Parcial 1a

Tuesday, February 25, 2020 8:51 AM

1 Consumidor
2 Insumos (L, K)
1 Bien Intermedio (X)
1 Bien Final (Y)

2013

$U(Y) = Y^{\alpha} \quad \alpha \in (0,1)$

$f_Y(L_Y, X) = L_Y^{1/2} X^{1/2}$

$f_X(L_X, K_X) = L_X^{1/2} K_X^{1/2}$

$\bar{K} = 1 \quad \bar{L} = 1 \quad w = 1$

Pruebe q' en EQ
$$\boxed{\begin{aligned} Y &= \frac{1+r}{2\sqrt{P_X}} \\ P_X &= 2r^{1/2} \\ P_Y &= 2P_X^{1/2} \end{aligned}}$$

Un EQ. Competitivo es: $\underbrace{(X^*, Y^*, L_X^*, L_Y^*, K_X^*)}_{\text{Asignación}}$ y $\underbrace{(P_Y^*, P_X^*, w^*, r^*)}_{\text{Precios}}$ T.Q:

1) Firmas Maximizan:

$$L_Y^*, X^* = \arg\max_{L_Y, X} \Pi_Y = P_Y^* L_Y^{1/2} X^{1/2} - L_Y w^* - X P_X^*$$

$$L_X^*, K_X^* = \arg\max_{L_X, K_X} \Pi_X = P_X^* L_X^{1/2} K_X^{1/2} - L_X w^* - K_X r^*$$

2) Consumidores Maximizan

$$Y^* = \arg\max_{Y} Y^{\alpha} \quad \text{s.a} \quad \underbrace{\underset{1}{\bar{L}} w^* + \underset{1}{\bar{K}} r^* + \Pi_X^* + \Pi_Y^*}_{\text{Ingreso}} \geq \underbrace{P_Y^* Y}_{\text{Gasto}}$$

3) Mercados se Vacían

$$\underbrace{L_X + L_Y}_{DD} = \underbrace{\bar{L}}_{OO} \longrightarrow L_X + L_Y = 1$$

$$K_X = \bar{K} \longrightarrow K_X = 1$$

$$\underbrace{X^O}_{\text{Oferta}} = X^D \Longrightarrow L_X^{1/2} K_X^{1/2} = X^*$$

$\longrightarrow X$ que Demanda

$$\underbrace{X^O}_{\text{OFERTA LA FIRMA QUE OFERTA } X} = X^D \Rightarrow L_x^{1/2} K_x^{1/2} = X$$

$X^D \Rightarrow$ X QUE DEMANDA LA FIRMA Q' PRODUCE Y

$$\underbrace{Y^D}_{\text{DD CONSUMIDOR}} = Y^O \Rightarrow Y^* = L_y^{1/2} X^{1/2}$$

$Y^O \Rightarrow$ FIRMA

Solucion:

FIRMA X

$$P_x L_x^{1/2} K_x^{1/2} - w L_x - r K_x$$

CPO

$$\frac{1 P_x}{2} L_x^{-1/2} K_x^{1/2} - w = 0$$

$$\frac{1}{2} P_x L_x^{1/2} K_x^{-1/2} - r = 0$$

$$\frac{K_x}{L_x} = \frac{w}{r}$$

$$\boxed{K_x = 1} \quad \boxed{w = 1}$$

$$\frac{1}{L_x} = \frac{1}{r}$$

$$\boxed{L_x = r}$$

$$P_x L_x^{1/2} K_x^{1/2} - w L_x - r K_x = 0$$

(EN EQ) $\Rightarrow$ POR RENDIMIENTOS CONSTANTES A ESCALA

$$f(\lambda L_x, \lambda K_x) = \lambda^{1/2} L_x^{1/2} \lambda^{1/2} K_x^{1/2}$$
$$= \lambda L_x^{1/2} K_x^{1/2}$$
$$= \lambda f(L_x, K_x)$$

$$P_x (r)^{1/2} 1 - 1(r) - r(1) = 0$$

$$P_x r^{1/2} - 2r = 0$$

$$\boxed{P_x = 2r^{1/2}}$$

FIRMA Y

$$P_y L_y^{1/2} X^{1/2} - w L_y - P_x X$$

CPO

$$P_y \frac{1}{2} L_y^{-1/2} X^{1/2} - w = 0$$

$$P_y \frac{1}{2} L_y^{1/2} X^{-1/2} - P_x = 0$$

$$\frac{X}{L_y} = \frac{w}{P_x} = \frac{1}{P_x}$$

EN EQ: $X = r^{1/2}$, $P_x = 2r^{1/2}$

$$\frac{r^{1/2}}{L_y} = \frac{1}{2r^{1/2}}$$

$$\boxed{2r = L_y}$$

EN EQ, POR RENDIMIENTOS CONSTANTES A ESCALA:

$$P_y L_y^{1/2} X^{1/2} - w L_y - X P_x = 0$$

$$P_y (2r)^{1/2} (r^{1/2})^{1/2} - 1 \cdot 2r - r^{1/2}(2r^{1/2}) = 0$$

$$P_y \, 2^{1/2} r^{1/2} r^{1/4} - 2r - 2r = 0$$

$\boxed{P_x = 2r^{1/2}}$

$X^o = r^{1/2} \, 1^{1/2} = \underbrace{L_x}_{r}^{1/2} \underbrace{K_x}_{1}^{1/2}$

$X^o = r^{1/2}$

$P_y \, 2^{1/2} r^{1/2} r^{1/4} - 2r - 2r = 0$

$P_y \, 2^{1/2} r^{3/4} = 4r$

$\boxed{P_y = 2^{3/2} r^{1/4}}$

$P_x = 2r^{1/2}$

$\sqrt{P_x} = \quad 2^{1/2} r^{1/4}$

$\boxed{P_y = 2 \cdot \sqrt{P_x}}$

$\boxed{Y^o = L_y^{1/2} X^{1/2} = (2r)^{1/2} (r^{1/2})^{1/2} = 2^{1/2} r^{3/4}}$

Consumidores

$$\max_{Y} \; Y^{\alpha} \quad s.a \quad w\bar{L} + r\bar{K} + \Pi_x + \Pi_y \geq P_y Y$$

$$Y^* = \frac{w\bar{L} + r\bar{K} + \Pi_x + \Pi_y}{P_y}$$

En EQ: $\Pi_x = 0 = \Pi_y$

$w = 1$

$P_y = 2\sqrt{P_x}$

$$Y^* = \frac{1 \cdot 1 + r \cdot 1}{2\sqrt{P_x}} = \frac{1 + r}{2\sqrt{P_x}}$$

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③ Mcdos Vacien

$\boxed{K_x = 1}$ ✓

$L_x + L_y = 1 = r + 2r = 1 \Rightarrow 3r = 1 \Longrightarrow r = 1/3$

$X^o = X^D$ ✓

$r = 1/3, \; L_x = 1/3, \; L_y = 2/3, \; K_x = 1$

$r = 1/3,\ L_x = 1/3,\ L_y = 2/3,\ K_x = 1$

$X = \left(\frac{1}{3}\right)^{1/2} \quad P_x = 2\left(\frac{1}{3}\right)^{1/2},\ P_y = 2\,2^{1/2}\left(\frac{1}{3}\right)^{1/4} = 2^{3/2}\left(\frac{1}{3}\right)^{1/4}$

$y^o = 2^{1/2}\left(\frac{1}{3}\right)^{3/4} = \frac{2^{1/2}}{3^{3/4}}$ ✓ $\quad y^D = \frac{1+1/3}{2\,2^{1/2}\left(\frac{1}{3}\right)^{1/4}} = \frac{4/3}{2^{3/2}\left(\frac{1}{3}\right)^{1/4}} = \frac{2^2\,3^{1/4}}{2^{3/2}(3)} = \frac{2^{1/2}}{3^{3/4}}$

LA LEY DE WALRAS ✓

PROBLEMA ⟹ O.P.

$$\max_{Y, L_y, L_x, X, K_x} y^{\alpha} \quad \text{s.a.} \quad Y = L_y^{1/2} X^{1/2}$$

$$X = L_x^{1/2} K_x^{1/2}$$

$$L_x + L_y = L$$

$$K_x = 1$$

$U(x,y,h) = xyh$

$T = 200 = h + L$

$f_x(L_x) = L_x^{1/2}$

$f_y(L_y) = L_y^{1/2}$

a) $L < 200$, ENCONTRAR FPP

MAX $\underbrace{L_y^{1/2}}_{f_y}$ s.a. $f_x(L_x) = L_x^{1/2} \geq \bar{X}$

$L = L_x + L_y < 200$

$X^2 = L_x$

$Y^2 = L_y$

$\Longrightarrow L_x + L_y = L < 200$

$X^2 + Y^2 < 200$

$Y \leq \sqrt{200 - X^2}$