Lecture 14

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Lecture14

Lecture 14: Game Theory // Nash equilibrium

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Lecture 14: Game Theory // Nash equilibrium

Mixed strategies

Examples

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Mixed strategies

Examples

Mixed strategies

Consider rock/paper/scissors

S_2

S_1

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

- This game is entirely stochastic (ability has nothing to do with your chances of winning)

Mixed strategies

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- The probability of winning with every strategy is the same
- Thus, people *tend* choose randomly which of the three options to play
- We would like the concept of Nash equilibrium to reflect this

Mixed strategies

$\sigma_1 : \{P_1, P_4, T_1\} \to \{0, 1\}$

Definition
A mixed strategy σ_i is a function $\sigma_i : S_i \to [0, 1]$ such that

$$\boxed{\sum_{s_i \in S_i} \sigma_i(s_i) = 1.}$$

$$\left.\begin{aligned} \sigma_1(P_1) &= 1/2 \\ \sigma_1(P_4) &= 1/4 \\ \sigma_1(T_1) &= 1/4 \end{aligned}\right\} \Sigma = 1$$

- $\sigma_i(s_i)$ represents the probability with which player i plays s_i

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$\sigma_1(T_1) = 1 \sim T_1$ SETZA

- We will denote the set of all mixed strategies of player i by $\boxed{\Sigma_i}$

Mixed strategies

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- $$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n).$$

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- The expected utility of playing "rock" is

$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

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- $$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n). = \mathbb{E}(U_i)$$
- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\boxed{\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})}$) U(Piedra, Papel)
- The expected utility of playing "rock" is

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U(Piedra, Tijera)

- If I'm randomizing over rock and scissors (i.e., $\sigma_i = (\frac{1}{2}, 0, \frac{1}{2})$) then

$$E(U_i(\sigma, \sigma_{-i})) = \underbrace{-1}_{\text{rock vs paper}}\boxed{\frac{1}{4}} + \underbrace{1}_{\text{rock vs scissors}}\boxed{\frac{1}{4}} + \underbrace{1}_{\text{scissors vs paper}}\boxed{\frac{1}{4}} + \underbrace{0}_{\text{scissors vs scissors}}\boxed{\frac{1}{4}} = \frac{1}{4}$$

Mixed strategies

Definition

A (possibly mixed) strategy profile $(\sigma_1^*, \sigma_2^*, \ldots, \sigma_n)^*$ is a Nash equilibrium if and only if for every i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

for all $\sigma_i \in \Sigma_i$.

Mixed strategies

Definition (Mixed Strategy Dominance Definition A)

Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all mixed strategies of the opponents, σ_{-i},

$$u_i(\sigma_i, \boxed{\sigma_{-i}}) > u_i(\sigma_i', \boxed{\sigma_{-i}}).$$

$\forall \sigma_i \ \Sigma_{-i}$

Mixed strategies

If σ_i is better than σ_i' no matter what **pure strategy** opponents play, then σ_i is also strictly better than σ_i' no matter what **mixed strategies** opponents play

Theorem

Let σ_i and σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if and only if for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof- Part 1

- Since $S_{-i} \subseteq \Sigma_{-i}$, if σ_i strictly dominates σ_i'

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- Then for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof - Part 2

- To prove the other direction, suppose that for all $s_{-i} \in S_{-i}$,

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- For any σ_{-i},

$$\begin{aligned} u_i(\sigma_i, \sigma_{-i}) &= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) \end{aligned}$$

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- So

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i', s_{-i}) = u_i(\sigma_i', \sigma_{-i})$$

Mixed strategies

Definition (Mixed Strategy Dominance Definition B)

Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all pure strategies of the opponents, $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

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Battle of the sexes

EN = {(G,G),(P,P)}

J2

J1

	G	P
G	2,1	0,0
P	0,0	1,2

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

- There are two pure strategy equilibria (G,G) and (P,P)

Battle of the sexes

	α	$1-\alpha$
	G	P
β G	2,1	0,0
$1-\beta$ P	0,0	1,2

- There are two pure strategy equilibria (G,G) and (P,P)
- We now look for Nash equilibria that involve randomizationby the players

