Lecture 20: Infinitely Repeated Games

Mauricio Romero

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Cournot n-firms

Bertrand n-firms

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  - Note: as N grows large, $p^* \to c$ and $\pi^* \to 0$, as in PC

- If firms cooperate: $\max_q = Nq(a - b(Nq) - c) \rightarrow q^c = \frac{(a-c)}{2bN}$

- $p^c = \frac{a+c}{2}$, higher than p^*.

- $\pi^c = \frac{(a-c)^2}{4bN}$, higher than π^*.

- But why can't each firm do this? Because NE condition is not satisfied: $\max_{q_i} \pi_i =$ $\max_{q_i} q_i \left(a - b\left((N-1)\frac{(a-c)}{2bN} + q_i\right) - c\right) \rightarrow q^d = \frac{(a-c)(n+1)}{4Nb}$

- So the profits from deviating are: $\pi^d = \frac{(n+1)^2(a-c)^2}{16bn^2}$

- What if we repeat the game?

2-period Cournot game

- Second period: unique NE in these subgames (play the NE)
- First period: Given that NE in $t = 2 \longrightarrow$ unique SPNE is to play the NE of the stage game in both periods.
- What about 3 periods?
- What about N periods?

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There are two relevant subgames for firm i

- After a period in which cheating (either by himself or the other firm) has occurred
- Proposed strategy prescribes playing q^* forever (by all firms)
- This is NE of the subgame: playing q^* is a best-response to other firms playing q^*
- This satisfies SPE conditions.

- After a period when no cheating has occurred
- Proposed strategy prescribes cooperating and playing q^c, with discounted PV of payoffs $= \pi^c/(1-\delta)$
- The best other possible strategy is to play $BR_1(q^c_{-i}) \equiv q^d_i$ this period, but then be faced with $q_2 = q^*$ forever
- This yields discounted PV $= \pi^d + \delta(\pi^*/(1-\delta))$
- In order for q_c to be NE of this subgame, require $\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))$

- $\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))$
- $\frac{(a-c)^2}{4bN(1-\delta)} > \frac{(n+1)^2(a-c)^2}{16bn^2} + \delta\left(\frac{(a-c)^2}{(N+1)^2 b(1-\delta)}\right)$
- $\delta > \frac{(n+1)^2}{n^2+6b+1}$
- This value increases with n (i.e., collusion is harder to maintain as the number of firms grows)

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  - Per-firm profits: $\pi^* = 0$

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- But why can't each firm do this? Because NE condition is not satisfied. If everyone else plays $p = \frac{a+c}{2}$, I charge ε less, and essentially get the monopoly earnings all for my self
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