Lecture12

Thursday, March 26, 2020 2:58 PM

Lecture12

Lecture 12: Game Theory // Nash equilibrium

Mauricio Romero

Lecture 12: Game Theory // Nash equilibrium

Lecture 12: Game Theory // Nash equilibrium

Beauty contest

- Consider the following game among 100 people. Each individual selects a number, s_i, between 20 and 60.
- Let a_{-i} be the average of the number selected by the other 99 people. i.e. $a_{-i} = \sum_{j \neq i} \frac{s_j}{99}$.
- The utility function of the individual i is $u_i(s_i, s_{-i}) = 100 - \underline{(s_i - \frac{3}{2}a_{-i})^2}$

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others
- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$

$\boxed{\in [20, 60]}$

$\in [30, 90] \leftarrow 1.5[20, 60]$

$30 \geq s_i$

Beauty contest

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others
- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$
- but $a_{-i} \in [20, 60]$

$\forall$

$30 \succ s_i$

$s_i \in [20, 30)$

Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others
- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$
- but $a_{-i} \in [20, 60]$

- Therefore $s_i = 20$ is dominated by $s_i = 30$

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)

- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)

$s_i \in [20, 60) \to a_{-i} \in [20, 60]$

DESPUES DE ELIMINAR LO DOMINADO

$s_i \in [30, 60] \to a_{-i} \in [30, 60] \to 1.5a_{-i} \in [45, 90]$

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45

45

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$) $\rightarrow 1.5\, a_{-i} \in [67.5, 90]$

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.

Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.
- The solution by means of iterated elimination of dominated strategies is $\underbrace{(60, 60, ..., 60)}_{\text{100 times}}$

NYI

$S_i \in [0, 100]$

$U_i = -\left(S_i - \frac{2}{3} a_{-i}\right)^2$

$a_{-i} \in [0, 100] \rightarrow \frac{2}{3} a_{-i} \in [0, 66.6]$

$66.6 \gg (66.6, 100]$

$S_i \in [0, 66.6]$

$\hookrightarrow a_{-i} \in [0, 66.6] \rightarrow \frac{2}{3} a_{-i} \in [0, 44.4]$

$44.4 \gg (44.4, 100)$

$\rightarrow S_i \in [0, 44.4]$

$\vdots$

$\rightarrow$ Solución $S_i = 0 \; \forall i$

---

s2

s1

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- There is no strictly dominated strategy

---

s2

s1

	a	b
A	3, 4	4, 3
~~B~~	~~5, 3~~	~~3, 5~~
C	5, 3	4, 3

- There is no strictly dominated strategy
- However, C always gives at least the same utility to player 1 as B

---

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- There is no strictly dominated strategy
- However, C always gives at least the same utility to player 1 as B
- It's tempting to think player 1 would never play B

	a	b
A	3, 4	4, 3
B	5, 3	3, 5
C	5, 3	4, 3

- There is no strictly dominated strategy
- However, C always gives at least the same utility to player 1 as B
- It's tempting to think player 1 would never play C
- However, if player 1 is sure that player two is going to play a he would be completely indifferent between playing B or C

Definition

s_i weakly dominates s_i' if for all opponent pure strategy profiles, $s_{-i} \in S_{-i}$,

$$\boxed{u_i(s_i, s_{-i}) \boxed{\geq} u_i(s_i', s_{-i})}$$

and there is at least one opponent strategy profile $s''_{-i} \in S_{-i}$ for which

$$u_i(s_i, s''_{-i}) \boxed{>} u_i(s_i', s''_{-i}).$$

- Given the assumptions we have, we can not eliminate a weakly dominated strategy

- Given the assumptions we have, we can not eliminate a weakly dominated strategy
- Rationality is not enough

- Given the assumptions we have, we can not eliminate a weakly dominated strategy
- Rationality is not enough
- Even so, it sounds "logical" to do so and has the potential to greatly simplify a game

- Given the assumptions we have, we can not eliminate a weakly dominated strategy
- Rationality is not enough
- Even so, it sounds "logical" to do so and has the potential to greatly simplify a game

- There is a problem, and that is that the order in which we eliminate the strategies matters

S_2

S_1

	a	b
A	[illegible]	[illegible]
B	[illegible]	[illegible]
C	5, 3	[illegible]

$C \succsim B$

$a \succsim b$

$C \not\succsim A$

(C, a)

- If we eliminate B (C dominates weakly), then a weakly dominates b and we can eliminate b and therefore player 1 would never play A. This leads to the result (C, a).

S_2

S_1

	a	b
A	[illegible]	4, 3
B	[illegible]	3, 5
C	5, 3	4, 3

$C \succsim A$

$b \succsim a.$

$C \not\succsim B$

(C, b)

- If we eliminate B (C dominates weakly), then a weakly dominates b and we can eliminate b and therefore player 1 would never play A. This leads to the result (C, a).

- If on the other hand, we notice that A is also weakly dominated by C then we can eliminate it in the first round, and this would eliminate a in the second round and therefore B would be eliminated. This would result in (C, b).

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Remember the definition of competitive equilibrium in a market economy.

