Lecture 7: Monopoly

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Lecture 7: Monopoly

- Firm is faced a problem like the following:

$$\max_{K,L} p_x f_x(L, K) - wL - rK.$$

- The firm's choice of L and K does not affect the prices p, w, r

- This is called *price-taking* behavior

- Justified if the the market is composed of many small firms

- In many markets there is a single firm

- Since supply is completely controlled by the firm, it can use this in its favor

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- If

$$c(x) = \min_{K,L} wL + rK \text{ such that } f_x(K,L) = x$$

  then the above is equivalent to:

$$\max_{x} px - c(x).$$

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- $p(x)$ is the **demand** function

- We can also represent the problem as:

$$\max_{p} pq(p) - c(q(p))$$

- $q(p)$ is the **inverse demand function**

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- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$
- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

Elasticities

- If $\varepsilon_{q,p} \in (-1, 0)$, the demand is *inelastic*
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\varepsilon_{q,p} < -1$, then demand is *elastic*
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- $q(p) = e^C p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

Elasticities

Whenever the demand function has constant elasticity κ

- $q(p)Ap^{\kappa}$ for some $A > 0$.
- Equivalently,

$$p(q) = \left(\frac{q}{A}\right)^{1/\kappa}.$$

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- This implies

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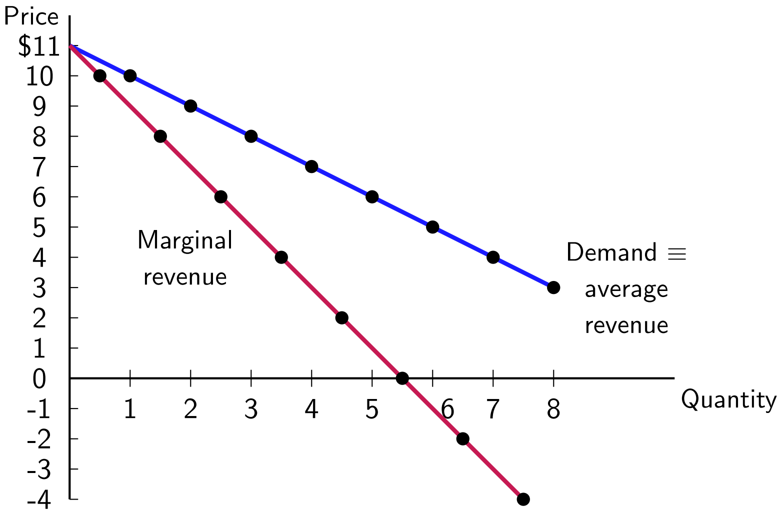
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits


Price
$11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
1
2
3
4
5
6
7
8
Quantity
Marginal revenue
Demand ≡ average revenue

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- The amount produced q is below the quantity where $p = MC$.

