# Lecture 12

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Lecture12

## Lecture 12: Game Theory // Nash equilibrium

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### Beauty contest

- Consider the following game among 100 people. Each individual selects a number, $s_i$, between 20 and 60.
- Let $a_{-i}$ be the average of the number selected by the other 99 people. i.e. $\boxed{a_{-i} = \sum_{j \neq i} \frac{s_j}{99}}$.
- The utility function of the individual $i$ is $\boxed{u_i(s_i, s_{-i}) = 100 - (\underbrace{s_i - \tfrac{3}{2}a_{-i}}_{+})^2}$

OBJETIVO:
ATINARLE 3/2
DE LO Q' DIGAN
LOS DEMAS

### Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

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## Beauty contest

$a_{-i}$

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- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$
- but $a_{-i} \in [20, 60]$ $\frac{3}{2}a_{-i} \in [\frac{3}{2}20, \frac{3}{2}60] = [30, 90]$

## Beauty contest

- Each individual maximizes his utility, FOC:

$$-2(s_i - \frac{3}{2}a_{-i}) = 0$$

- Individuals would prefer to select a number that is exactly equal to 1.5 times the average of the others
- That is they would like to choose $s_i = \frac{3}{2}a_{-i}$
- but $a_{-i} \in [20, 60]$
- Therefore $s_i = 20$ is dominated by $s_i = 30$

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- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45

$\to a_{-i} \in [30, 60]$

$\frac{3}{2} a_{-i} \in [45, 90]$

## Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)

$\to \frac{3}{2} a_{-i} \in [\frac{3}{2} 45, \frac{3}{2} 60]$

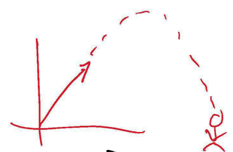
$\frac{3}{2} a_{-i} \in [67.5, 90]$

## Beauty contest

- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.

## Beauty contest

$(20, 60)$



- The same goes for any number between 20 (inclusive) and 30 (not included)
- Knowing this, all individuals believe that everyone else will select a number between 30 and 60 (i.e., $a_{-i} \in [30, 60]$)
- Playing a number between 30 and 45 (not including) would be strictly dominated by playing 45
- Knowing this, all individuals believe that everyone else will select a number between 45 and 60 (i.e., $a_{-i} \in [45, 60]$)
- 60 would dominate any other selection and therefore all the players select 60.
- The solution by means of iterated elimination of dominated strategies is $\underbrace{(60, 60, \ldots, 60)}_{100 \text{ times}}$

$\to \boxed{S = (60, 60, \ldots, 60)}$ No es O.P.

$$P.C \to \max \; 100 - \left(s_1 - \tfrac{3}{2} a_{-1}\right)^2 \quad s.a \quad \begin{aligned} 100 - \left(s_2 - \tfrac{3}{2} a_{-2}\right)^2 &\geq \bar{U}_2 : \lambda_2 \\ 100 - \left(s_3 - \tfrac{3}{2} a_{-3}\right)^2 &\geq \bar{U}_3 : \lambda_3 \\ &\vdots \\ 100 - \left(s_{100} - \tfrac{3}{2} a_{-100}\right)^2 &\geq \bar{U}_{100} : \lambda_{100} \end{aligned}$$

$$a_{-2} = \frac{s_1 + s_3 + \ldots + s_{100}}{99}$$

$$\frac{\partial y}{\partial s_1} = -2\left(s_1 - \tfrac{3}{2} a_{-1}\right) - \lambda_2\left(-2\left(s_2 - \tfrac{3}{2} a_{-2}\right)\left(-\tfrac{3}{2} \cdot \tfrac{1}{99}\right)\right) = 0 \longrightarrow -2\left(s - \tfrac{3}{2} s\right) - \lambda_2\left(-2\left(s - \tfrac{3}{2} s\right)\left(\tfrac{-3}{2 \cdot 99}\right)\right) = 0 \to s =$$

$$\frac{\partial y}{\partial s_2} \quad -2\left(s_1 - \tfrac{3}{2} a_{-1}\right)\left(-\tfrac{3}{2} \cdot \tfrac{1}{99}\right) + \lambda_3\left(-2\left(s_3 - \tfrac{3}{2} a_{-3}\right)\right) = 0$$