Definition

A competitive equilibrium in a market economy is a vector of prices and baskets x_i such that: 1) x_i maximizes the utility of each individual given the price vector i.e.

$$x_i = \arg \max_{p \ cdot x_i \leq p \cdot w_i} u(x_i)$$

2) the markets empty.

$$\sum_i x_i = \sum_i w_i$$

- 1) means that given the prices, individuals have no incentive to demand a different amount

- 1) means that given the prices, individuals have no incentive to demand a different amount

- The idea is to extend this concept to strategic situations

Best response

We denote $BR_i(s_{-i})$ (best response) as the set of strategies of individual i that maximize her utility given that other individuals follow the strategy profile s_{-i}. Formally,

Best response

We denote $BR_i(s_{-i})$ (best response) as the set of strategies of individual i that maximize her utility given that other individuals follow the strategy profile s_{-i}. Formally,

Definition

Given a strategy profile of opponents s_{-i}, we can define the best response of player i:

$$BR_i(s_{-i}) = \arg\max_{s_i' \in S_i} u_i(s_i', s_{-i}).$$

FIJO

- $s_i \in BR_i(s_{-i})$ if and only if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$

ES UN CONJUNTO (NO NECESARIAMENTE DE TAMAÑO 1)

Best response

We denote $BR_i(s_{-i})$ (best response) as the set of strategies of individual i that maximize her utility given that other individuals follow the strategy profile s_{-i}. Formally,

Definition

Given a strategy profile of opponents s_{-i}, we can define the best response of player i:

$$BR_i(s_{-i}) = \arg\max_{s_i' \in S_i} u_i(s_i', s_{-i}).$$

FIJO

- $s_i \in BR_i(s_{-i})$ if and only if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$

- There could be multiple strategies in $BR_i(s_{-i})$ but all such strategies give the same utility to player i if the opponents are indeed playing according to s_{-i}

Nash equilibrium

Definition

Suppose that we have a game $(I = \{1, 2, \ldots n\}, S_1, \ldots, S_n, u_1, \ldots, u_n)$. Then a strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a pure strategy **Nash equilibrium** if for every i and for every $s_i \in S_i$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*).$$

↳ PARA TODO INDIVIDUO

$s_i^* \in MR_i(s_{-i}^*) \quad \forall i$

$BR_i(s_{-i}^*)$

Nash equilibrium

Definition

Suppose that we have a game $(I = \{1, 2, \ldots, n\}, S_1, \ldots, S_n, u_1, \ldots, u_n)$. Then a strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a **pure strategy** Nash equilibrium if for every i, $s_i^* \in BR_i(s_{-i}^*)$.

- Analogous to that of a competitive equilibrium in the sense that nobody has unilateral incentives to deviate

Nash equilibrium

Definition

Suppose that we have a game $(I = \{1, 2, \ldots, n\}, S_1, \ldots, S_n, u_1, \ldots, u_n)$. Then a strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a **pure strategy** Nash equilibrium if for every i, $s_i^* \in BR_i(s_{-i}^*)$.

- Analogous to that of a competitive equilibrium in the sense that nobody has unilateral incentives to deviate

- once this equilibrium is reached, nobody has incentives to move from there

Nash equilibrium

Definition

Suppose that we have a game $(I = \{1, 2, \ldots, n\}, S_1, \ldots, S_n, u_1, \ldots, u_n)$. Then a strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a **pure strategy** Nash equilibrium if for every i, $s_i^* \in BR_i(s_{-i}^*)$.

- Analogous to that of a competitive equilibrium in the sense that nobody has unilateral incentives to deviate
- once this equilibrium is reached, nobody has incentives to move from there

- This is a concept of stability, but there is no way to ensure, or predict, that the game will reach this equilibrium

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Beauty contest

- Consider the following game among 2 people. Each individual selects a number, s_i, between 20 and 60.

Beauty contest

- Consider the following game among 2 people. Each individual selects a number, s_i, between 20 and 60.
- Let s_{-i} be the number selected by the other individual.

Beauty contest

- Consider the following game among 2 people. Each individual selects a number, s_i, between 20 and 60.
- Let s_{-i} be the number selected by the other individual.
- The utility function of the individual i is $u_i(s_i, s_{-i}) = 100 - (s_i - \frac{3}{2}s_{-i})^2$

$$U_i = 100 - \left(S_i - \frac{3}{2}S_{-i}\right)^2$$

$$\frac{\partial u_i}{\partial S_i} = -2\left(S_i - \frac{3}{2}S_{-i}\right) = 0$$

$$S_i = \frac{3}{2}\underline{S_{-i}}$$

$$S_{-i} \in [20, 60]$$

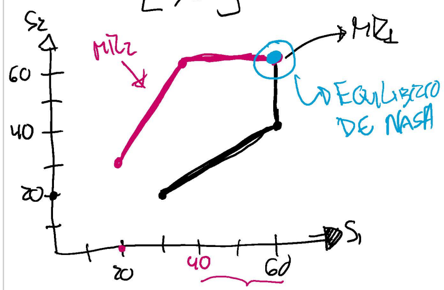
$$S_i \in [20, 60]$$

Beauty contest

The best response of an individual is given by

$$s_i(s_{-i})^* = \begin{cases} \frac{3}{2}s_{-i} & \text{if } s_{-i} \leq 40 \\ 60 & \text{if } s_{-i} > 40 \end{cases}$$

The Nash equilibrium is where both BR functions intersect (i.e., when both play 60)



Prisoner's dilemma

S_2

S_1

	C	NC
C	5,5	0,10
NC	10,0	2,2

$MR_1(C_2) = NC$

$MR_1(NC_2) = NC$

$MR_1(S_2) = NC$

$MR_2(S_1) = NC$

$EN = (NC, NC)$

Utilidad asociada de $(2,2)$

G.P.
(C,C)
(C,NC)
(NC,C)

Prisoner's dilemma

	C	NC
C	5,5	0,10
NC	10,0	2,2

The best response functions are:

$$BR_i(s_{-i}) = \begin{cases} NC & \text{if } s_{-i} = C \\ NC & \text{if } s_{-i} = NC \end{cases}$$

The Nash equilibrium is where both BR functions intersect (i.e., when both play NC, i.e., (NC, NC))

