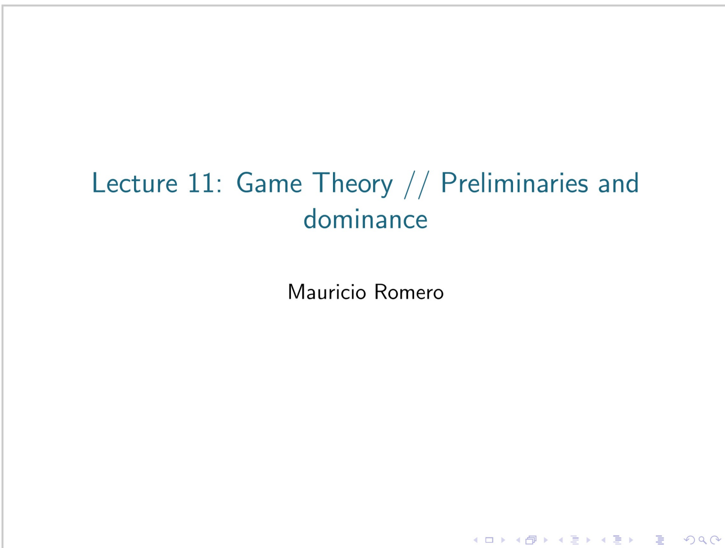
Lecture11.pdf

Tuesday, March 24, 2020 10:58 AM

Lecture11....



Introduction - Continued

Static games with complete information

Lecture 11: Game Theory // Preliminaries and dominance

Introduction - Continued

Static games with complete information

- We will represent games in two different ways

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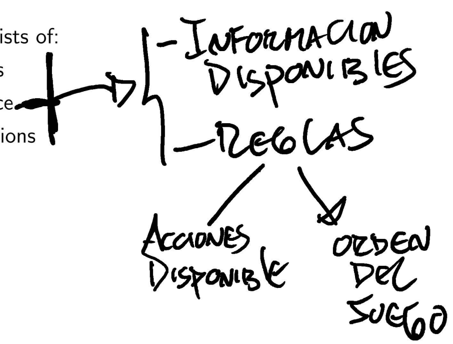
- This is just a way to schematizing the game and in general it makes the analysis simpler

Normal form

The normal form consists of:

- The list of players
- The strategy space
- The pay-off functions



Normal form

The normal form consists of:

- The list of players
- The strategy space
- The pay-off functions

There is no mention of rules or available information. Where is this hidden?

When there a few players (2 or 3) a matrix is used to represent the game in the normal form.

	s_2	s_2'
s_1	$(u_1(s_1, s_2), u_2(s_1, s_2))$	$(u_1(s_1, s_2'), u_2(s_1, s_2'))$
s_1'	$(u_1(s_1', s_2), u_2(s_1', s_2))$	$(u_1(s_1', s_2'), u_2(s_1', s_2'))$
s_1''	$(u_1(s_1'', s_2), u_2(s_1'', s_2))$	$(u_1(s_1'', s_2'), u_2(s_1'', s_2'))$

LISTA JUGADORES = {J1, J2}

$S_1 = \{s_1, s_1', s_1''\}$

$S_2 = \{s_2, s_2'\}$

Matching-Pennies (Pares y Nones) – Simultaneous

Forma Normal

Jugadores = {Ana, Beto}

$S_A = \{1,2\}$

$S_B = \{1,2\}$

$U_A(S_A,S_B) = \begin{cases} 1000 & S_A+S_B \text{ es Par} \\ -1000 & S_A+S_B \text{ es Impar} \end{cases}$

$U_B = \begin{cases} 1000 & S_A+S_B \text{ Impar} \\ -1000 & S_A+S_B \text{ Par} \end{cases}$

Both players play at the same time

A / B

	1_B	2_B
1_A	(1000,-1000)	(-1000,1000)
2_A	(-1000,1000)	(1000,-1000)