$$\cancel{-2\left(s - \tfrac{3}{2} s\right)} = \lambda_2\left(\cancel{-2\left(s - \tfrac{3}{2} s\right)}\left(\tfrac{-3}{2 \cdot 99}\right)\right)$$

$$\vdots$$

$$\frac{\partial y}{\partial s_{100}} = -2\left(s_1 - \tfrac{3}{2} a_{-1}\right)\left(-\tfrac{3}{2} \tfrac{1}{99}\right) + \lambda_{100}\left(-2\left(s_{100} - \tfrac{3}{2} a_{-100}\right)\right) = 0$$

$$\lambda_2 = \frac{2 \cdot 99}{-2}$$

## Lecture 12: Game Theory // Nash equilibrium

Dominance
- Weakly dominated strategies

Nash equilibrium

Some examples

Relationship to dominance

Examples
- Cournot Competition
- Cartels

| | a | b |
|---|---|---|
| A | 3, 4 | 4, 3 |
| B | 5, 3 | 3, 5 |
| C | 5, 3 | 4, 3 |

- There is no strictly dominated strategy

| | a | b |
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- There is no strictly dominated strategy
- However, $C$ always gives at least the same utility to player 1 as $B$

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- It's tempting to think player 1 would never play $C$

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- There is no strictly dominated strategy
- However, $C$ always gives at least the same utility to player 1 as $B$
- It's tempting to think player 1 would never play $C$
- However, if player 1 is sure that player two is going to play $a$ he would be completely indifferent between playing $B$ or $C$

Definition

$s_i$ weakly dominates $s_i'$ if for all opponent pure strategy profiles, $s_{-i} \in S_{-i}$,

$$u_i(s_i, s_{-i}) \boxed{\geq} u_i(s_i', s_{-i})$$

and there is at least one opponent strategy profile $s''_{-i} \in S_{-i}$ for which

$$u_i(s_i, s''_{-i}) > u_i(s_i', s''_{-i}).$$

- Given the assumptions we have, we can not eliminate a weakly dominated strategy

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- Rationality is not enough

$$\frac{\partial y}{\partial s_{100}} = -2\left(s_1 - \frac{3}{2}a_{-1}\right)\left(-\frac{3}{2}\frac{1}{99}\right) + \lambda_{100}\left(-2\left(s_{100} - \frac{3}{2}a_{-100}\right)\right) - \ldots$$

$$\boxed{s_1 = s_2 = \cdots = s_{100} = s}$$

$$\lambda_2 = \frac{2.99}{-3}$$

$$S = (20, 20, \ldots, 20)$$

$$U_i(20, \ldots, 20) \geq U_i(60, 60, \ldots, 60)$$

- Given the assumptions we have, we can not eliminate a weakly dominated strategy
- Rationality is not enough
- Even so, it sounds "logical" to do so and has the potential to greatly simplify a game

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- Given the assumptions we have, we can not eliminate a weakly dominated strategy
- Rationality is not enough
- Even so, it sounds "logical" to do so and has the potential to greatly simplify a game
- There is a problem, and that is that the order in which we eliminate the strategies matters

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| | a | b |
|---|---|---|
| [redacted] | [redacted] | [redacted] |
| C | 5, 3 | 4, 3 |

$C \geq B \Rightarrow a \geq b \Rightarrow C \gg A \rightarrow \boxed{Sol = (C, a)}$

$C \geq A \Rightarrow b \geq a \Rightarrow C \gg B \rightarrow \boxed{Sol = (C, b)}$

$C \geq A$ & $C \geq B \Rightarrow a \sim b \rightarrow$ No Soluble.

- If we eliminate $B$ ($C$ dominates weakly), then $a$ weakly dominates $b$ and we can eliminate $b$ and therefore player 1 would never play A. This leads to the result $(C, a)$.

