \frac{1}{4}A \cdot \frac{A}{2} = \frac{A^2}{8}, \pi_2^s = \frac{1}{4}A \cdot \frac{A}{4} = \frac{A^2}{16}.$$

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- Thus, the firms' payoffs in the SPNE is:

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- But by choosing something optimal, firm 1 will be able to do even better

Lecture 16: Applications of Subgame Perfect Nash Equilibrium

Ultimatum Game

Alternating offers

Stackelberg Competition

Entry Deterrence in Quantity Competition

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  3. If potential entrant enters, she chooses $q_E \in [0, A]$.
  4. The price P is determined according to the inverse demand function $P = A - q_M - q_E$ and firms receive their payoffs, where there are no other costs to production than the one just mentioned above.

If the monopolist faced no entrant, then he would solve the following maximization problem:

$$\max_{q_M}(A - q_M)q_M \Longrightarrow q_M^m = \frac{A}{2}.$$

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- The entrant's best response function is:

$$BR_E(q_M) = \begin{cases} 0 & \text{if } \frac{(A-q_M)^2}{4} < F, \\ \{0, (A-q_M)/2\} & \text{if } \frac{(A-q_M)^2}{4} = F, \\ (A-q_M)/2 & \text{if } \frac{(A-q_M)^2}{4} > F. \end{cases}$$

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- This simplifies to

$$BR_E(q_M) = \begin{cases} 0 & \text{if } q_M > A - 2\sqrt{F}, \\ \{0, (A - q_M)/2\} & \text{if } q_M = A - 2\sqrt{F}, \\ (A - q_M)/2 & \text{if } q_M < A - 2\sqrt{F}. \end{cases}$$

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- Both lead to the same equilibrium outcome which is for the monopolist to choose $q_M = A/2$ and for the entrant to choose $q_E = 0$

- If the fixed cost is too high for entry, then even when the monopolist chooses the monopolist quantity, the entrant still wants to stay out

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- For simplicity we will look for equilibria in which the entrant chooses $(A - q_M)/2$ if $q_M = A - 2\sqrt{F}$

- As a result,

$$q_E^*(q_M) = \begin{cases} 0 & \text{if } q_M > A - 2\sqrt{F}, \\ \frac{A-q_M}{2} & \text{if } q_M \leq A - 2\sqrt{F}. \end{cases}$$

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- The monopolist must maximize the following utility function given that the entrant chooses according to q_E^*:

$$\max_{q_M \geq 0} u_M(q_M, q_E^*(\cdot)) = \max_{q_M \geq 0} \begin{cases} (A - q_M)q_M & \text{if } q_M > A - 2\sqrt{F} \\ \frac{q_M(A - q_M)}{2} & \text{if } q_M \leq A - 2\sqrt{F}. \end{cases}$$

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