Prisoner's dilemma – A trick

Best response of 1 to 2 playing C

S_2

S_1

	C	NC
C	5,5	0,10
NC	10,0	2,2

→ EN = (NC,NC)

Prisoner's dilemma – A trick

Best response of 1 to 2 playing NC

	C	NC
C	5,5	0,10
NC	10,0	2,2

Prisoner's dilemma – A trick

Best response of 2 to 1 playing C

	C	NC
C	5,5	0,10
NC	10,0	2,2

Prisoner's dilemma – A trick

Best response of 2 to 1 playing NC

	C	NC
C	5,5	0,10
NC	10,0	2,2

When underlined for both players, it is a Nash equilibrium (both are doing their BR)

Battle of the sexes

S_2

S_1

	G	P
G	2,1	0,0
P	0,0	1,2

$EN = \{(G,G),(P,P)\}$

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

$$BR_i(s_{-i}) = \begin{cases} G & \text{if } s_{-i} = G \\ P & \text{if } s_{-i} = P \end{cases}$$

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

$$BR_i(s_{-i}) = \begin{cases} G & \text{if } s_{-i} = G \\ P & \text{if } s_{-i} = P \end{cases}$$

Thus, (G, G) y (P, P) are both Nash equilibrium

Matching pennies (Pares o Nones) – Simultaneous

S_2

S_1

	1	2
1	(1000,-1000)	(-1000,1000)
2	(-1000,1000)	(1000,-1000)

Matching pennies (Pares o Nones) – Simultaneous

	1	2
1	$(\underline{1000},-1000)$	$(-1000,\underline{1000})$
2	$(-1000,\underline{1000})$	$(\underline{1000},-1000)$

Matching pennies (Pares o Nones) – Simultaneous

	1	2
1	$(\underline{1000},-1000)$	$(-1000,\underline{1000})$
2	$(-1000,\underline{1000})$	$(\underline{1000},-1000)$

$$BR_1(s_2) = \begin{cases} 1 & \text{if } s_2 = 1 \\ 2 & \text{if } s_2 = 2 \end{cases}$$

$$BR_2(s_1) = \begin{cases} 2 & \text{if } s_1 = 1 \\ 1 & \text{if } s_2 = 2 \end{cases}$$

There is no Nash equilibrium in pure strategies

Lecture 12: Game Theory // Nash equilibrium

- Dominance
- Nash equilibrium
- Some examples
- Relationship to dominance
- Examples

Lecture 12: Game Theory // Nash equilibrium

- Dominance
- Nash equilibrium
- Some examples
- Relationship to dominance
- Examples

Nash equilibrium survive IDSDS

Theorem

Every Nash equilibrium survives the iterative elimination of strictly dominated strategies

Proof

By contradiction:

- Suppose it is not true

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^* $= (s_1^*, \ldots, s_N^*)$

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*
- Without loss of generality say we eliminated the strategy s_i^* of individual i

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*
- Without loss of generality say we eliminated the strategy s_i^* of individual i
- It must have been that

$$\boxed{u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}}$$

$s_i \gg s_i^*$

DOMINA

.

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*
- Without loss of generality say we eliminated the strategy s_i^* of individual i
- It must have been that

$$u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}$$

.
- In particular

$$\boxed{u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)}$$

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*
- Without loss of generality say we eliminated the strategy s_i^* of individual i
- It must have been that

$$u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}$$

.

- In particular

$$u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)$$

- But this means s_i^* is not the best response of individual i to s_{-i}^*

Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium s^*
- Lets zoom in in the round where we first eliminate a strategy that is part of s^*
- Without loss of generality say we eliminated the strategy s_i^* of individual i
- It must have been that

$$u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}$$

.

- In particular

$$u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)$$

- But this means s_i^* is not the best response of individual i to s_{-i}^*
- And this is a contradiction!

Nash equilibrium survive IDSDS

Theorem

If the process of IDSDS comes to a single solution, that solution is a Nash Equilibrium and is unique.

Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

Proof.

By contradiction:

- Suppose that the results from IDSDS (s^*) is not a Nash Equilibrium

□

Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

Proof.

By contradiction:

- Suppose that the results from IDSDS (s^*) is not a Nash Equilibrium
- For some individual i there exits s_i such that

$$u_i(s_i, s_{-i}^*) > u_i(s_i^*, s_{-i}^*)$$

□

Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

Proof.
By contradiction:

- Suppose that the results from IDSDS (s^*) is not a Nash Equilibrium
- For some individual i there exits s_i such that

$$u_i(s_i, s^*_{-i}) > u_i(s^*_i, s^*_{-i})$$

- But then s_i could not have been eliminated

□

Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

Proof.
By contradiction:

- Suppose that the results from IDSDS (s^*) is not a Nash Equilibrium
- For some individual i there exits s_i such that

$$u_i(s_i, s^*_{-i}) > u_i(s^*_i, s^*_{-i})$$

- But then s_i could not have been eliminated
- And this is a contradiction!

□

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Lecture 12: Game Theory // Nash equilibrium

Dominance

Nash equilibrium

Some examples

Relationship to dominance

Examples

Lecture 12: Game Theory // Nash equilibrium

Cournot Competition

- We will apply the concept of pure Nash equilibrium to analyze oligopoly markets

Cournot Competition

- We will apply the concept of pure Nash equilibrium to analyze oligopoly markets
- Suppose that there are two firms that produce the same product have zero marginal cost of production.

Cournot Competition

- We will apply the concept of pure Nash equilibrium to analyze oligopoly markets
- Suppose that there are two firms that produce the same product have zero marginal cost of production.
- If firm 1 and 2 produce q_1 and q_2 units of the commodity respectively, the inverse demand function is given by:

$$\boxed{P(Q) = 120 - Q,}\ \underline{Q = q_1 + q_2}.$$

Cournot Competition

- We will apply the concept of pure Nash equilibrium to analyze oligopoly markets
- Suppose that there are two firms that produce the same product have zero marginal cost of production.
- If firm 1 and 2 produce q_1 and q_2 units of the commodity respectively, the inverse demand function is given by:

$$P(Q) = 120 - Q, Q = q_1 + q_2.$$

- Strategy space is $S_i = [0, +\infty)$

Cournot Competition

- We will apply the concept of pure Nash equilibrium to analyze oligopoly markets
- Suppose that there are two firms that produce the same product have zero marginal cost of production.
- If firm 1 and 2 produce q_1 and q_2 units of the commodity respectively, the inverse demand function is given by:

$$P(Q) = 120 - Q, Q = q_1 + q_2.$$

- Strategy space is $S_i = [0, +\infty)$

- The utility function of player i is given by:

$$\boxed{\begin{array}{l}\pi_1(q_1, q_2) = (120 - (q_1 + q_2))q_1 = P(Q)q_1 - \cancel{Cq_1} \\ \pi_2(q_1, q_2) = (120 - (q_1 + q_2))q_2 = P(Q)q_2 - \cancel{Cq_2}\end{array}}$$

$C = 0$

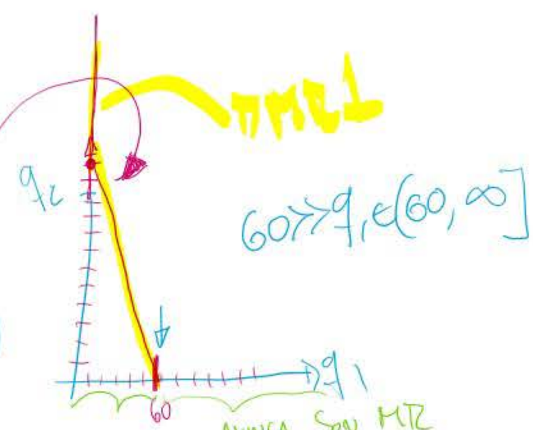
Cournot Competition

- Are there any strictly dominant strategies?

$$\boxed{\pi_1 = (120 - q_1 - q_2)q_1}$$

$$\frac{\partial \pi_1}{\partial q_1} = 120 - 2q_1 - q_2 = 0$$

$$\boxed{\boxed{\frac{120 - q_2}{2}} = \boxed{q_1}}$$



Cournot Competition

- Are there any strictly dominant strategies?

Cournot Competition

- Are there any strictly dominant strategies? The answer is no, why?
- Are there any strictly dominated strategies?

Cournot Competition

- Are there any strictly dominant strategies? The answer is no, why?
- Are there any strictly dominated strategies?
- The strategies $q_i \in (120, +\infty)$ are strictly dominated by the strategy 0

Cournot Competition

- Are there any strictly dominant strategies? The answer is no, why?
- Are there any strictly dominated strategies?
- The strategies $q_i \in (120, +\infty)$ are strictly dominated by the strategy 0
- Are there any others? given q_{-i},

$$\frac{d\pi_i}{dq_i}(120 - q_i - q_{-i})q_i = 120 - 2q_i - q_{-i}$$

Cournot Competition

- Are there any strictly dominant strategies? The answer is no, why?
- Are there any strictly dominated strategies?
- The strategies $q_i \in (120, +\infty)$ are strictly dominated by the strategy 0
- Are there any others? given q_{-i},

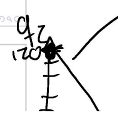
$$\frac{d\pi_i}{dq_i}(120 - q_i - q_{-i})q_i = 120 - 2q_i - q_{-i}$$

- Therefore 60 strictly dominates any $q_i \in (60, 120]$

$s_i \in [0, 60]$

$BR_1(q_2)$

$(\pi_1(40,40), \pi_2(40,40))$



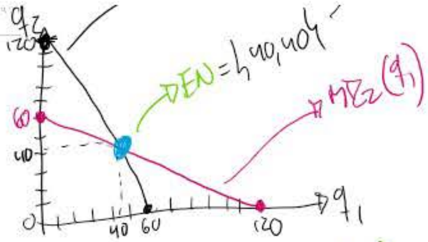
$NE = (40, 40)$

$BR_2(q_1)$

Cournot Competition

Cournot Competition

- $$\boxed{BR_i(q_{-i})} = \boxed{\frac{120 - q_{-i}}{2}}.$$



$$\frac{120-q_2}{2} = q_1 = MR_1(q_2)$$

$$\frac{120-q_1}{2} = q_2 = MR_2(q_1)$$

$$60 - \frac{q_2}{2} - q_1 = 0$$

$$60 - \frac{q_1}{2} - q_2 = 0$$

$\times(-2)$

$$-120 + q_2 + 2q_1 = 0$$

$$-60 + \frac{3}{2}q_1 = 0$$

$$\boxed{q_1^* = \frac{120}{3} = 40 \quad q_2^* = 40}$$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- for any $q_i \in [0, 60]$, there exists some $q_{-i} \in [0, +\infty)$ such that $BR_i(q_{-i}) = q_i$