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| | a | b |
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| A | 3, 4 | 4, 3 |
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| C | 5, 3 | 4, 3 |

- If we eliminate $B$ ($C$ dominates weakly), then $a$ weakly dominates $b$ and we can eliminate $b$ and therefore player 1 would never play A. This leads to the result $(C, a)$.
- If on the other hand, we notice that $A$ is also weakly dominated by $C$ then we can eliminate it in the first round, and this would eliminate $a$ in the second round and therefore $B$ would be eliminated. This would result in $(C, b)$.

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Remember the definition of competitive equilibrium in a market economy.

**Definition**

A competitive equilibrium in a market economy is a vector of prices and baskets $x_i$ such that: 1) $x_i$ maximizes the utility of each individual given the price vector i.e.

$$x_i = \arg \max_{p \ cdot x_i \leq p \cdot w_i} u(x_i)$$

2) the markets empty.

$$\sum_i x_i = \sum_i w_i$$

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- 1) means that given the prices, individuals have no incentive to demand a different amount

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- 1) means that given the prices, individuals have no incentive to demand a different amount
- The idea is to extend this concept to strategic situations

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## Best response

$MR_i(s_{-i})$

We denote $\boxed{BR_i(s_{-i})}$ (best response) as the set of strategies of individual $i$ that maximize her utility given that other individuals follow the strategy profile $s_{-i}$. Formally,

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**Definition**

Given a strategy profile of opponents $s_{-i}$, we can define the best response of player $i$:

$$\boxed{BR_i(s_{-i}) = \arg \max_{s_i' \in S_i} u_i(s_i', s_{-i})}$$

- $s_i \in BR_i(s_{-i})$ if and only if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$

$$\max_x f(x) = f(x^*)$$

$$\arg \max_x f(x) = x^*$$

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- $s_i \in BR_i(s_{-i})$ if and only if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$
- There could be multiple strategies in $BR_i(s_{-i})$ but all such strategies give the same utility to player $i$ if the opponents are indeed playing according to $s_{-i}$

## Nash equilibrium

Normal

**Definition**
Suppose that we have a game $(I = \{1, 2, \ldots n\}, S_1, \ldots, S_n, u_1, \ldots, u_n)$. Then a strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a pure strategy **Nash equilibrium** if for every $i$ and for every $s_i \in S_i$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*).$$

$$\boxed{s_i^* \in MR_i(s^*_{-i})}$$

## Nash equilibrium

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- Analogous to that of a competitive equilibrium in the sense that nobody has unilateral incentives to deviate

## Nash equilibrium

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- once this equilibrium is reached, nobody has incentives to move from there

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- Analogous to that of a competitive equilibrium in the sense that nobody has unilateral incentives to deviate
- once this equilibrium is reached, nobody has incentives to move from there
- This is a concept of stability, but there is no way to ensure, or predict, that the game will reach this equilibrium

## Lecture 12: Game Theory // Nash equilibrium



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## Beauty contest

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- Let $s_{-i}$ be the number selected by the other individual.
- The utility function of the individual $i$ is $\boxed{u_i(s_i, s_{-i}) = 100 - (s_i - \frac{3}{2}s_{-i})^2}$

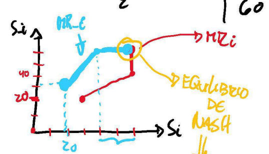
$U_1(S_1,S_2) = 100 - (S_1 - \frac{3}{2}S_2)^2$

$U_2(S_1,S_2) = 100 - (S_2 - \frac{3}{2}S_1)^2$

$MR_i(S_{-i}) \Rightarrow$ JUGADA ÓPTIMA

$$\frac{\partial U(S_i,S_{-i})}{\partial S_i} = -2\left(S_i - \frac{3}{2}S_{-i}\right) = 0$$

$$S_i^* = \frac{3}{2}S_{-i} \Rightarrow S_i^* = \begin{cases} \frac{3}{2}S_{-i} & \text{si } S_{-i} \in [20,40] \\ 60 & \text{si } S_i \geq 40 \end{cases}$$




$$S_{-i}^* = \begin{cases} \frac{3}{2}S_i & \text{si } S_i \in [20,40] \\ 60 & \text{si } S_i \geq 40 \end{cases}$$

$$U_{-i}(S_i,S_{-i}) = 100 - \left(S_{-i} - \frac{3}{2}S_i\right)^2$$

## Beauty contest

The best response of an individual is given by

$$s_i(s_{-i})^* = \begin{cases} \frac{3}{2}s_{-i} & \text{if } s_{-i} \leq 40 \\ 60 & \text{if } s_{-i} > 40 \end{cases}$$

