Lecture10

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Lecture10

Lecture 10: Game Theory // Preliminaries

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Introduction

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s.t.

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- Game theory is a formal methodology and a set of techniques to study: rational agents in strategic settings. ← CLAVE
- 'Rational': maximizing over well-defined objectives
- 'strategic': agents care about the actions taken by other agents (MIS ACCIONES LE IMPORTAN A OTROS)
- In general equilibrium theory, agents are price takers and solve

$$\max_x u(x) \longrightarrow u(x, x_{-i})$$

x_{-i} → DEMAS

s.t.

$$p \cdot x \leq p \cdot w,$$

- Agents decisions do not affect p, and thus there is no **strategic** interaction
- Although p is determined from the interaction of all agents (aggregate supply = aggregate demand)

Definition (Strategic Interaction)

There is *strategic interaction* when an agent takes into account how her actions affect other individuals and how other's action affect her

- Originally, game theory was developed to design optimal strategies in games like chess or poker

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$x_i \rightarrow U_{-i}$

$x_{-i} \rightarrow U_i$

- Originally, game theory was developed to design optimal strategies in games like chess or poker

- However, it allows to study a wide range of situations that were did not fit in traditional microeconomics theory

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History in one slide

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- In the early 1950's, John Nash made his seminal contributions to non-zero-sum games and started bargaining theory
- In 1967–1968, John Harsanyi formalized methods to study games of incomplete information
- In the 1970s, game theory became part of main stream economics (and other social sciences)

Strategic situations and their representation

A game is the description of a strategic situation. To describe a game we need to describe the following elements:

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- The information available to each player

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- The rule of the game: a) What actions are available to each player (at each decision point), and b) the order in which players take those actions
- The information available to each player
- How the results of the game depends on the actions taken by each individual
- How individuals value the results of the game

2 FIRMAS COMPITEN PUBLICIDAD

A few examples

Example (Matching pennies (pares y nones) – Sequential)
Two players, Ana & Bart, choose whether to show one or two fingers. First, Ana shows fingers to Bart, then Bart, after observing Ana's play, chooses how many fingers to show. If the total number of fingers is even, then Bart pays Ana 1,000 MXN. If the total number of fingers is odd, then Ana pays Bart 1,000 MXN.

A few examples

Example (Matching pennies (pares y nones) – Simultaneous)
Two players, Ana & Bart, choose whether to show one or two fingers simultaneously. If the total number of fingers is even, then Bart pays Ana 1,000 MXN. If the total number of fingers is odd, then Ana pays Bart 1,000 MXN.

Lecture 10: Game Theory // Preliminaries

Introduction
- Assumptions
- Notation
- Strategies Vs Actions

- We assume agents maximize their expected utility

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  - Have a well defined utility function

- We assume agents maximize their expected utility
  - Have a well defined utility function
  - Under uncertainty they maximize the expected utility

SUPUESTO FUERTE!

$\mathbb{E}(U_i)$

$\cancel{\neq U(\mathbb{E}())}$

- Not a trivial assumption

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- $$x^* = \arg\max_{x \cdot p \leq w \cdot p} u(x) = \arg\max_{x \cdot p \leq w \cdot p} f(u(x)),$$
  for any increasingly monotone f

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  for any increasingly monotone f
- If x^* solves
  $$\max_{x \cdot p \leq w \cdot p} \mathbb{E}u(x)$$
  it does not necessarily solve
  $$\max_{x \cdot p \leq w \cdot p} \mathbb{E}f(u(x))$$

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- In other words, the specific utility function has important repercussions

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- The first pays 10 with probability 0.5 y 0 with probability 0.5 and costs 5
- The second pays 100 with probability 0.5 y 0 with probability 0.5 and costs 50

$1/2 \to 5 = (10-5)$

$1/2 \to -5 = (0-5)$

$\mathbb{E}(x) = \frac{1}{2}5 + \frac{1}{2}(-5) = 0$

$1/2$ $\quad$ $1/2$

$50 = 100-50$ $\quad$ $-50 = (0-50)$

$\to \mathbb{E}(x) = 0 = \frac{1}{2}50 + \frac{1}{2}(-50) = 0$

$\cancel{U(\mathbb{E}(x))}$

$\cancel{(1)\ U_1(0) = \ln(0+51)}$

$\cancel{(2)\ U_2(0) = 0+51}$

$\cancel{(3)\ U_3(0) = e^{0+51}}$

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- Assume there are three agents with utility functions: $u^1(x) = \ln(x+51), u^2(x) = x+51, u^3(x) = e^{x+51}$