Matching-Pennies (Pares y Nones) – Sequential

J = {Ana, Beto}

$S_A = \{1,2\}$

$S_B = \{(1,1); (1,2); (2,1); (2,2)\}$

Es lo que hago si Ana 1 dedo — Lo que hago si Ana 2 dedos

A plays first, then B

A / B

	(1, 1)	(1, 2)	(2, 1)	(2, 2)
1_A	(1000,-1000)	(1000,-1000)	(-1000,1000)	(-1000,1000)
2_A	(-1000,1000)	(1000,-1000)	(-1000,1000)	(1000,-1000)

Prisoner's Dilemma

There are two players $I = \{1, 2\}$ that are members of a drug cartel who are both arrested an imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack enough evidence to convict the pair on the principal charge so they must settle for a lesser charge. Simultaneously, the prosecutor offers each prisoner a deal. Each prisoner is given the opportunity to either 1) betray the other by testifying the other committed the crime or to 2) cooperate with the other prisoner and stay silent.

Prisoner's Dilemma

The strategies of player 1:

$$S_1 = \{\text{betray}_1, \text{silent}_1\}.$$

Prisoner's Dilemma

The strategies of player 1:

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The strategies of player 2:

$$S_2 = \{\text{betray}_2, \text{silent}_2\}.$$

Prisoner's Dilemma

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The strategies of player 2:

$$S_2 = \{\text{betray}_2, \text{silent}_2\}.$$

The utility function of the players is given by:

$$u_1(b_1, b_2) = -2, \; u_2(b_1, b_2) = -2$$
$$u_1(b_1, s_2) = 0, \; u_2(b_1, s_2) = -3$$
$$u_1(s_1, b_2) = -3, \; u_2(s_1, b_2) = 0$$
$$u_1(s_1, s_2) = -1, \; u_2(s_1, s_2) = -1.$$

Prisoner's Dilemma

Prisoner's Dilemma

	s_2	b_2

- J_1 va primero
- J_2 segundo (después observar q' hizo J_1)

$S_1 = \{s_1, b_1\}$.

$S_2 = \{(s_2, s_2); (s_2, b_2); (b_2, s_2); (b_2, b_2)\}$

J_2

	(s_2, s_2)	(s_2, b_2)	(b_2, s_2)	(b_2, b_2)

Prisoner's Dilemma

	s_2	b_2
s_1	$-1, -1$	$-3, 0$
b_1	$0, -3$	$-2, -2$

$S_2 = \{(s_2, s_2); (s_2, b_2); (b_2, s_2); \ldots$

	(s_2, s_2)	(s_2, b_2)	(b_2, s_2)	(b_2, b_2)
s_1	$(-1,-1)$	$(-1,-1)$	$(-3,0)$	$(-3,0)$
b_1	$(0,-3)$	$(-2,-2)$	$(0,-3)$	$(-2,-2)$

Lecture 10: Game Theory // Preliminaries and dominance

- This is in many case the most natural way to represent a way, but always not the most useful

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  - The actions available to each player in each point in time

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  - The information available to each player in each point in time
  - The actions available to each player in each point in time
  - The pay-off functions

EQUIVALENTE A ESTRATEGIAS

- The extensive form is usually accompanied by a visual representation call the “game tree”

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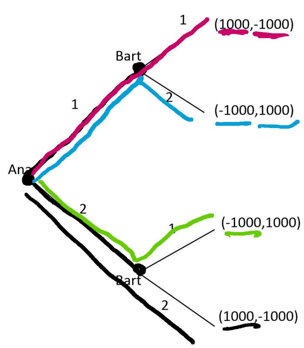
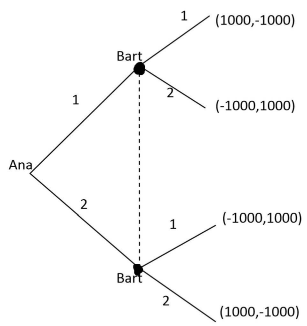
- Each node where a branch begins is a decision node, where a player needs to choose an action
- If two nodes are connected by a dotted line, it means they are in the same information set (i.e., the player is not sure in which node she is in)