The Nash equilibrium is where both BR functions intersect (i.e., when both play 60)

$$\boxed{\begin{matrix} S^* = (60, 60) \\ \quad\ \uparrow \quad\ \ \uparrow \\ \quad\ S_1^* \quad S_2^* \end{matrix}}$$

$\boxed{U_i(20,20) > U(60,60)}$

$U_1(30,20) > U_1(20,20) \rightarrow$ SÍ INCENTIVOS UNILATERALES A MOVERSE

$U_1(S_i, 60) \leq U_1(60,60)$

## Prisoner's dilemma

$S_2$

$S_1$

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

$MR_1(S_2) = NC$

$\boxed{MR_2(S_1) = NC}$

$MR_1(S_2 = C) = NC$

$MR_1(S_2 = NC) = NC$

$\boxed{EN = (NC, NC)}$

## Prisoner's dilemma

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

The best response functions are:

$$BR_i(s_{-i}) = \begin{cases} NC & \text{if } s_{-i} = C \\ NC & \text{if } s_{-i} = NC \end{cases}$$

The Nash equilibrium is where both BR functions intersect (i.e., when both play NC, i.e., $(NC, NC)$)

## Prisoner's dilemma – A trick

Best response of 1 to 2 playing C

J2

J1

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

$S^* = (NC, NC)$

E.N.

## Prisoner's dilemma – A trick

Best response of 1 to 2 playing NC

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

## Prisoner's dilemma – A trick

Best response of 2 to 1 playing C

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

## Prisoner's dilemma – A trick

Best response of 2 to 1 playing NC

| | C | NC |
|---|---|---|
| C | 5,5 | 0,10 |
| NC | 10,0 | 2,2 |

When underlined for both players, it is a Nash equilibrium (both are doing their BR)

## Battle of the sexes

J2

J1

| | G | P |
|---|---|---|
| G | 2,1 | 0,0 |
| P | 0,0 | 1,2 |

EN = { (G,G); (P,P) }

## Battle of the sexes

| | G | P |
|---|---|---|
| G | 2,1 | 0,0 |
| P | 0,0 | 1,2 |

$$BR_i(s_{-i}) = \begin{cases} G & \text{if } s_{-i} = G \\ P & \text{if } s_{-i} = P \end{cases}$$

## Battle of the sexes

| | G | P |
|---|---|---|
| G | 2,1 | 0,0 |
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$$BR_i(s_{-i}) = \begin{cases} G & \text{if } s_{-i} = G \\ P & \text{if } s_{-i} = P \end{cases}$$

Thus, $(G, G)$ y $(P, P)$ are both Nash equilibrium

## Matching pennies (Pares o Nones) – Simultaneous

$s_2$

$s_1$

| | 1 | 2 |
|---|---|---|
| 1 | (1000,-1000) | (-1000,1000) |
| 2 | (-1000,1000) | (1000,-1000) |

## Matching pennies (Pares o Nones) – Simultaneous

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| 1 | (1000,-1000) | (-1000,1000) |
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## Matching pennies (Pares o Nones) – Simultaneous

| | 1 | 2 |
|---|---|---|
| 1 | (1000,-1000) | (-1000,1000) |
| 2 | (-1000,1000) | (1000,-1000) |

$$BR_1(s_2) = \begin{cases} 1 & \text{if } s_2 = 1 \\ 2 & \text{if } s_2 = 2 \end{cases}$$

$$BR_2(s_1) = \begin{cases} 2 & \text{if } s_1 = 1 \\ 1 & \text{if } s_2 = 2 \end{cases}$$

There is no Nash equilibrium in pure strategies

## Lecture 12: Game Theory // Nash equilibrium



## Lecture 12: Game Theory // Nash equilibrium

## Nash equilibrium survive IDSDS

**Theorem**
*Every Nash equilibrium survives the iterative elimination of strictly dominated strategies*

TODO ES ELIMINADO

## Proof

By contradiction:

- Suppose it is not true

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- Without loss of generality say we eliminated the strategy $s_i^*$ of individual $i$

## Proof

By contradiction:

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- Lets zoom in in the round where we first eliminate a strategy that is part of $s^*$
- Without loss of generality say we eliminated the strategy $s_i^*$ of individual $i$
- It must have been that

$$u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \boxed{\forall s_{-i} \in S_{-i}}$$

## Proof

By contraction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium $s^*$
- Lets zoom in in the round where we first eliminate a strategy that is part of $s^*$
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$$u_i(s_i^*, s_{-i}) < u_i(s_i, s_{-i}) \boxed{\forall s_{-i} \in S_{-i}}$$

Si Domina a $S_i^*$

.

- In particular

$$u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)$$

$\Rightarrow S_i^* \notin MR_i(S_{-i}^*)$

## Proof

By contradiction:

- Suppose it is not true
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- Lets zoom in in the round where we first eliminate a strategy that is part of $s^*$
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$$u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)$$

- But this means $s_i^*$ is not the best response of individual $i$ to $s_{-i}^*$

## Proof

By contradiction:

- Suppose it is not true
- Then we must have eliminated some strategy in the Nash equilibrium $s^*$
- Lets zoom in in the round where we first eliminate a strategy that is part of $s^*$
- Without loss of generality say we eliminated the strategy $s_i^*$ of individual $i$
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.

- In particular

$$u_i(s_i^*, s_{-i}*) < u_i(s_i, s_{-i}^*)$$

- But this means $s_i^*$ is not the best response of individual $i$ to $s_{-i}^*$
- And this is a contradiction!

## Nash equilibrium survive IDSDS

**Theorem**

*If the process of IDSDS comes to a single solution, that solution is a Nash Equilibrium and is unique.*

Los juegos es Soluble
Por IDSDS

## Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

**Proof.**

By contradiction:

- Suppose that the results from IDSDS ($s^*$) is not a Nash Equilibrium

$\square$

## Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

**Proof.**

By contradiction:

- Suppose that the results from IDSDS ($s^*$) is not a Nash Equilibrium
- For some individual $i$ there exits $s_i$ such that

$$u_i(s_i, s_{-i}^*) > u_i(s_i^*, s_{-i}^*)$$

$\square$

## Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

**Proof.**
By contradiction:

- Suppose that the results from IDSDS ($s^*$) is not a Nash Equilibrium
- For some individual $i$ there exits $s_i$ such that

$$u_i(s_i, s^*_{-i}) > u_i(s^*_i, s^*_{-i})$$

- But then $s_i$ could not have been eliminated

□

## Proof

First let's proof its a Nash Equilibrium. The fact that is unique is trivial by the previous theorem.

**Proof.**
By contradiction:

- Suppose that the results from IDSDS ($s^*$) is not a Nash Equilibrium
- For some individual $i$ there exits $s_i$ such that

$$u_i(s_i, s^*_{-i}) > u_i(s^*_i, s^*_{-i})$$

- But then $s_i$ could not have been eliminated
- And this is a contradiction!

□

## Lecture 12: Game Theory // Nash equilibrium



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- Strategy space is $S_i = [0, +\infty)$ → CANTIDADES.
- The utility function of player $i$ is given by:

$$\pi_1(q_1, q_2) = (120 - (q_1 + q_2))q_1, \Rightarrow \boxed{P(Q)}q_1 - \cancel{CT(q_1)} \to 0$$
$$\pi_2(q_1, q_2) = (120 - (q_1 + q_2))q_2.$$

$$\Rightarrow \frac{\partial \pi_1}{\partial q_1} = 120 - 2q_1 - q_2 \Rightarrow \boxed{\frac{120 - q_2}{2} = q_1^*}$$

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$$MR_i \Rightarrow \frac{d\pi_i}{dq_i}(120 - q_i - q_{-i})q_i = 120 - 2q_i - q_{-i} \rightarrow \boxed{\frac{120 - q_{-i}}{2} = q_i}$$

