Lecture 18

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Lecture18

Lecture 18: Repeated Games

Mauricio Romero

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Recap from last class

More than one NE in the stage game

Example 1

Example 2

Theorem

Suppose that the stage game G has exactly one NE, $(a_1^*, a_2^*, \dots, a_n^*)$. Then for any $\delta \in (0,1]$ and any T, the T-times repeated game has a unique SPNE in which all players i play a_i^* at all information sets.

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- We concentrate just on the payoffs in the future. Thus in period $T-1$, player i simply wants to maximize:

$$\max_{a_i \in A_i} \delta^{T-2} u_i(a_i, a_{-i}^{T-1}) + \delta^{T-1} u_i(a^*).$$

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- Thus again, for this to be a Nash equilibrium, we need $a_1^{T-1} = a_1^*, \ldots, a_n^{T-1} = a_n^*$.
- Following exactly this induction, we can conclude that every player must play a_i^* at all times and all histories

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- What would happen if there are more than one NE of the stage game?

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- Suppose instead that the stage game looks as follows

Normal Form

	A_2	B_2	C_2
A_1	1, 1	0, 0	0, 0
B_1	0, 0	4, 4	1, 5
C_1	0, 0	5, 1	3, 3

$EN = \{(A_1, A_2), (C_1, C_3)\}$

(B_1, B_2) es un O.P.

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- If the game is only played once

- There are two pure strategy Nash equilibria: (A_1, A_2) and (C_1, C_2).

- (B_1, B_2) is not a Nash equilibrium if the game is only played once

- In the one-shot game, the Nash equilibria are inefficient because they are Pareto dominated by (B_1, B_2)

- Playing the NE of the stage game in every period is a SPNE in the repeated game

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$U_T = U_{T-1} + \delta^{T-1} G + \cancel{\text{Futuro}}$ NO HAY

$U_1(a_i^T, a_{-i}^T),\ U_2(a_i^T, a_{-i}^T)$

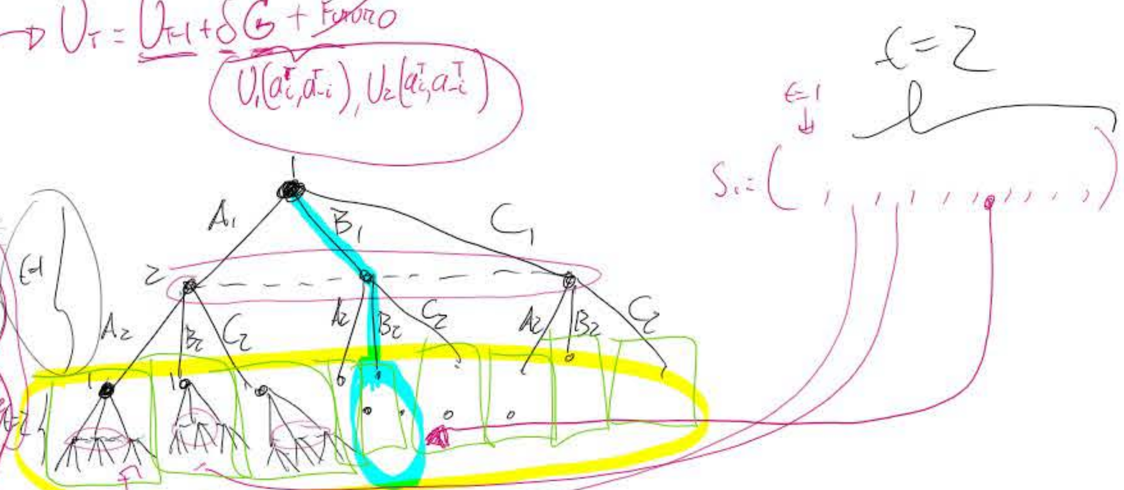
- The logic is the same as when there is a single NE

- $\boxed{\text{Always playing } (A_1, A_2) \text{ is a SPNE}}$

$t=1 \quad t=2$

$S_1 = (A_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1)$

$S_2 = (A_2, A_2, A_2, A_2, A_2, A_2, A_1, A_2, A_1, A_2)$



S_1 SON VECTORES 10 ENTRADAS $= S_2$ SON VECTORES 10 ENTRADAS

$|S_1| = 3^{10}$ $\quad |S_2| = 3^{10}$

- Always playing (A_1, A_2) is a SPNE
- Player 1's strategy is given by:
  1. Play A_1 in period 1;
  2. Play A_1 at all histories in period 2.
- Player 2's strategy is given by:
  1. Play A_2 in period 1;
  2. Play A_2 at all histories in period 2.

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- Always playing (C_1, C_2) is a SPNE
- Player 1's strategy is given by:
  1. Play C_1 in period 1
  2. Play C_1 at all histories in period 2.
- Player 2's strategy is given by:
  1. Play C_2 in period 1
  2. Play C_2 at all histories in period 2.

But are there more?

- Combining *NE* of the stage game is also a SPNE

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- The logic is the same as before

- Playing (A_1, A_2) in $t = 1$ and (C_1, C_2) in $t = 2$ is a SPNE

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- Similarly, playing (C_1, C_2) in $t = 1$ and (A_1, A_2) in $t = 2$ is a SPNE

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$t=1$ $\quad$ $t=2$

$$\left.\begin{aligned} S_1 &= (C_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1, A_1) \\ S_2 &= (C_2, A_2, A_2, A_2, A_2, A_2, A_2, A_2, A_2, A_2) \end{aligned}\right\} = \text{EPS}$$

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$$\left.\begin{aligned} S_1 &= (A_1, C_1, \ldots, C_1) \\ S_2 &= (A_2, C_2, \ldots, C_2) \end{aligned}\right\} = \text{EPS}$$

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- What makes a repeated game interesting is when players play strategies in SPNE that condition on what happened in the past
- This could not happen when the stage game had a unique NE
- In the last period, all players were required to play the unique NE action after all histories! Why?

Proof

- To see this, suppose that a history (a_1, a_2) was played in period 1 resulting in payoffs from period 1 of (x, y)

$t=1 \to (B_1, B_2)$
$t=2 \to (C_1, C_2)$

OBJETIVO

$$S_1 = \begin{cases} B_1 & \text{EN } t=1 \\ C_1 & \text{EN } t=2 \text{ SI } t=1\ (B_1,B_2) \\ A_1 & \text{EN } t=2 \text{ SI } t=1 \text{ NO SE JUGO } (B_1,B_2) \end{cases} \Rightarrow S_1 = (B_1, A_1, A_1, A_1, A_1, C_1, A_1, A_1, A_1, A_1)$$

$$S_2 = \begin{cases} B_2 & \text{EN } t=1 \\ C_2 & \text{EN } t=2 \text{ SI } t=1\ (B_1,B_2) \\ A_2 & \text{EN } t=2 \text{ SI } t=1 \text{ NO SE JUGO } (B_1,B_2) \end{cases} \Rightarrow S_2 = (B_2, A_2, A_2, A_2, A_2, C_2, A_2, A_2, A_2, A_2)$$

SUBJUEGO DONDE SE JUGO (B_1, B_2)

COMPROBAR QUE (S_1, S_2) ES UN EPS

⇒ 10 SUBJUEGOS

$E=2$

SON 9

⇒ E.N. SON (A_1,A_2) ó (C_1,C_2) DEL JUEGO BASE

⇒ $U(S_1,S_2)$ ES E.N. EN LOS SUBJUEGOS

$E=1$

JUEGO COMPLETO

$$U_1(S_1,S_2) = \underbrace{\boxed{4}}_{\substack{U(B_1,B_2)\\ E=1}} + \delta \cdot \underbrace{3}_{\substack{U(C_1,C_2)\\ E=2}}$$

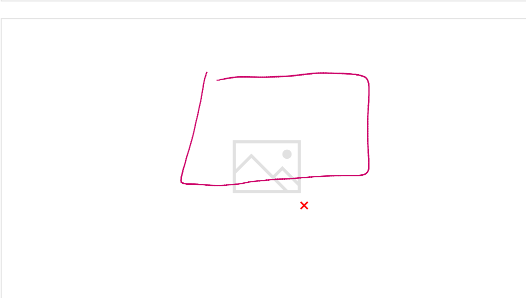
$$U_1(\text{DESVIARME}) = \underbrace{\boxed{5}}_{\substack{U(C_1,B_2)\\ E=1}} + \delta \cdot \underbrace{1}_{\substack{U(A_1,A_2)\\ E=2}}$$

NO ME DESVÍO SI: $4+3\delta \geq 5+\delta$

(SIGO LA ESTRATEGIA)

$$2\delta \geq 1$$

$$\boxed{\delta \geq 1/2}$$



- Are there any other action profiles that can be played in the first period?

Normal Form

	A_2	B_2	C_2
A_1	$1,1$	$0,0$	$0,0$
B_1	$0,0$	$4,4$	$1,5$
C_1	$0,0$	$5,1$	$3,3$

- Suppose that the players were to play (A_1, B_2) in the first period

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- Are there any other action profiles that can be played in the first period?

(C_1, B_2) ev $\epsilon=1$

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- Suppose that the players were to play (A_1, B_2) in the first period
- Can this occur? The answer is **no**

- Are there any other action profiles that can be played in the first period?

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(A_1, B_2) no $\epsilon=1$
(A_1, C_2) ''
(B_1, A_2) ''
(C_1, A_2) ''

- Suppose that the players were to play (A_1, B_2) in the first period
- Can this occur? The answer is **no**
- Remember either (A_1, A_2) or (C_1, C_2) must be played in any pure strategy SPNE after a history

$\underbrace{\quad}_{0}$ $\underbrace{\quad}_{3}$

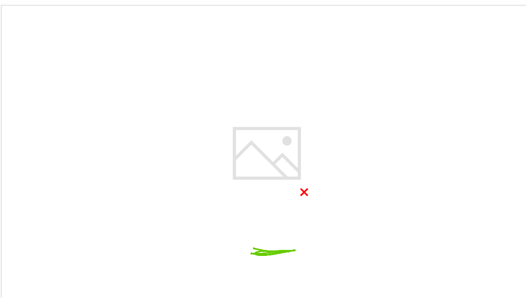
$\underbrace{\quad}_{5}$ $\underbrace{\quad}_{1}$

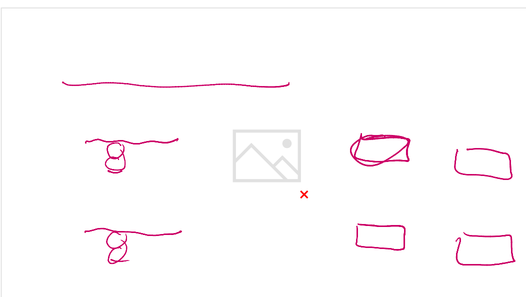
no Desvio si

$3\delta > 5 + \delta$

$2\delta > 5$

$\boxed{\delta > 5/2 > 1}$ Imposible.





The normal form of this game (conditional on what happens in $T = 2$) is:

Normal Form

	A_2	B_2	C_2
A_1	$1+3\delta, 1+3\delta$	$3\delta, 3\delta$	$3\delta, 3\delta$
B_1	$3\delta, 3\delta$	$4+3\delta, 4+3\delta$	$1+3\delta, 5+3\delta$
C_1	$3\delta, 3\delta$	$5+3\delta, 1+3\delta$	$3+\delta, 3+\delta$

$B_1 \succsim C_1$

$1+3\delta \geq 3+\delta$

$2\delta \geq 2$

$\boxed{\delta \geq 1}$

$\boxed{\begin{matrix} C_1 \succ B_1 \\ \delta < 1 \end{matrix}}$

$\boxed{\begin{matrix} B_1 \sim C_1 \\ \delta = 1 \end{matrix}}$

- In this game the best response for player i is:

$$BR_1(s_2) = \begin{cases} A_1 & \text{if } s_2 = A_2 \\ C_1 & \text{if } s_2 = B_2 \\ C_1 & \text{if } s_2 = C_2 \\ B_1 & \text{if } s_2 = C_2 \text{ \& } \delta = 1 \end{cases}$$

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- In this game the best response for player 2 is:

$$BR_2(s_1) = \begin{cases} A_2 & \text{if } s_1 = A_1 \\ C_2 & \text{if } s_1 = B_1 \\ C_2 & \text{if } s_1 = C_1 \\ \boxed{B_2 \quad \text{if } s_1 = C_1 \text{ \& } \delta = 1} \end{cases}$$

(C_1, B_2)

EN E=1

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- An equilibrium outcome of this game is to play (C_1, B_2) in period 1 and (C_1, C_2) in period 2 if $\boxed{\delta = 1}$

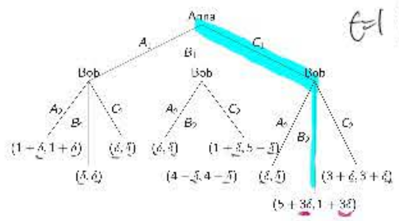
- There are other SPNE that results in the same equilibrium outcome
- For example consider the following SPNE
- Player 1's strategy is:
  1. Play C_1 in period 1.
  2. Play A_1 in period 2 if the first period action profile was anything other than (C_1, B_2).
  3. Play C_1 in period 2 if the first period action profile was (C_1, B_2).

  1. Player 2's strategy is:
  2. Play B_2 in period 1.
  3. Play A_2 in period 2 if the first period action profile was anything other than (C_1, B_2).
  4. Play C_2 in period 2 if the first period action profile was (C_1, B_2).

- We know that the strategy is a *NE* in the subgames that start in $t = 2$

- We know that the strategy is a NE in the subgames that start in $t = 2$
- But what about the whole game?

So we can simplify the game which gives the following game tree.



$t=1$ (CONDICIONAL A lo q' PASA EN $t=2$)

The normal form of this game (conditional on what happens in $t = 2$) is:

Normal Form

	A_2	B_2	C_2
A_1	$1-\delta, 1-\delta$	$3\delta, \delta$	δ, δ
B_1	δ, δ	$4+\delta, 4-\delta$	$1+\delta, 5+\delta$
C_1	δ, δ	$5+3\delta, 1-3\delta$	$3+\delta, 3+\delta$

$B_2 \succsim C_2$
$1+3\delta \geq 3+\delta$
$2 \geq 2\delta$
$\delta \leq 1$

$B_2 \sim C_2$
$\delta = 1$

$C_2 > B_2$
$\delta < 1$

- In this game the best response for player 1 is:

$$\boxed{BR_1(s_2) = \begin{cases} A_1 & \text{if } s_2 = A_2 \\ C_1 & \text{if } s_2 = B_2 \\ C_1 & \text{if } s_2 = C_2 \end{cases}}$$

- In this game the best response for player 2 is:

$$BR_2(s_1) = \begin{cases} A_2 & \text{if } s_1 = A_1 \\ C_2 & \text{if } s_1 = B_1 \\ C_2 & \text{if } s_1 = C_1 \\ B_2 & \text{if } s_1 = C_1 \,\&\, \delta = 1 \end{cases}$$

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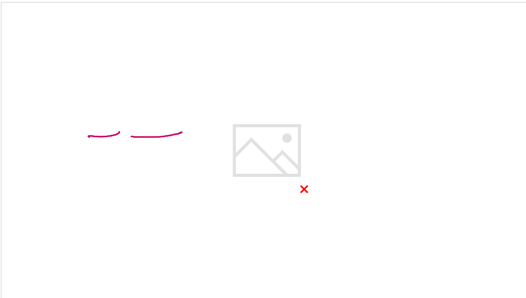
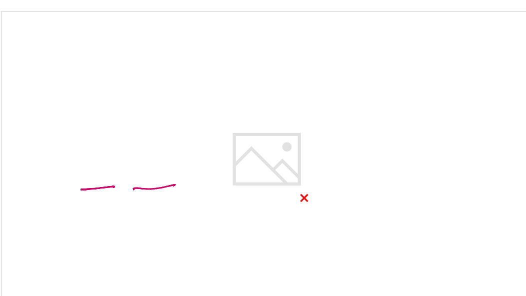
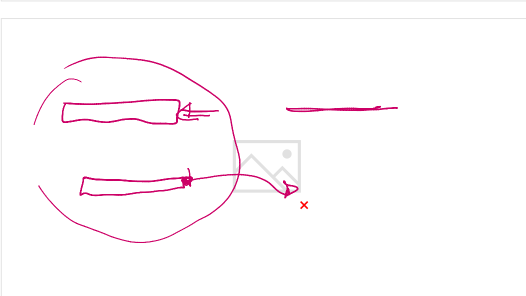
ULTIMO
PERIODO

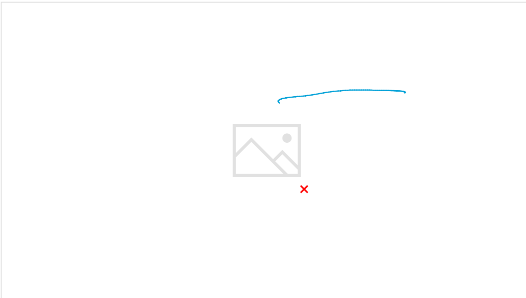
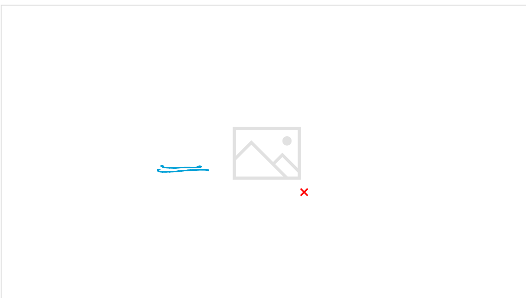


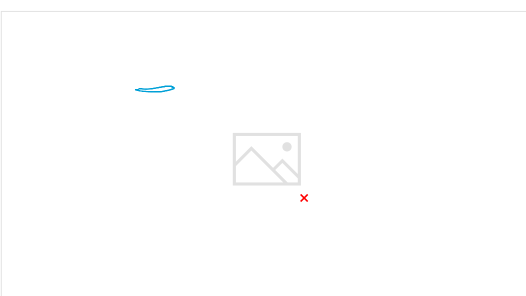
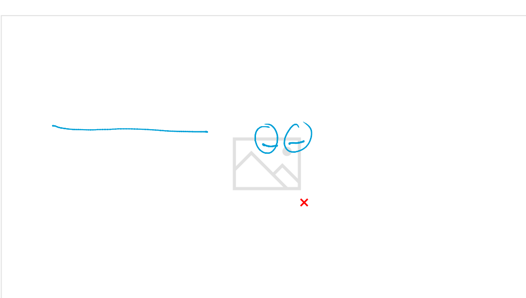
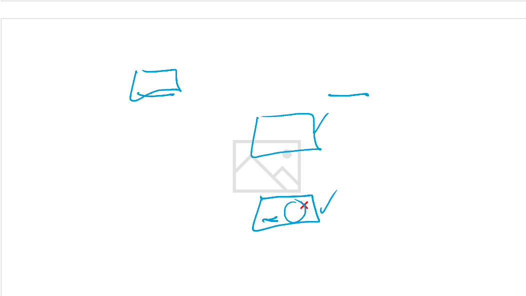




$EN = \{(B_1, B_2), (C_1, C_2)\}$

10+

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no me desvio si

$10+3\delta \geq 11+\delta$

$2\delta \geq 1$

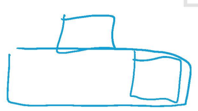
$\boxed{\delta \geq 1/2}$

SIGUE $S1$

$10+\delta \geq 9+3\delta$

$1 \geq 2\delta$

$\boxed{\delta \leq 1/2}$



- The key here is that player 2 by breaking the agreement in period 1 moves the period 2 play to his favored stage game NE of (C_1, C_2)

- Suppose we flipped the roles of B and C and considered the following strategy profile

- Player 1 plays the following strategy:
  1. A_1 in period 1;
  2. C_1 in period 2 if (A_1, A_2) was played in period 1;
  3. B_1 in period 2 if (A_1, A_2) was not played in period 1.

- Player 2 plays the following strategy:
  1. A_2 in period 1;
  2. C_2 in period 2 if (A_1, A_2) was played in period 1;
  3. B_2 in period 2 if (A_1, A_2) was not played in period 1.