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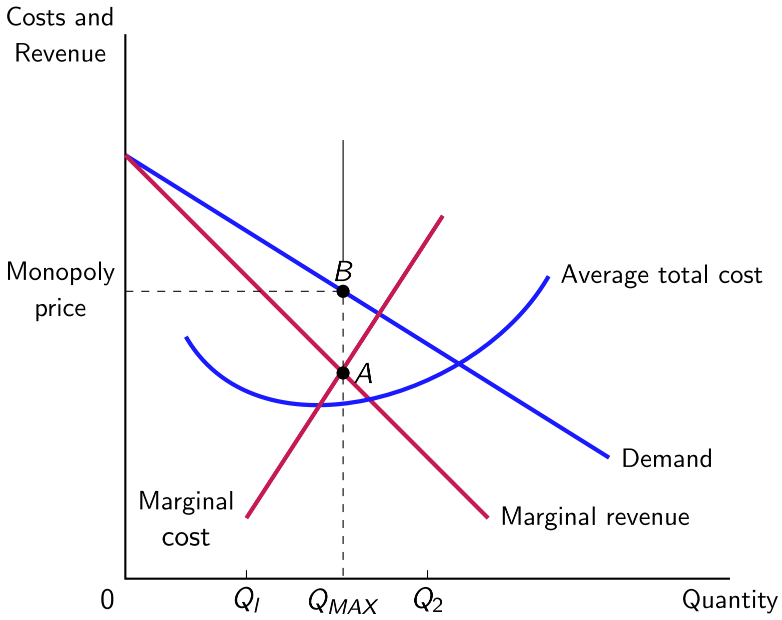
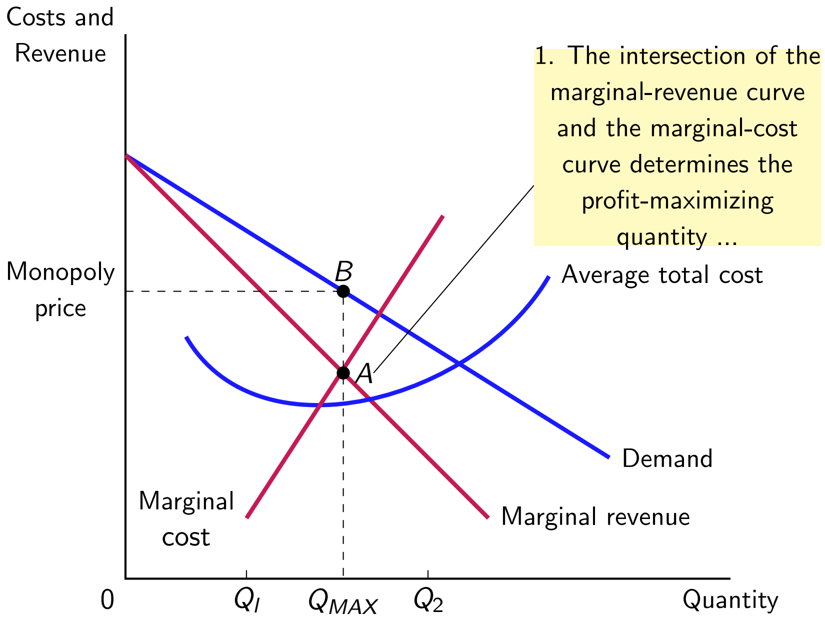
- Both consumer surplus and total surplus is less than is socially optimal
- Thus the pricing policies used by monopolies are inefficient, leading to what is called “dead-weight loss”


Costs and
Revenue
Monopoly
price
B
A
Average total cost
Demand
Marginal
cost
Marginal revenue
0
Q_I
Q_{MAX}
Q_2
Quantity


Costs and Revenue
1. The intersection of the marginal-revenue curve and the marginal-cost curve determines the profit-maximizing quantity ...
Monopoly price
B
A
Average total cost
Demand
Marginal cost
Marginal revenue
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Q_I
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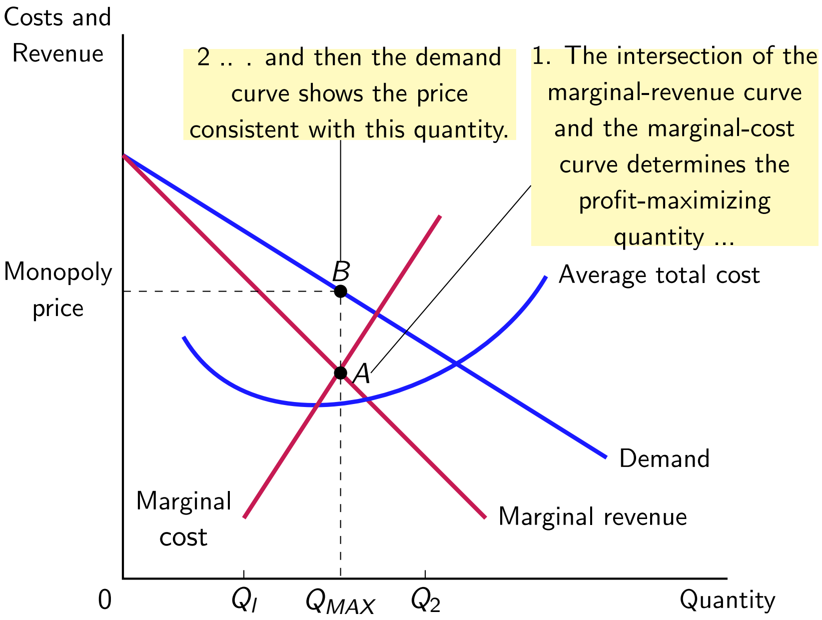
Quantity