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G

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- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$

Battle of the sexes

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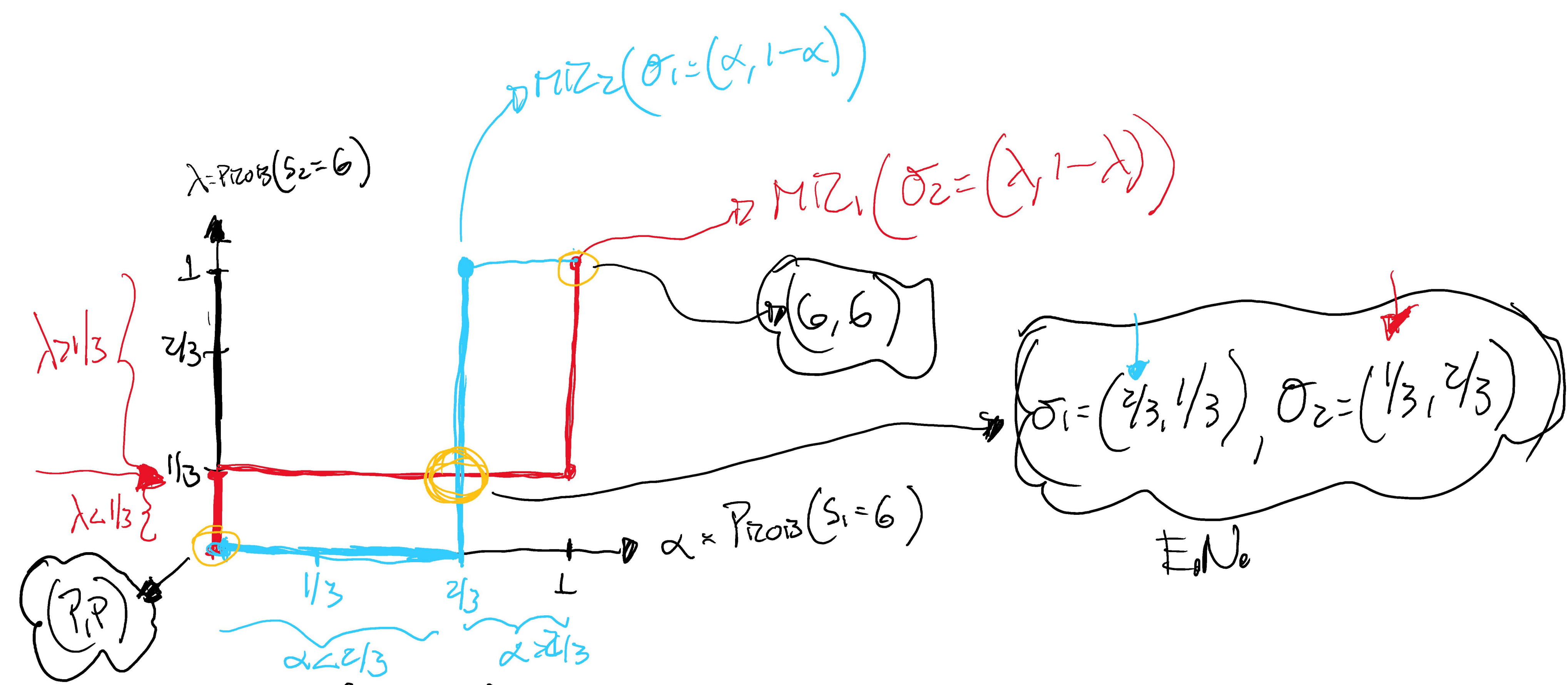
-

S_2

S_1	G	P
G	2,1	0,0
P	0,0	1,2

Prob(G) Prob(P)

$\sigma_2 = (\lambda, 1-\lambda)$



$\boxed{S_1}$ $U_1(G, \sigma_2) = 2\lambda + (1-\lambda)0 = 2\lambda = \mathbb{E}(U_1(G,\sigma_2))$