Cournot Competition

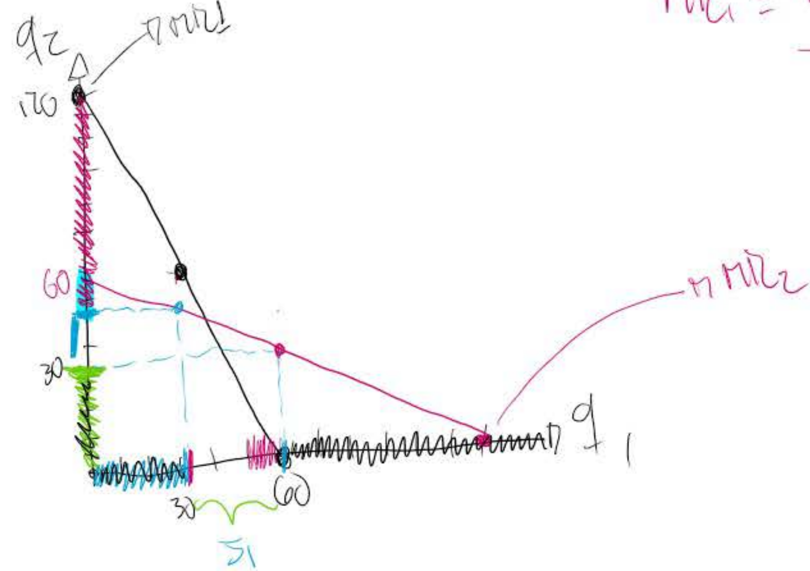
- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- for any $q_i \in [0, 60]$, there exists some $q_{-i} \in [0, +\infty)$ such that $BR_i(q_{-i}) = q_i$
- Such a q_i can never be strictly dominated

$$MR_1 = \frac{120 - q_2}{2}$$

$$q_2 \in (0, 60) \Downarrow q_1(0) = 60 \quad q_1(60) = 30$$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- for any $q_i \in [0, 60]$, there exists some $q_{-i} \in [0, +\infty)$ such that $BR_i(q_{-i}) = q_i$
- Such a q_i can never be strictly dominated
- After one round of deletion of strictly dominated strategies, we are left with: $S_i = [0, 60]$



Cournot Competition

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$

31

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [0, 60]$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [0, 60]$
- Therefore $q_i \in [0, 30)$ are strictly dominated by $q_i = 30$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [0, 60]$
- Therefore $q_i \in [0, 30)$ are strictly dominated by $q_i = 30$
- After two rounds of deletion of strictly dominated strategies, we are left with: $S_i = [30, 60]$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [30, 60]$
- 45 strictly dominates all strategies $q_i \in (45, 60]$
- After three rounds of deletion of strictly dominated strategies, we are left with: $S_i = [30, 45]$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [30, 45]$
- 37.5 strictly dominates all strategies $q_i \in [30, 37.5]$
- After four rounds of deletion of strictly dominated strategies, we are left with: $S_i = [37.5, 45]$

Cournot Competition

- After (infinitely) many iterations, the only remaining strategies are $S_i = 40$
- The unique solution by IDSDS is $q_1^* = q_2^* = 40$.

Cournot Competition

- There will also be a unique Nash equilibrium

Cournot Competition

$$\boxed{P = 120 - Q}$$

- There will also be a unique Nash equilibrium
- $$\boxed{BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.}$$

$$\Pi_i = (120 - \underbrace{q_1 - q_2}_{Q = q_1 + q_2})\, q_i$$

$$q_1 = \frac{120 - q_2}{2} = MR_1$$

$$q_2 = \frac{120 - q_1}{2} = MR_2$$

Cournot Competition

- There will also be a unique Nash equilibrium
- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- At any Nash equilibrium, we must have: $q_1^* \in BR_1(q_2^*)$ and $q_2^* \in BR_2(q_1^*)$.

Cournot Competition

- There will also be a unique Nash equilibrium
- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- At any Nash equilibrium, we must have: $q_1^* \in BR_1(q_2^*)$ and $q_2^* \in BR_2(q_1^*)$.
- $$q_1^* = \frac{120 - q_2^*}{2}, q_2^* = \frac{120 - q_1^*}{2}.$$

Cournot Competition

- There will also be a unique Nash equilibrium
- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- At any Nash equilibrium, we must have: $q_1^* \in BR_1(q_2^*)$ and $q_2^* \in BR_2(q_1^*)$.
- $$q_1^* = \frac{120 - q_2^*}{2}, q_2^* = \frac{120 - q_1^*}{2}.$$
- We can solve for q_1^* and q_2^* to obtain:
$$q_1^* = 40, q_2^* = 40, Q^* = 80, \Pi_1^* = \Pi_2^* = \boxed{1600.}$$

Cournot Competition vs Monopoly (cartel)

- In a perfectly competitive market, price equals marginal cost and the total quantity produced will be $Q = 120$.

$MC = 0$

$\hookrightarrow P = 120 - Q$

Cournot Competition vs Monopoly (cartel)

- In a perfectly competitive market, price equals marginal cost and the total quantity produced will be $Q = 120$.

- A monopolist would solve the following maximization problem:

$$\Pi_m = \max_Q \underbrace{(120 - Q)}_{P} Q \Rightarrow Q^* = 60, P^* = 60, \Pi^m = 3600.$$

$$\frac{\partial \Pi_m}{\partial Q} = 120 - 2Q = 0 \qquad \boxed{Q^m = 60}$$

$P^m = 60$

$\pi^m = 3{,}600$

$\boxed{1800}$ π_1 $\boxed{1800}$ π_2

Cournot Competition vs Monopoly (cartel)

- In a perfectly competitive market, price equals marginal cost and the total quantity produced will be $Q = 120$.
- A monopolist would solve the following maximization problem:

$$\max_Q (120 - Q)Q \Rightarrow Q^* = 60, P^* = 60, \Pi^m = 3600.$$

- The profits to each firm in the Cournot Competition is less than half of the monopoly profits