$\boxed{x = \$}$

→ Mismas Pref. Neoclásicas

$e^{U^1} = U^2 \qquad e^{U_2} = U_3$

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- The first pays 10 with probability 0.5 y 0 with probability 0.5 and costs 5
- The second pays 100 with probability 0.5 y 0 with probability 0.5 and costs 50
- The only difference is the monetary units they use
- Assume there are three agents with utility functions: $u^1(x) = \ln(x+51), u^2(x) = x+51, u^3(x) = e^{x+51}$
- All 3 agents have the "same preferences"

$\frac{1}{2}U(5) + \frac{1}{2}U(-5) = \mathbb{E}(U(\text{Lotería } 1))$

$\frac{1}{2}\ln(56) + \frac{1}{2}\ln(46)$

$\frac{1}{2}U(50) + \frac{1}{2}U(-50)$

Utility	Lottery 1	Lottery 2
$\mathbb{E}u^1$	$0.5\ln(56) + 0.5\ln(46) \approx 3.92$	$0.5\ln(101) + 0.5\ln(1) \approx 2.3$
$\mathbb{E}u^2$	$0.5(56) + 0.5(46) = 51$	$0.5(101) + 0.5(1) = 51$
$\mathbb{E}u^3$	$0.5e^{56} + 0.5e^{46} \approx 1.04 \times 10^{24}$	$0.5e^{101} + 0.5e^{1} \approx 3.65 \times 10^{43}$

→ ÚTILES

- If $x^* = \arg\max_{x\in\Gamma} \mathbb{E}u(x)$

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- Then $x^* = \arg\max_{x\in\Gamma} \mathbb{E}au(x) + b$

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- Then $x^* = \arg\max_{x\in\Gamma} \boxed{\mathbb{E}au(x) + b}$
- Proof that linear (or afine) transformations of the utility function represent the same preferences under uncertainty.

$\cancel{e^{xy} \to xy}$

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- All hats are white, but no one knows their own color (just that it's black or white)

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- Now they go around trying to guess their own color. If they get it correctly they earn all sorts of riches, but if they don't they **die**. They can either guess or pass

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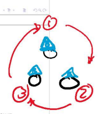
- What happens?



1. PASO
2. PASO
3. PASO
1. PASO
⋮

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- Suppose there are 3 players and "god" places a hat over them
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- All hats are white, but no one knows their own color (just that it's black or white)
- Now they go around trying to guess their own color. If they get it correctly they earn all sorts of riches, but if they don't they **die**. They can either guess or pass
- What happens?
- They go around for ever saying "pass"

- Now suppose "god" says: There is at least one white hat

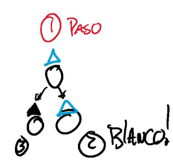
1. PASO
2. PASO
3. BLANCO

3 "mmm, y si mi sombrero es negro"

↳ 1 PASO

2 mmm, como 1 no dijo blanco → mio blanco

3 como 2 paso → 3 es blanco



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- They already knew everyone knew there was at least a white hat

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- What happens?
- The first two pass, the third says "white"
- Why?
- They already knew there was at least a white hat (they knew there were at least two)
- They already knew everyone knew there was at least a white hat
- Now they all now, that everyone knows, that everyone knows (ad infinitum) that there is a white hat.

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- We say Y is common knowledge when all players know Y, and they all know that everyone knows Y, and they all know that everyone knows that everyone knows Y.... ad infinitum

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- This highlights the difference between *mutual knowledge* e common knowledge
- We say Y is common knowledge when all players know Y, and they all know that everyone knows Y, and they all know that everyone knows that everyone knows Y.... ad infinitum
- We will always assume things are common knowledge (there are some extensions to the cases when utility functions are not common knowledge)

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Introduction
Assumptions
Notation
Strategies Vs Actions

We will use the following notation:

- Game participants (players) will be denoted by index i, where $i = 1, .., N$ and there are N players.
- A_i is the space of possible actions for individual i. $a_i \in A_i$ is an action.
- If we have a vector $a = (a_1, ..., a_{i-1}, a_i, a_{i+1}, ..., a_N)$, then we will denote by $a_{-i} := (a_1, ..., a_{i-1}, a_{i+1}, ..., a_N)$ y $a = (a_i, a_{-i})$. → DEMAS → YO
- S_i is the strategy space for individual i. $s_i \in S_i$ is a strategy.
- A strategy is a complete action plan. i.e., is an action for every possible contingency of the game a player may face.
- u^i is the utility of player i. $u_i(s_i, s_{-i})$, i.e., the utility of player i may depend on her strategy, as well as the strategy of other players.

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- Think of matching pennies – Sequential.
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- A strategy for Ana is an action (she chooses first, and thus faces a single contingency) $S_{ana} = A_{ana}$

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- Think of matching pennies – Sequential.
- The actions for both individuals are $A_i = \{1, 2\}$
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- For Bart, a strategy has an action for the two contingencies he may face (1) if Ana chooses 1 finger, (2) if Ana chooses 2 fingers

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- *The difference between strategy and actions is* **VERY** *important*
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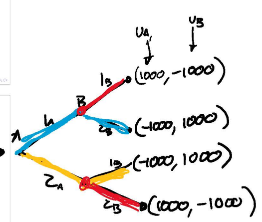
- The actions for both individuals are $A_i = \{1, 2\}$
- A strategy for Ana is an action (she chooses first, and thus faces a single contingency) $S_{ana} = A_{ana}$
- For Bart, a strategy has an action for the two contingencies he may face (1) if Ana chooses 1 finger, (2) if Ana chooses 2 fingers
- $S_{Bart} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$



$A_A = \{1_A, 2_A\}$

$A_B = \{1_B, 2_B\}$

$S_A = \{1_A, 2_A\}$.

$S_B = \{(1_B, 1_B); \boxed{(1_B, 2_B)}; \boxed{(2_B, 1_B)}; (2_B, 2_B)\}$.

→ "ÓPTIMA"

→ "PEOR"

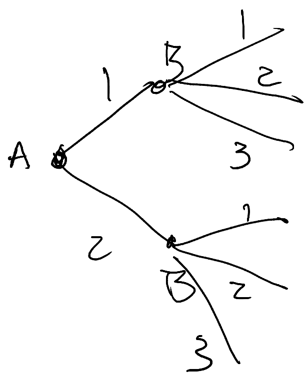
Q' HACE BART

Q' HACE SI ANA

O' hace BART si ANA JUEGA 1

O' hace si ANA JUEGA 2

"PEOR"



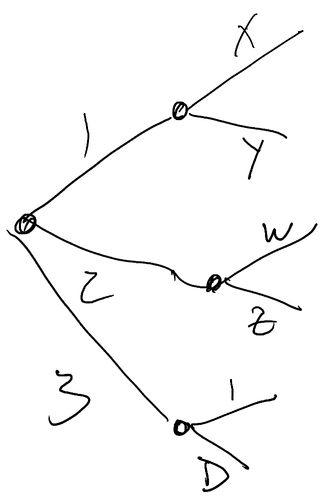
$S_A = \{1, 2\}$

ANA JUGO 1 — ANA JUGO 2

$S_B = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) \}$

(,)

3op X 3op = 9



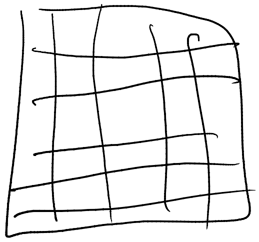
$S_A = \{1, 2, 3\}$

$S_B = \{ (X, W, I), (X, W, D), (Y, W, I), (Y, W, D), (X, Z, I), \boxed{(X, Z, D)}, (Y, Z, I), (Y, Z, D) \}$

SI ANA 1 — SI ANA 2 — SI ANA 3

2op X 2op X 2op

$2^3 = 8$



2 RONDAS

$$|S_B| = 20$$

20 OPCIONES 20 OPCIONES

$$S_N \overset{E.G.}{=} (\underbrace{\_, \_, \_, \ldots, \_}_{20 \text{ ENTRADAS}})$$

$$20 \times 20 \times 20 \cdots \times 20$$

$$|S_N| = 20^{20}$$