Matching-Pennies (Pares y Nones) – Sequential



Pares o Nones I

Matching-Pennies (Pares y Nones) – Simultaneous



Pares o Nones II

Lecture 10: Game Theory // Preliminaries and dominance

- Introduction - Continued
  - Normal or extensive form
  - Extensive form
  - Some important remarks
  - Some examples
  - What's next
- Static games with complete information
  - Dominance of Strategies

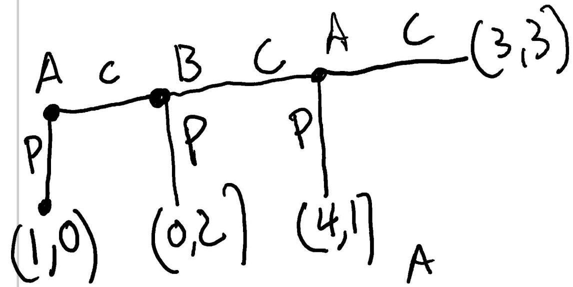
Theorem

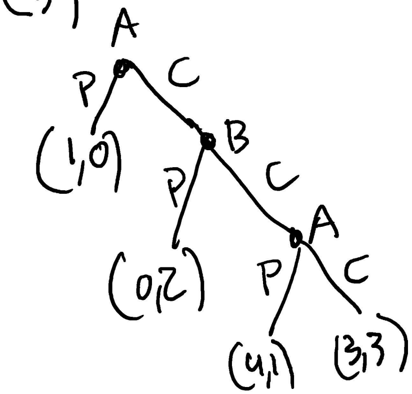
Every game can be represented in both forms (extensive and normal). The representation you choose will not alter the analysis, but it may be simpler to do the analysis with one form or another. A normal form game may have several extensive representations (but every extensive form has a single normal form equivalent to it); however, all of the results we will see/use are robust to the representation used.

Lecture 10: Game Theory // Preliminaries and dominance

Centipede Game

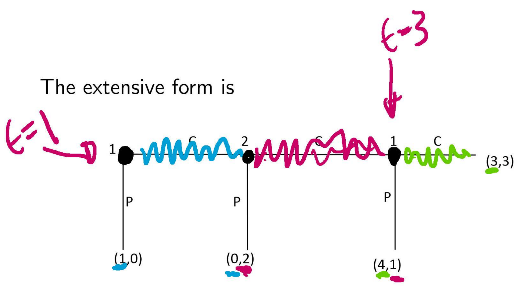
Suppose there are two individuals Ana and Bernardo. Ana is given a chocolate. She can stop the game and keep the chocolate or she can continue. If she continues, Ana's chocolate is taken away and Bernardo is given two. Bernardo can then stop the game and keep two chocolates (and Ana will get zero) or can continue. If he continues, a chocolate is taken away from him and Ana is given four. Ana can stop the game and keep 4 chocolates (and Bernardo will keep one), or she can continue, in which case the game ends with three chocolates for each one.





Centipede Game

The extensive form is



NORMAL

$J = \{A, B\}$

$S_A = \{(C, C); (C, P); (P, C); (P, P)\}.$

C HACE EN t=1

C HACE EN t=3

$S_B = \{P, C\}.$

Centipede Game

The normal form is

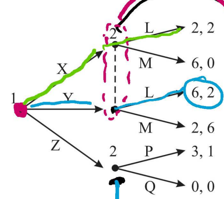
J_B

	C	P
C,C	3,3	0,2
C,P	4,1	0,2
P,C	1,0	1,0
P,P	1,0	1,0

J_A

LO ACABA EN t=1

Consider the following game in extensive form:



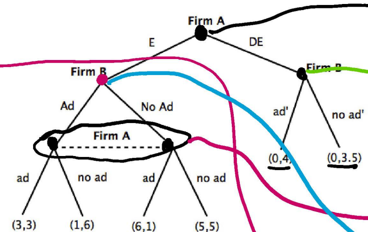
$S_1 = \{X, Y, Z\}$

$S_2 = \{(L, P); (L, Q); (M, P); (M, Q)\}$

The normal form is:

1 \ 2	LP	LQ	MP	MQ
X	2, 2	2, 2	6, 0	6, 0
Y	6, 2	6, 2	2, 6	2, 6
Z	3, 1	0, 0	3, 1	0, 0

Consider the following game in extensive form



$S_A = \{(E, Ad); (E, NAd); (DE, Ad); (DE, NAd)\}$

$S_B = \{(Ad, Ad'); (Ad, NoAd'); (NoAd, Ad'); (NoAd, NoAd')\}$

The normal form is:

	Ad, ad'	$Ad, no\ ad'$	$No\ Ad, ad'$	$No\ Ad, no\ ad'$
(E, ad)	3,3	3,3	6,1	6,1
$(E, no\ ad)$	1,6	1,6	5,5	5,5
(DE, ad)	0, 4	0,3.5	0,4	0,3.5
$(DE, no\ ad)$	0, 4	0,3.5	0,4	0,3.5

Lecture 10: Game Theory // Preliminaries and dominance

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- This is a concept equivalent to general equilibrium, where given market prices, everyone is optimizing, markets empty, and therefore no one has incentives to deviate, but nobody told us how we got there .. . pause (the Walrasian auctioneer?)

Lecture 11: Game Theory // Preliminaries and dominance

Introduction - Continued

Static games with complete information

Lecture 11: Game Theory // Preliminaries and dominance

Introduction - Continued

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Static games with complete information

- Games where all players move simultaneously and only once
- If players move sequentially, but can not observe what other people played, it's equivalent to a static game
- Only consider games of complete information (all players know the objective functions of their opponents)
- These are very restrictive conditions but they will allow us to present very important concepts that will be easy to extend to more complex games
- As each player faces one contingency, the strategies are identical to the actions.

Lecture 10: Game Theory // Preliminaries and dominance

Dominance

- Intuitively if a strategy s_i always results in a greater utility than s_i', regardless of the strategy followed by the other players then the strategy s_i' should never be chosen by individual i

Dominance

COMPARA 2 ESTRATEGIAS

s_i **strictly dominates** s_i' if no matter what the opponent does, s_i gives a better payoff to i than s_i'

Definition

Let s_i, s_i' be two pure strategies. Then we say that s_i strictly dominates s_i' if for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

$S_{-i} = \{s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N\}$.

Dominance

GLOBAL

A pure strategy s_i is **strictly dominant** if s_i strictly dominates every other strategy s_i'

Definition

Let s_i be a pure strategy of player i. Then s_i is strictly dominant if for all $s_i' \neq s_i$, s_i strictly dominates s_i'.

Dominance

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- We can eliminate any strategy that is strictly dominated

Dominance in the prisoners dilemma

s_2

s_1

	C	NC
C	5,5	0,10
NC	10,0	2,2

- NC dominates C for both individuals

NC >> C

Nadie juega C

Cooperar

No Coopera

Dominance in the prisoners dilemma

	C	NC
C	5,5	0,10
NC	10,0	2,2

- *NC* dominates *C* for both individuals
- (NC, NC) is not a Pareto Optimum.

Dominance in the prisoners dilemma

	C	NC
C	5,5	0,10
NC	10,0	2,2

- *NC* dominates *C* for both individuals
- (NC, NC) is not a Pareto Optimum.
- What happened to the first welfare theorem? Is it incorrect?

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*
- Player 1 would foresee this...

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- *B* now dominates *A* for player 1

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- *B* now dominates *A* for player 1
- Player 2 would foresee this (that player 1 foresees that 2 will not play a, and thus he will not play B)

Dominance (iterated)

	[redacted]	c
B	[redacted]	4, 5

- Player 2 would play *c* and player 1 would play *B*

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c) → $(U_1, U_2) = (4, 5)$

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c)

- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c)
- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)
- The equilibrium is the set of strategies, not the payoff!

IDSDS

Definition (Solvable by IDSDS)

A game is solvable by **Iterated Deletion of Strictly Dominated Strategies** if the result of the iteration is a single strategy profile (one strategy for each player)

IDSDS

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- Is the order of elimination of the strategies important? **No**

- Not all games are solvable by IDSDS

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

- No strategy is dominated for either player