Cournot Competition vs Monopoly (cartel)

- In a perfectly competitive market, price equals marginal cost and the total quantity produced will be $Q = 120$.
- A monopolist would solve the following maximization problem:

$$\max_Q (120 - Q)Q \Rightarrow Q^* = 60, P^* = 60, \Pi^m = 3600.$$

$\hookrightarrow (q_1 = 30, q_2 = 30)$

- The profits to each firm in the Cournot Competition is less than half of the monopoly profits

- In a duopoly, externalities are imposed on the other firm

$q_2^m = 30$

$$\Pi_1 = (120 - q_2 - q_1) q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = 120 - 30 - 2q_1 = 0$$

$$\frac{90}{2} = q_1$$

$$\boxed{45 = q_1}$$

$$\Pi_1 = (120 - \underset{q_2}{30} - 45) 45$$

$$= 45^2 = \boxed{2{,}025}$$

$$\Rightarrow 45 = q_1 = q_2$$

$$\Pi_1 = (120 - 45 - 45) 45$$

$$= 30 \cdot 45 = \boxed{1{,}350}$$

Cournot Competition - General case

- n firms are competing a la Cournot → COMPITEN EN CANTIDAD

Cournot Competition - General case

- n firms are competing a la Cournot
- The inverse demand function is given by:

$$\boxed{P(q_1 + q_2 + \cdots q_n).}$$

Cournot Competition - General case

- n firms are competing a la Cournot
- The inverse demand function is given by:

$$P(q_1 + q_2 + \cdots q_n).$$

- Suppose that the cost function is $c_i(q_i)$ for firm i

Cournot Competition - General case

- n firms are competing a la Cournot
- The inverse demand function is given by:

$$P(q_1 + q_2 + \cdots q_n).$$

- Suppose that the cost function is $c_i(q_i)$ for firm i
- To simplify notation, let $\boxed{Q_{-i} = \sum_{j\neq i} q_j} = Q - q_i = Q_{-i} \Rightarrow Q = Q_{-i} + q_i$

Cournot Competition - General case

- n firms are competing a la Cournot
- The inverse demand function is given by:

$$P(q_1 + q_2 + \cdots q_n).$$

- Suppose that the cost function is $c_i(q_i)$ for firm i
- To simplify notation, let $Q_{-i} = \sum_{j \neq i} q_j$
-

$$\boxed{\max_{q_i} p(q_i + Q_{-i})q_i - c_i(q_i)}$$

Cournot Competition - General case

$$\Pi_i \; \max_{q_i} p(q_i + Q_{-i})q_i - c_i(q_i)$$

$$Q = Q_{-i} + q_i$$

- First order condition implies:

$$q_i \frac{dP}{dQ}(q_i + Q_{-i}) + P(q_i + Q_{-i}) = \frac{dc_i}{dq_i}(q_i)$$

$$\boxed{q_i \frac{dP}{dQ}(Q)} + P(Q) = \frac{dc_i}{dq_i}(q_i)$$

$$P(Q) - \frac{dc_i}{dq_i}(q_i) = -q_i \frac{dP}{dQ}(Q)$$

$$\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} = -\frac{q_i}{Q} \boxed{\frac{Q}{P(Q)} \frac{dP}{dQ}(Q)}$$

$$\boxed{\frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = \varepsilon_{Q,P}}$$

$$\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} = -\frac{q_i}{Q} \frac{1}{\varepsilon_{Q,P}(Q)}$$

Cournot Competition - General case

$$\boxed{\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} = -\frac{q_i}{Q} \frac{1}{\varepsilon_{Q,P}(Q)}}$$

$$\frac{q_i}{Q} \to 0$$

SI HAY INFINITAS FIRMAS

$$\frac{P - CMg}{P} = 0$$

$$\Rightarrow P = CMg$$

- Therefore in a pure strategy Nash equilibrium $(q_1^*, q_2^*, \ldots, q_n^*)$ with $Q^* = q_1^* + q_2^* + \cdots q_n^*$, we must have:

$$\frac{P(Q^*) - \frac{dc_1}{dq_1}(q_1^*)}{P(Q^*)} = -\frac{q_1^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)},$$

$$\frac{P(Q^*) - \frac{dc_2}{dq_2}(q_2^*)}{P(Q^*)} = -\frac{q_2^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)},$$

$$\vdots$$

$$\frac{P(Q^*) - \frac{dc_n}{dq_n}(q_n^*)}{P(Q^*)} = -\frac{q_n^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)}.$$

N ECUACIONES

N INCOGNITAS

Cournot Competition - General case

- Suppose that all firms have exactly the same cost function c

$$\Pi_i = P(Q)q_i - c(q_i)$$

$$\frac{P(Q^*) - \frac{dc}{dq_1}(q_1^*)}{P(Q^*)} = -\frac{q_1^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)},$$

$$\frac{P(Q^*) - \frac{dc}{dq_2}(q_2^*)}{P(Q^*)} = -\frac{q_2^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)},$$

$$\vdots$$

$$\frac{P(Q^*) - \frac{dc}{dq_n}(q_n^*)}{P(Q^*)} = -\frac{q_n^*}{Q^*} \frac{1}{\varepsilon_{Q,P}(Q^*)}.$$

$$Q^* = q_1^* + q_2^* + \cdots + q_N^*$$

$$Q^* = n q^*$$

$$\boxed{\frac{1}{n} = \frac{q^*}{Q^*}}$$

Cournot Competition - General case

- Let us conjecture that there exists a pure strategy Nash equilibrium that is **symmetric**, in which $q_1^* = q_2^* = \cdots q_n^* = q^*$