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$$\frac{d\pi_i}{dq_i}(120 - q_i - q_{-i})q_i = 120 - 2q_i - q_{-i}$$

- Therefore 60 strictly dominates any $\boxed{q_i \in (60, 120]}$ $\Rightarrow q_i, q_{-i} \in [0, 60]$

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- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [0, 60]$
- Therefore $q_i \in [0, 30)$ are strictly dominated by $q_i = 30$
- After two rounds of deletion of strictly dominated strategies, we are left with: $S_i = [30, 60]$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [30, 60]$
- 45 strictly dominates all strategies $q_i \in (45, 60]$
- After three rounds of deletion of strictly dominated strategies, we are left with: $S_i = [30, 45]$

Cournot Competition

- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- $q_{-i} = [30, 45]$
- 37.5 strictly dominates all strategies $q_i \in [30, 37.5]$
- After four rounds of deletion of strictly dominated strategies, we are left with: $S_i = [37.5, 45]$

Cournot Competition

- After (infinitely) many iterations, the only remaining strategies are $S_i = 40$
- The unique solution by IDSDS is $q_1^* = q_2^* = 40$. $\longrightarrow$ EN

Cournot Competition

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- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2} = MR_i(q_{-i})$$

$q_1^* = \frac{120 - q_2^*}{2}$ $\Rightarrow$ 2 ECUACIONES

$q_2^* = \frac{120 - q_1^*}{2}$ 2 INCOGNITAS

$\Rightarrow q_2^* = 60 - \frac{q_1^*}{2}$

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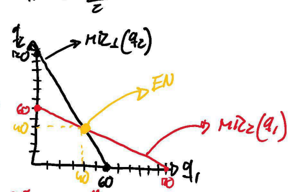


Cournot Competition

- There will also be a unique Nash equilibrium
- $$BR_i(q_{-i}) = \frac{120 - q_{-i}}{2}.$$
- At any Nash equilibrium, we must have: $q_1^* \in BR_1(q_2^*)$ and $q_2^* \in BR_2(q_1^*)$.

$q_1^* = \frac{120 - q_2^*}{2} \longrightarrow 0 = 120 - q_2^* - 2q_1^*$

$q_2^* = \frac{120 - q_1^*}{2} \longrightarrow 0 = 120 - 2q_2^* - q_1^*$

$\times(-2)$

$0 = -240 + 2q_2^* + 4q_1^*$

$0 = -120 + 0q_2^* + 3q_1^*$

$\Rightarrow 120 = q^* \Rightarrow \boxed{40 = q_1^*}$

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-
$$q_1^* = \frac{120 - q_2^*}{2}, q_2^* = \frac{120 - q_1^*}{2}.$$
- We can solve for $q_1^*$ and $q_2^*$ to obtain:
$$\boxed{q_1^* = 40, q_2^* = 40, Q^* = 80, \Pi_1^* = \Pi_2^* = 1600.}$$

## Cournot Competition vs Monopoly (cartel)

- In a perfectly competitive market, price equals marginal cost and the total quantity produced will be $\boxed{Q = 120.}$

$\boxed{P=0}$

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- A monopolist would solve the following maximization problem:
$$\max_{Q} \underbrace{(120 - Q)}_{P} Q \Rightarrow \boxed{Q^* = 60, P^* = 60, \Pi^m = 3600.}$$

$\Pi = (120 - Q)Q$

$\frac{\partial \Pi}{\partial Q} = 120 - 2Q = 0 \rightarrow Q = 60$
$P = 60$

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- The profits to each firm in the Cournot Competition is less than half of the monopoly profits
- In a duopoly, externalities are imposed on the other firm

## Cournot Competition - General case

- $n$ firms are competing a la Cournot

$0 = -120 + 0q_2 + 3q_1^*$

$\Rightarrow \frac{120}{3} = q_1^* \Rightarrow \boxed{40 = q_1^*}$

$\boxed{q_2^* = \frac{120 - q_1^*}{2} = \frac{120 - 40}{2} = \frac{80}{2} = 40}$

$\boxed{Q_1 = Q_2 = 30 \quad ?.Q \quad Q = 60 = Q^M}$

$\Pi_1(q_1, q_2 = 30) = (120 - q_1 - 30)\, q_1$

$\frac{\partial \Pi}{\partial q_1} = 90 - 2q_1 = 0$

$\boxed{q_1^* = 45}$

$\partial q_1$

$$\boxed{q_1^* = 45}$$

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-

$$\max_{q_i} p(q_i + Q_{-i})q_i - c_i(q_i)$$

## Cournot Competition - General case

$$\max_{q_i} p(q_i + Q_{-i})q_i - c_i(q_i)$$

- First order condition implies:

$$\begin{aligned}
q_i \frac{dP}{dQ}(q_i + Q_{-i}) + P(q_i + Q_{-i}) &= \frac{dc_i}{dq_i}(q_i) \\
q_i \frac{dP}{dQ}(Q) + P(Q) &= \frac{dc_i}{dq_i}(q_i) \\
P(Q) - \frac{dc_i}{dq_i}(q_i) &= -q_i \frac{dP}{dQ}(Q) \\
\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} &= -\frac{q_i}{Q} \frac{Q}{P(Q)} \frac{dP}{dQ}(Q) \\
\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} &= -\frac{q_i}{Q} \frac{1}{\varepsilon_{Q,P}(Q)}
\end{aligned}$$

## Cournot Competition - General case

$$\frac{P(Q) - \frac{dc_i}{dq_i}(q_i)}{P(Q)} = -\frac{q_i}{Q}\frac{1}{\varepsilon_{Q,P}(Q)}$$

- Therefore in a pure strategy Nash equilibrium $(q_1^*, q_2^*, \ldots, q_n^*)$ with $Q^* = q_1^* + q_2^* + \cdots q_n^*$, we must have:

$$\frac{P(Q^*) - \frac{dc_1}{dq_1}(q_1^*)}{P(Q^*)} = -\frac{q_1^*}{Q^*}\frac{1}{\varepsilon_{Q,P}(Q^*)},$$
$$\frac{P(Q^*) - \frac{dc_2}{dq_2}(q_2^*)}{P(Q^*)} = -\frac{q_2^*}{Q^*}\frac{1}{\varepsilon_{Q,P}(Q^*)},$$
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## Cournot Competition - General case

- Suppose that all firms have exactly the same cost function $c$

$$\frac{P(Q^*) - \frac{dc}{dq_1}(q_1^*)}{P(Q^*)} = -\frac{q_1^*}{Q^*}\frac{1}{\varepsilon_{Q,P}(Q^*)},$$
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- Rewriting

$$P(Q^*) = \frac{1}{1 + \frac{1}{n}\frac{1}{\varepsilon_{Q,P}(Q^*)}}\frac{\partial c}{dq}\left(\frac{Q^*}{n}\right).$$

## Lecture 12: Game Theory // Nash equilibrium

Dominance
  Weakly dominated strategies

Nash equilibrium

Some examples

Relationship to dominance

Examples
  Cournot Competition
  Cartels

## Cartels

- Suppose there are three firms who face zero marginal cost
- The inverse demand function is given by:

$$p(q_1 + q_2 + q_3) = 1 - q_1 - q_2 - q_3 = 1 - Q$$


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- In a Nash equilibrium we must have:

$$q_1^* = \frac{1 - q_2^* - q_3^*}{2}$$
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- $q_1^* = q_2^* = q_3^* = \frac{1}{4}$
- Price is $p^* = 1/4$ and all firms get the same profits of $1/16$

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- Therefore

$$\begin{aligned} q_A^* &= \frac{1 - q_B^*}{2} \\ q_B^* &= \frac{1 - q_A^*}{2}. \end{aligned}$$

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- Firms 1 and 2 suffered, while firm 3 is better off!
- Firm 3 is obtaining a disproportionate share of the joint profits (more than $1/3$)

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- Firm 3 clearly wants to stay out

## Cartels

There are many ifficulties associated with sustaining collusive agreements (e.g., the OPEC cartel)