- This is not a SPNE either because now player 1 has a definitive incentive to deviate from (A_1, A_2) in period 1

Stage Game

	A_2	B_2	C_2
A_1	$(10,10)$	$(0,9)$	$(0,9)$
B_1	$(11,-1)$	$(3,1)$	$(0,0)$
C_1	$(11,-2)$	$(0,0)$	$(1,3)$

- This is not a SPNE either because now player 1 has a definitive incentive to deviate from (A_1, A_2) in period 1

Stage Game

	A_2	B_2	C_2
A_1	$(10,10)$	$(0,9)$	$(0,9)$
B_1	$(11,-1)$	$(3,1)$	$(0,0)$
C_1	$(11,-2)$	$(0,0)$	$(1,3)$

- Player 1:

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Stage Game

	A_2	B_2	C_2
A_1	$(10,10)$	$(0,9)$	$(0,9)$
B_1	$(11,-1)$	$(3,1)$	$(0,0)$
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- Player 1:
  - If he follows: $u_1 = 10 + \delta$

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Stage Game

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A_1	$(10,10)$	$(0,9)$	$(0,9)$
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Stage Game

	A_2	B_2	C_2
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- Player 1:
  - If he follows: $u_1 = 10 + \delta$
  - If he defects: $u_1 = 11 + 3\delta$
  - Always defects

- So how do we construct a SPNE with (A_1, A_2) played in period 1?

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- So how do we construct a SPNE with (A_1, A_2) played in period 1?
- The key here is to notice that player 2 does not need to be punished in period 2 from breaking the agreement in period 1
- This is because in period 1 player 2 is best responding **myopically** at (A_1, A_2) already
- In other words, need to be punished **only if** the player has a deviation that benefits him **myopically** or in the short term

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- Player 1 plays the following strategy:
  1. A_1 in period 1;
  2. B_1 in period 2 if player 1 played A_1;
  3. C_1 in period 2 if player 1 played B_1 or C_1.
- Player 2 plays the following strategy:
  1. A_2 in period 1;
  2. B_2 in period 2 if player 1 played A_1;
  3. C_2 in period 2 if player 1 played B_1 or C_1.

$\Rightarrow$ ES EPS

$\delta \geq 1/2$

Stage Game

	A_2	B_2	C_2
A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
B_1	$(11, -1)$	$(3, 1)$	$(0, 0)$
C_1	$(11, -2)$	$(0, 0)$	$(1, 3)$

- Player 1:

Stage Game

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A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
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- Player 1:
  - If he follows: $u_1 = 10 + 3\delta$

Stage Game

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- Player 1:
  - If he follows: $u_1 = 10 + 3\delta$
  - If he defects: $u_1 = 11 + \delta$

Stage Game

	A_2	B_2	C_2
A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
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- Player 1:
  - If he follows: $u_1 = 10 + 3\delta$
  - If he defects: $u_1 = 11 + \delta$
  - Follows if $\delta \geq \frac{1}{2}$

Stage Game

	A_2	B_2	C_2
A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
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- Player 1:
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  - Follows if $\delta \geq \frac{1}{2}$
- Player 2:

Stage Game

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C_1	$(11, -2)$	$(0, 0)$	$(1, 3)$

- Player 1:
  - If he follows: $u_1 = 10 + 3\delta$
  - If he defects: $u_1 = 11 + \delta$
  - Follows if $\delta \geq \frac{1}{2}$
- Player 2:
  - If he follows: $u_2 = 10 + \boxed{X\delta}$

Stage Game

	A_2	B_2	C_2
A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
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- Player 2:
  - If he follows: $u_2 = 10 + X\delta$
  - If he defects: $u_2 = 9 + X\delta$

$10 + x\delta \geq 9 + x\delta$

Stage Game

	A_2	B_2	C_2
A_1	$(10, 10)$	$(0, 9)$	$(0, 9)$
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  - Follows if $\delta \geq \frac{1}{2}$
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  - If he follows: $u_2 = 10 + X\delta$
  - If he defects: $u_2 = 9 + X\delta$
  - Follows