Cournot Competition - General case

- Let us conjecture that there exists a pure strategy Nash equilibrium that is **symmetric**, in which $q_1^* = q_2^* = \cdots q_n^* = q^*$
- In this case $Q^* = nq^*$

$$\boxed{\frac{P(nq^*) - \frac{dc}{dq_1}(q^*)}{P(nq^*)} = -\frac{1}{n}\frac{1}{\varepsilon_{Q,P}(nq^*)}}$$

$\boxed{n=1} \rightarrow$ Monopolio

$\boxed{n \to \infty} \Rightarrow \to 0 \Rightarrow \frac{P(Q) - Cmg}{P} = 0 \Rightarrow P^* = Cmg$

Cournot Competition - General case

- Let us conjecture that there exists a pure strategy Nash equilibrium that is **symmetric**, in which $q_1^* = q_2^* = \cdots q_n^* = q^*$
- In this case $Q^* = nq^*$

$$\frac{P(nq^*) - \frac{dc}{dq_1}(q^*)}{P(nq^*)} = -\frac{1}{n}\frac{1}{\varepsilon_{Q,P}(nq^*)}$$

- Rewriting

$$P(Q^*) = \frac{1}{1 + \frac{1}{n}\frac{1}{\varepsilon_{Q,P}(Q^*)}}\frac{\partial c}{dq}\left(\frac{Q^*}{n}\right).$$

Lecture 12: Game Theory // Nash equilibrium

Cartels

- Suppose there are three firms who face zero marginal cost
- The inverse demand function is given by:

Un EQ Simetrico

MAL

Cartels

- Suppose there are three firms who face zero marginal cost
- The inverse demand function is given by:

$$p(q_1 + q_2 + q_3) = 1 - q_1 - q_2 - q_3 = 1 - Q$$

$$\boxed{\Pi_1 = (1 - q_1 - q_2 - q_3)q_1}$$

$$\frac{\partial \Pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - q_3 = 0$$

$$\boxed{\frac{1 - q_2 - q_3}{2} = q_1} = MR_1$$

RACIONALIDAD

$$\boxed{MR_2 = \frac{1 - q_1 - q_3}{2} = q_2}$$

$$\boxed{MR_3 = \frac{1 - q_1 - q_2}{2} = q_3}$$

$\Rightarrow$ BUSCAR UN EQ SIMETRICO $q_1^* = q_2^* = q_3^* = q^*$

$$\boxed{\text{UN EQ SIMETRICO } q_1^* = q_2^* = q_3^* = q^*}$$

$$\Rightarrow \frac{1 - q^* - q^*}{2} = q^*$$

$$1 - 2q^* = 2q^*$$

$$1 = 4q^*$$

$$\boxed{\frac{1}{4} = q^* = q_1^* = q_2^* = q_3^*}$$

$$\boxed{Q^* = \frac{3}{4}}$$

$\neq$

MAL

$$\Pi_1 = \cancel{(1 - q_1 - q_2 - q_3)q_1}$$

$$= \cancel{(1 - q - q - q)q}$$

$$= \cancel{(1 - 3q)q}$$

$$\cancel{\frac{\partial \Pi_1}{\partial q}} = 1 - 6q = 0$$

$$\boxed{q = 1/6}$$

Cartels

- Suppose there are three firms who face zero marginal cost
- The inverse demand function is given by:

$$p(q_1 + q_2 + q_3) = 1 - q_1 - q_2 - q_3 = 1 - Q$$

- The first order condition gives

$$1 - 2q_i - Q_{-i} = 0 \Longrightarrow q_i = \frac{1 - Q_{-i}}{2} \Longrightarrow BR_i(Q_{-i}) = \frac{1 - Q_{-i}}{2}.$$

Cartels

- Suppose there are three firms who face zero marginal cost
- The inverse demand function is given by:

$$p(q_1 + q_2 + q_3) = 1 - q_1 - q_2 - q_3 = 1 - Q$$

- The first order condition gives

$$1 - 2q_i - Q_{-i} = 0 \Longrightarrow q_i = \frac{1 - Q_{-i}}{2} \Longrightarrow BR_i(Q_{-i}) = \frac{1 - Q_{-i}}{2}.$$

- In a Nash equilibrium we must have:

$$q_1^* = \frac{1 - q_2^* - q_3^*}{2}$$
$$q_2^* = \frac{1 - q_1^* - q_3^*}{2}$$
$$q_3^* = \frac{1 - q_1^* - q_2^*}{2}.$$

Cartels

- The easiest way to solve this first, let us add the three equations to get:

$$Q^* = \frac{3}{2} - Q^* \Longrightarrow Q^* = \frac{3}{4}.$$

$EN = (q_1^* = 1/4, q_2^* = 1/4, q_3^* = 1/4)$

$\Pi_i = 1/4$

Cartels

Cartels

- The easiest way to solve this first, let us add the three equations to get:

$$Q^* = \frac{3}{2} - Q^* \Longrightarrow \boxed{Q^* = \frac{3}{4}}.$$

- Note that

$$q_1^* = \frac{1}{2} - \frac{q_2^* - q_3^*}{2} \Longrightarrow \frac{q_1^*}{2} = \frac{1}{2} - \frac{Q^*}{2} \Longrightarrow \boxed{q_1^* = \frac{1}{4}}.$$

$EN = (q_1^* = 1/4, q_2^* = 1/4, q_3^* = 1/4)$

$p^* = 1 - q_1 - q_2 - q_3 = 1 - 3/4 = 1/4$

$\boxed{\pi_1^* = (1/4)(1/4) = 1/16 = \pi_2^* = \pi_3^*}$

Cartels

- The easiest way to solve this first, let us add the three equations to get:

$$Q^* = \frac{3}{2} - Q^* \Longrightarrow Q^* = \frac{3}{4}.$$

- Note that

$$q_1^* = \frac{1}{2} - \frac{q_2^* - q_3^*}{2} \Longrightarrow \frac{q_1^*}{2} = \frac{1}{2} - \frac{Q^*}{2} \Longrightarrow q_1^* = \frac{1}{4}.$$

- $q_1^* = q_2^* = q_3^* = \frac{1}{4}$

Cartels

- The easiest way to solve this first, let us add the three equations to get:

$$Q^* = \frac{3}{2} - Q^* \Longrightarrow Q^* = \frac{3}{4}.$$

- Note that

$$q_1^* = \frac{1}{2} - \frac{q_2^* - q_3^*}{2} \Longrightarrow \frac{q_1^*}{2} = \frac{1}{2} - \frac{Q^*}{2} \Longrightarrow q_1^* = \frac{1}{4}.$$

- $\boxed{q_1^* = q_2^* = q_3^* = \frac{1}{4}}$
- Price is $p^* = 1/4$ and all firms get the same profits of 1/16

Cartels

- Two of the firms merge into firm A, while one of the firms remains single, call that firm B

Cartels

- Two of the firms merge into firm A, while one of the firms remains single, call that firm B
- Each firm then again faces the profit maximization problem:

$$\Pi_i = \max_{q_i} \underbrace{(1 - q_i - q_{-i})}_{P} q_i \Longrightarrow BR_i(q_{-i}) = \frac{1 - q_{-i}}{2}.$$

$$\frac{\partial \Pi_i}{\partial q_i} = 1 - 2q_i - q_{-i} = 0 \Rightarrow \frac{1 - q_{-i}}{2} = q_i = MR_i(q_{-i})$$

Cartels

- Two of the firms merge into firm A, while one of the firms remains single, call that firm B
- Each firm then again faces the profit maximization problem:

$$\max_{q_i} (1 - q_i - q_{-i}) q_i \Longrightarrow BR_i(q_{-i}) = \frac{1 - q_{-i}}{2}.$$

- Therefore

$$\boxed{\begin{aligned} q_A^* &= \frac{1 - q_B^*}{2} \\ q_B^* &= \frac{1 - q_A^*}{2}. \end{aligned}}$$

$$q_A^* = q_B^* = q^* \Rightarrow q^* = \frac{1 - q^*}{2}$$

$$2q^* = 1 - q^*$$

$$\boxed{q^* = 1/3}$$

$$p^* = 1 - 1/3 - 1/3 = 1/3$$

$$\boxed{\pi^* = 1/3(1/3) = 1/9}$$

$$\Pi_A = 1/9 \qquad \boxed{\Pi_B = 1/9}$$

$$\Pi_{A_1} = \frac{1}{18} \qquad \Pi_{A_2} = \frac{1}{18}$$

Cartels

- Solving this:

$$q_A^* = q_B^* = \frac{1}{3}.$$

Cartels

- Solving this:

$$q_A^* = q_B^* = \frac{1}{3}.$$

- The price is then $p^* = 1/3$

Cartels

- Solving this:
$$q_A^* = q_B^* = \frac{1}{3}.$$
- The price is then $p^* = 1/3$
- If the profits are shared equally among firms 1 and 2 who have merged, then profits of firms 1 and 2 are 1/18 whereas firm 3 obtains a profit of 1/9

Cartels

- Solving this:
$$q_A^* = q_B^* = \frac{1}{3}.$$
- The price is then $p^* = 1/3$
- If the profits are shared equally among firms 1 and 2 who have merged, then profits of firms 1 and 2 are 1/18 whereas firm 3 obtains a profit of 1/9
- Firms 1 and 2 suffered, while firm 3 is better off!

Cartels

- Solving this:
$$q_A^* = q_B^* = \frac{1}{3}.$$
- The price is then $p^* = 1/3$
- If the profits are shared equally among firms 1 and 2 who have merged, then profits of firms 1 and 2 are 1/18 whereas firm 3 obtains a profit of 1/9
- Firms 1 and 2 suffered, while firm 3 is better off!
- Firm 3 is obtaining a disproportionate share of the joint profits (more than 1/3)

Cartels

- You might expect that 3 may want to join the cartel as well...

Cartels

- You might expect that 3 may want to join the cartel as well...
- In the monopolist problem, we solve:

$$\Pi^m = \max_Q \underbrace{(1-Q)}_{P} Q \implies \boxed{Q^* = \frac{1}{2}.} \longrightarrow q_1 = q_2 = q_3 = \frac{1}{6}$$

$$P^m = 1/2$$

$$\pi^m = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$\pi_1 = \frac{1}{12} \quad \pi_2 = \frac{1}{12} \quad \pi_3 = \frac{1}{12}$$

Cartels

- You might expect that 3 may want to join the cartel as well...
- In the monopolist problem, we solve:

$$\max_Q (1-Q)Q \implies Q^* = \frac{1}{2}.$$

- Total profits then are given by $\frac{1}{4}$ which means that each firm obtains a profit of $\frac{1}{12} < \frac{1}{9}$

Cartels

- You might expect that 3 may want to join the cartel as well...
- In the monopolist problem, we solve:

$$\max_Q (1-Q)Q \implies Q^* = \frac{1}{2}.$$

- Total profits then are given by $\frac{1}{4}$ which means that each firm obtains a profit of $\frac{1}{12} < \frac{1}{9}$
- Firm 3 clearly wants to stay out

Cartels

There are many ifficulties associated with sustaining collusive agreements (e.g., the OPEC cartel)