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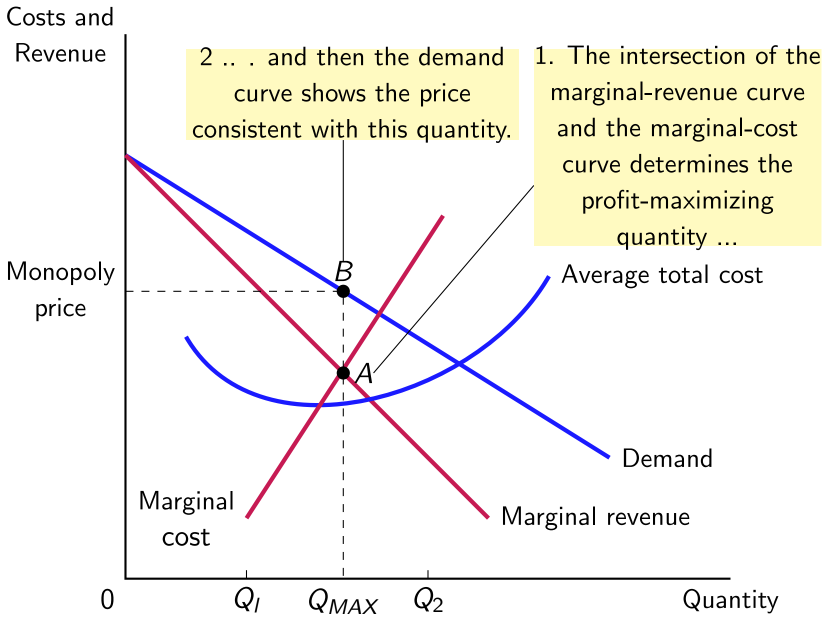
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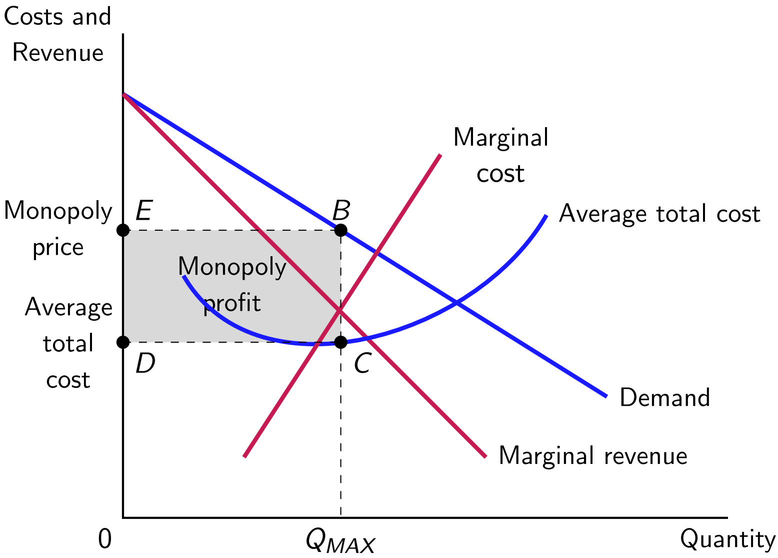
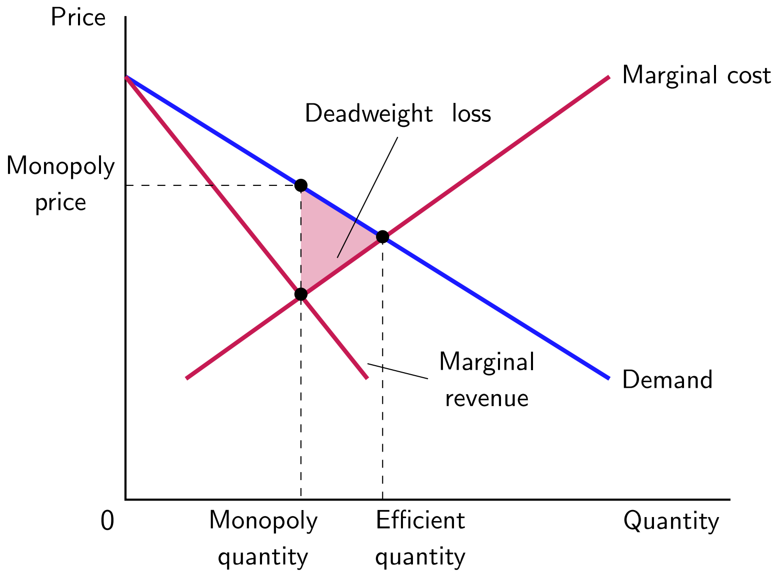

Costs and Revenue
Marginal cost
Monopoly price
E
B
Average total cost
Monopoly profit
Average total cost
D
C
Demand
Marginal revenue
0
Q_{MAX}
Quantity


Price
Marginal cost
Deadweight loss
Monopoly
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Efficient
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- If marginal costs are constant at c
- $$p = \frac{c}{1 + \frac{1}{\kappa}} \Longrightarrow q(p) = A \left(\frac{c}{1 + \frac{1}{\kappa}} \right)^{\kappa}.$$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc.
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply.