Lecture 14

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Lecture14

Lecture 14: Game Theory // Nash equilibrium

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Lecture 14: Game Theory // Nash equilibrium

Mixed strategies

Examples

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Mixed strategies

Consider rock/paper/scissors

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

- This game is entirely stochastic (ability has nothing to do with your chances of winning)

Mixed strategies

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- The probability of winning with every strategy is the same
- Thus, people *tend* choose randomly which of the three options to play
- We would like the concept of Nash equilibrium to reflect this

Mixed strategies

$\sigma_i(S_i) \to$ PROBABILIDAD DE JUGAR S_i

Definition
A mixed strategy σ_i is a function $\sigma_i : S_i \to [0, 1]$ such that

$$\boxed{\sum_{s_i \in S_i} \sigma_i(s_i) = 1.}$$

- $\sigma_i(s_i)$ represents the probability with which player i plays s_i

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- A **pure strategy** is simply a mixed strategy σ_i that plays some strategy $s_i \in S_i$ with probability one

$\boxed{S_i}$ ó $\boxed{\sigma_i(S_i) = 1}$

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- $\sigma_i(s_i)$ represents the probability with which player i plays s_i
- A **pure strategy** is simply a mixed strategy σ_i that plays some strategy $s_i \in S_i$ with probability one
- We will denote the set of all mixed strategies of player i by $\boxed{\Sigma_i}$

Mixed strategies

- Given a mixed strategy profile $\boxed{\sigma_1, \sigma_2, \ldots, \sigma_n}$, we need a way to define how players evaluate payoffs of mixed strategy profiles

Mixed strategies

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$$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n).$$

→ VALOR ESPERADO
ESPERADO
DE LA
UTILIDAD

Mixed strategies

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- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)

→ $PROB(S_{-i} = ROCA)$
→ $PROB(S_{-i} = PAPEL)$
→ $PROB(S_{-i} = TIJERA)$

Mixed strategies

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- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)
- The expected utility of playing "rock" is

$U(ROCA, ROCA) = 0$ → $PROB = 0$

PROB

$$E(U_i(rock, \sigma_{-i})) = -1\boxed{\frac{1}{2}} + 1\boxed{\frac{1}{2}} = 0$$

OPERADOR DE ESPERANZA

$U(ROCA, S_{-i} = PAPEL)$ $U(ROCA, TIJERAS)$

Mixed strategies

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$$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n).$$
- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)
- The expected utility of playing "rock" is
$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$
- If I'm randomizing over rock and scissors (i.e., $\sigma_i = (\frac{1}{2}, 0, \frac{1}{2})$) then

$$E(U_i(\sigma, \sigma_{-i})) = \underbrace{-1\frac{1}{4}}_{\text{rock vs paper}} + \underbrace{1\frac{1}{4}}_{\text{rock vs scissors}} + \underbrace{1\frac{1}{4}}_{\text{scissors vs paper}} + \underbrace{0\frac{1}{4}}_{\text{scissors vs scissors}} = \boxed{\frac{1}{4}}$$

Mixed strategies

Definition
A (possibly mixed) strategy profile $(\sigma_1^*, \sigma_2^*, \ldots, \sigma_n)^*$ is a Nash equilibrium if and only if for every i,

$$\boxed{u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)}$$

for all $\sigma_i \in \Sigma_i$.

Mixed strategies

Definition (Mixed Strategy Dominance Definition A)
Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all mixed strategies of the opponents, σ_{-i},

$$u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma_i', \sigma_{-i}). \quad \forall \sigma_{-i} \in \Sigma_{-i}$$

Mixed strategies

If σ_i is better than σ_i' no matter what **pure strategy** opponents play, then σ_i is also strictly better than σ_i' no matter what **mixed strategies** opponents play

Theorem
Let σ_i and σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if and only if for all $s_{-i} \in S_{-i}$, $\Rightarrow$ $\Leftarrow$

$$\boxed{u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).}$$

Proof- Part 1

$(\Rightarrow)$ σ_i DOMINA A σ_i'

$$U_i(\sigma_i, \sigma_{-i}) > U_i(\sigma_i', \sigma_{-i})$$

$\forall \sigma_{-i}$

- Since $\boxed{S_{-i} \subseteq \Sigma_{-i}}$, if σ_i strictly dominates σ_i'

Proof- Part 1

- Since $S_{-i} \subseteq \Sigma_{-i}$, if σ_i strictly dominates σ_i'
- Then for all $s_{-i} \in S_{-i}$,
$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof - Part 2

- To prove the other direction, suppose that for all $s_{-i} \in S_{-i}$,
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$$\boxed{u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).}$$
- For any σ_{-i},
$$\begin{aligned} \boxed{u_i(\sigma_i, \sigma_{-i})} &= \boxed{\sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i})} \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \boxed{\sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i})} \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \boxed{u_i(\sigma_i, s_{-i})} \end{aligned}$$

$$\geq \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) u_i(\sigma_i', s_{-i})$$

Proof - Part 2

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$$\boxed{u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).}$$
- For any σ_{-i},
$$\begin{aligned} u_i(\sigma_i, \sigma_{-i}) &= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) \end{aligned}$$
- So
$$u_i(\sigma_i, \sigma_{-i}) = \boxed{\sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i})} > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i', s_{-i}) = u_i(\sigma_i', \sigma_{-i})$$

$\forall \sigma_{-i}$

Mixed strategies

Definition (Mixed Strategy Dominance Definition B)

Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all pure strategies of the opponents, $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Lecture 14: Game Theory // Nash equilibrium

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Examples

Battle of the sexes

S_2

S_1

	G	P
G	2,1	0,0
P	0,0	1,2

$\Rightarrow EN = \{(G,G),(P,P)\}$.

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

- There are two pure strategy equilibria (G, G) and (P, P)

Battle of the sexes

	G	P
G	2,1	0,0
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- There are two pure strategy equilibria (G, G) and (P, P)
- We now look for Nash equilibria that involve randomizationby the players

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G

$\sigma_1 = (\lambda, 1-\lambda)$ $\sigma_2 = (q, 1-q)$

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G
-

$$E(u_1(\lambda, q)) = \boxed{\underbrace{2}_{\to U(G,G)}\lambda q + (1-\lambda)(1-q)\cdot \underbrace{1}_{\to U(P,P)}} + \overset{U(G,P)}{0}\lambda(1-q) + \underbrace{0}_{\to U(P,G)}\cdot(1-\lambda)q$$

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G
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- $U_1(\sigma_1, \sigma_2) = 2\lambda q + (1-\lambda)(1-q)$

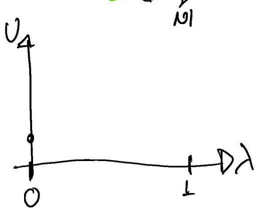
$= 2\lambda q + 1 - \lambda - q + \lambda q$

$= 3\lambda q - \lambda - q + 1$

$$U_1(\sigma_1, \sigma_2) = \lambda \underbrace{(3q-1)}_{N1} + \underbrace{1-q}_{N2}$$

$\to$ LINEA RECTA

$\downarrow$ PENDIENTE



$N_1 > 0$ $(3q - 1 > 0) \Rightarrow \lambda = 1$

$\boxed{N_1 = 0}$ $(3q - 1 = 0) \Rightarrow \lambda \in [0,1]$

$N_1 < 0$ $(3q - 1 < 0) \Rightarrow \lambda = 0$

P(GOLFITO) P(PELICULAS)

$\sigma_1 = (1, 0)$ $q > 1/3$

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G
- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$
- **Case 1:** If $q > 1/3$, then $2q > 2/3 > 1-q$ and therefore, the best response is $\lambda = 1$

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- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$

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- **Case 2:** if $q = 1/3$, then $2q = 2/3 = 1-q$ and therefore, the best response is $\lambda \in [0,1]$
- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$

- Thus, the best response function is given by:

$$BR_1(q) = \begin{cases} 1 & \text{if } q > 1/3 \\ [0,1] & \text{if } q = 1/3 \\ 0 & \text{if } q < 1/3. \end{cases}$$

$N_1 < 0 \; (3q-1<0) \Rightarrow \lambda = 0$

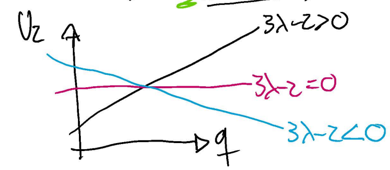
$$MR_1(\sigma_2=(q,1-q)) = \begin{cases} \sigma_1=(1,0) & q>1/3 \\ \sigma_1=(\lambda,1-\lambda)\ \lambda\in(0,1) & q=1/3 \\ \sigma_1=(0,1) & q<1/3 \end{cases}$$

$$U_2(\sigma_1=(\lambda,1-\lambda),\sigma_2=(q,1-q)) = \underbrace{1}_{U_2(G,G)}\cdot\lambda q + \underbrace{0}_{U_2(G,P)}\lambda(1-q) + \underbrace{0}_{U_2(P,G)}(1-\lambda)q + \underbrace{2}_{U_2(P,P)}(1-\lambda)(1-q)$$

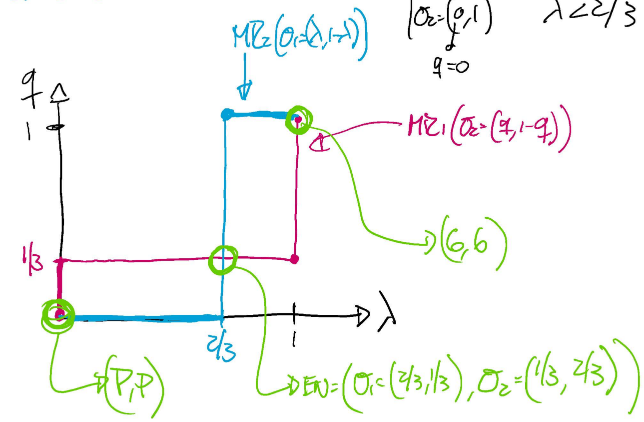
$$= \lambda q + 2 - 2\lambda - 2q + 2\lambda q$$

$$= 3\lambda q - 2\lambda + 2 - 2q$$

$$U_2(\sigma_1,\sigma_2) = q(3\lambda-2) + 2 - 2\lambda$$



$$MR_2(\sigma_1=(\lambda,1-\lambda)) = \begin{cases} \sigma_2=(1,0)\ (q=1) & \lambda>2/3 \\ \sigma_2=(q,1-q)\ q\in[0,1] & \lambda=2/3 \\ \sigma_2=(0,1)\ (q=0) & \lambda<2/3 \end{cases}$$

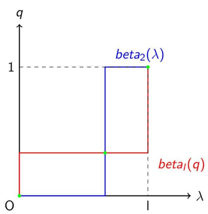


Battle of the sexes

Similarly we can calculate the best response function for player 2 and we get:

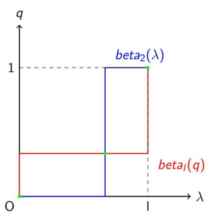
$$BR_2(\lambda) = \begin{cases} 1 & \text{if } \lambda > 2/3 \\ [0,1] & \text{if } \lambda = 2/3 \\ 0 & \text{if } \lambda < 2/3. \end{cases}$$

Battle of the sexes



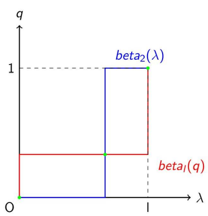
- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$

Battle of the sexes



- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$
- First two are the pure strategy NE we had found before

Battle of the sexes



- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$
- First two are the pure strategy NE we had found before
- Last is a strictly mixed NE: both players randomize

Consider the following game

S_2

S_1

	E	~~F~~	G
A	5, 10	~~5, 3~~	3, 4
~~B~~	[illegible]	[illegible]	[illegible]
C	4, 2	~~8, 4~~	3, 8
D	2, 4	~~1, 3~~	8, 4

G DOMINA F

D DOMINA B

	E	G
A	5, 10	3, 4
C	4, 2	3, 8
D	2, 4	8, 4

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$

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- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$

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- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- Then $BR_2(\sigma_1) = \{(p, 0, 1-p), p \in [0,1]\}$

- G dominates F (player 2)

- G dominates F (player 2)

- D dominates B (player 1)

Reduced game

	E	G
A	5, 10	3, 4
D	2, 4	8, 4

$\sigma_1 = (p, \underbrace{0}_{\text{Prob}(C)}, 1-p)$

σ_1 Domina a C.

$\boxed{U_1(\sigma_1, E) > U_1(C, E) \quad y \quad U_1(\sigma_1, G) > U_1(C, G)}$

$5p + 2(1-p) > 4 \quad y \quad 3p + 8(1-p) > 3$

$5p + 2 - 2p > 4 \quad y \quad 3p + 8 - 8p > 3$

$3p > 2 \qquad 5 > 5p$

$\boxed{p > 2/3} \qquad \boxed{p < 1}$

$\sigma_1 = (p, 0, 1-p) \quad \boxed{p \in (2/3, 1)}$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\begin{aligned} \mathbb{E}U(\sigma_1, E) &> U(C, E) \\ 3p + 2 &> 4 \\ p &> \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbb{E}U(\sigma_1, G) &> \mathbb{E}U(C, G) \\ 8 - 5p &> 3 \\ p &< \frac{5}{5} = 1 \end{aligned}$$

S_2

J_1

Reduced game

	E	G
A	5, 10	3, 4
D	2, 4	8, 4

$\sigma_1 = (p, 1-p)$

$\sigma_2 = (q, 1-q)$

$U_1(A, \sigma_2) = 5 \cdot q + 3(1-q) = 3 + 2q$

$U_1(D, \sigma_2) = 2 \cdot q + 8(1-q) = 8 - 6q$

$MR_1(\sigma_2)$ es A si $3 + 2q > 8 - 6q$

$8q > 5$

$q > 5/8$

$MR_1(\sigma_2)$ es D si $3 + 2q < 8 - 6q$

$q < 5/8$

$MR_1(\sigma_2)$ es Indiferente entre A y D $\quad \boxed{q = 5/8}$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$

$\sigma_1 = (\underbrace{1}_{\text{Pr}(A)}, 0) = A \qquad q > 5/8$

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- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$
- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$
- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$

$$MR_1(\sigma_2) = \begin{cases} \boxed{\sigma_1 = (1,0)} = A & q > 5/8 \\ \boxed{\sigma_1 = (p, 1-p),\ p \in [0,1]} & \boxed{q = 5/8} \\ \sigma_1 = (0,1) = D & q < 5/8 \end{cases}$$

$$U_2(\sigma_1 = (p, 1-p), E) = 10p + 4(1-p) = 4 + 6p$$

$$U_2(\sigma_1 = (p, 1-p), G) = 4p + 4(1-p) = 4$$

$MR_2(\sigma_1)$ E si $4 + 6p > 4$

$\boxed{p > 0}$

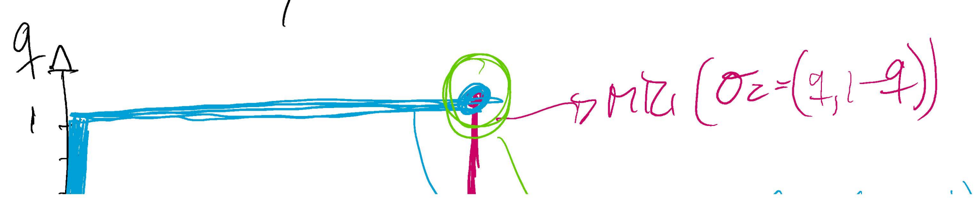
$MR_2(\sigma_1)$ G si $4 > 4 + 6p$

NUNCA

$MR_2(\sigma_1)$ INDIFERENTE ENTRE E y G si $4 = 4 + 6p$

$\boxed{p = 0}$

$$MR_2(\sigma_1) = \begin{cases} \sigma_2 = (1,0) & p > 0 \\ \sigma_2 = (q, 1-q)\ q \in [0,1] & \boxed{p = 0} \end{cases}$$



- Lets find $BR_1(\sigma_2 = (q, 1-q))$
- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$
- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$
- Thus

$$BR_1(q, 1-q) = \begin{cases} \sigma_1 = (0,1) & \text{if } 0 \leq q < \frac{5}{8} \\ \sigma_1 = (1,0) & \text{if } \frac{5}{8} < q \leq 1 \\ \sigma_1 = (p, 1-p) & \text{if } \frac{5}{8} = q \end{cases}$$

- Lets find $BR_2(\sigma_1 = (p, 1-p))$

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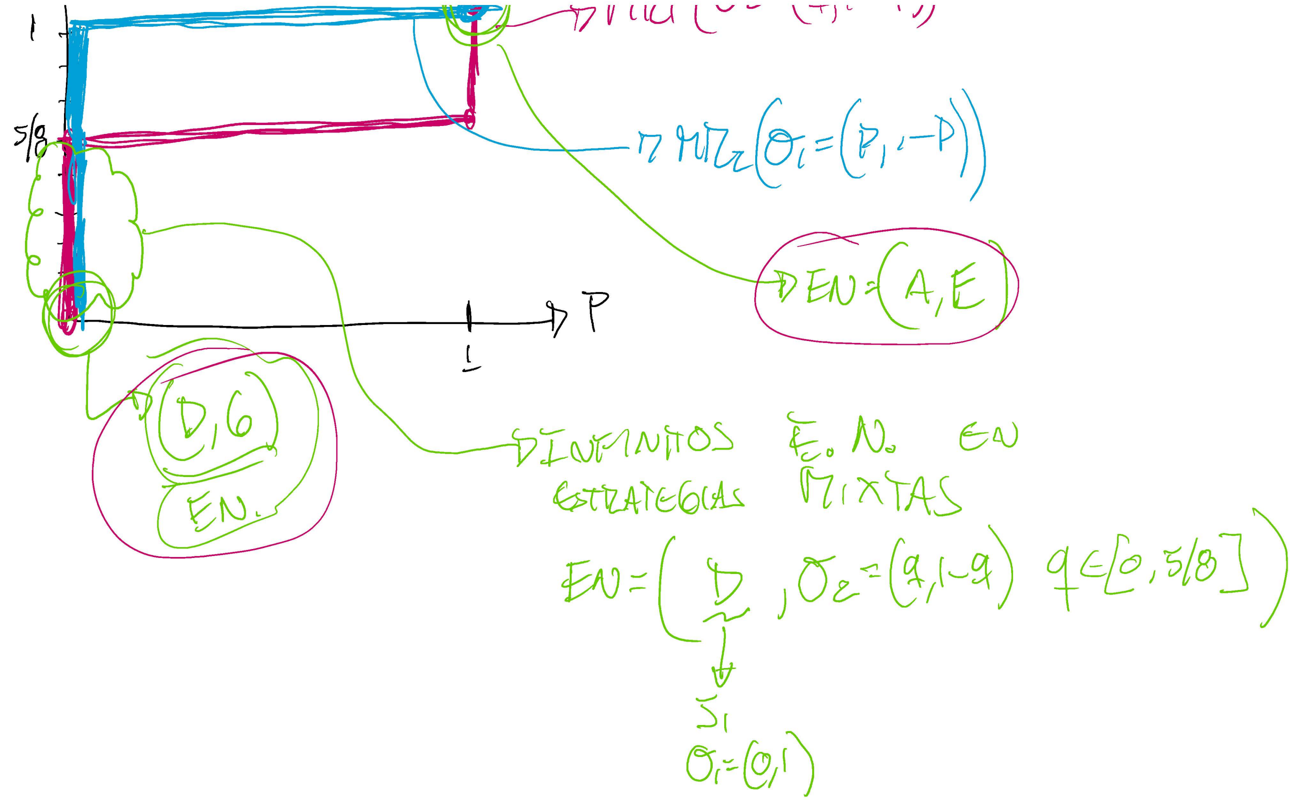

- Lets find $BR_2(\sigma_1 = (p, 1-p))$
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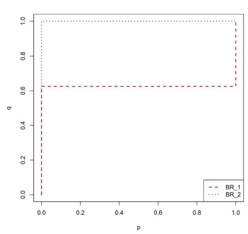
- $6p + 4 > 4$ if $p > 0$
- $6p + 4 < 4$ if $p < 0$.
- Thus

$$BR_2(p, 1-p) = \begin{cases} \sigma_2 = (1, 0) & \text{if } p > 0 \\ \sigma_2 = (q, 1-q) & \text{if } p = 0 \end{cases}$$

Best responses



$NE = \{(A, E), (D, \sigma_2^q)\}$ where $\sigma_2^q = (q, 1-q)$ and $0 \leq q \leq \frac{5}{8}$