$U_1(P, \sigma_2) = \lambda 0 + (1-\lambda)1 = 1-\lambda = \mathbb{E}(U_1(P,\sigma_2))$

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G
- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$
- **Case 1:** If $q > 1/3$, then $2q > 2/3 > 1-q$ and therefore, the best response is $\lambda = 1$
- **Case 2:** if $q = 1/3$, then $2q = 2/3 = 1-q$ and therefore, the best response is $\lambda \in [0,1]$
- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$
- Thus, the best response function is given by:

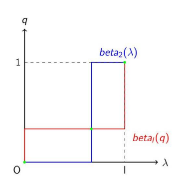
$$BR_1(q) = \begin{cases} 1 & \text{if } q > 1/3 \\ [0,1] & \text{if } q = 1/3 \\ 0 & \text{if } q < 1/3. \end{cases}$$

Battle of the sexes

Similarly we can calculate the best response function for player 2 and we get:

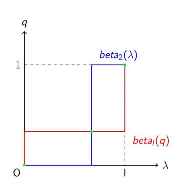
$$BR_2(\lambda) = \begin{cases} 1 & \text{if } \lambda > 2/3 \\ [0,1] & \text{if } \lambda = 2/3 \\ 0 & \text{if } \lambda < 2/3. \end{cases}$$

Battle of the sexes



- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$

Battle of the sexes



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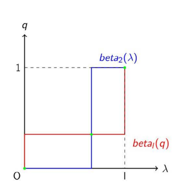

- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$
- First two are the pure strategy NE we had found before
- Last is a strictly mixed NE: both players randomize

Consider the following game

S_2

$\beta_E \quad \beta_F \quad 1-\beta_E-\beta_F$ $\quad EN = \{(A,E); (D,G)\}$.

S_1

		E	F	G
α_A	A	5, 10	5, 3	3, 4
α_B	B	1, 4	[illegible]	7, 6
α_C	C	4, 2	8, 4	3, 8
$\alpha_D = 1-\alpha_B-\alpha_C$	D	2, 4	[illegible]	8, 4

$\boxed{G \succ_2 F}$

$\boxed{D \succ_1 B}$

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$

$\boxed{S_1}$ $U_1(G, \sigma_2) = 2\lambda + (1-\lambda)0$

$U_1(P, \sigma_2) = \lambda \cdot 0 + (1-\lambda) \cdot 1 = 1-\lambda = \mathbb{E}(U_1(P, \sigma_2))$

$\boxed{G > P}$ $\quad$ $G < P$ $\quad$ $\boxed{G \sim P}$

$2\lambda > 1-\lambda$ $\quad$ $2\lambda < 1-\lambda$ $\quad$ $2\lambda = 1-\lambda$

$3\lambda > 1$ $\quad$ $\lambda < 1/3$ $\quad$ $3\lambda = 1$

$\lambda > 1/3$ $\quad$ $\lambda = 1/3$

$$MR_1(\sigma_2 = (\lambda, 1-\lambda)) = \begin{cases} G & \text{si } \lambda > 1/3 \\ G \sim P & \text{si } \lambda = 1/3 \rightarrow \sigma_1 = (\alpha, 1-\alpha) \text{ si } \lambda = 1/3, \ \alpha \in [0,1] \\ P & \text{si } \lambda < 1/3 \end{cases}$$

S_2 $\boxed{\alpha = PROB(S_1 = G)}$

$U_2(\sigma_1 = (\alpha, 1-\alpha), G) = 1 \cdot \alpha + 0 \cdot (1-\alpha) = \alpha$

$U_2(\sigma_1 = (\alpha, 1-\alpha), P) = 0 \cdot \alpha + 2(1-\alpha) = 2 - 2\alpha$

$\boxed{\begin{array}{l} G > P \\ \alpha > 2-2\alpha \\ 3\alpha > 2 \\ \alpha > 2/3 \end{array}}$ $\quad$ $P > G$ $\quad$ $P \sim G$

$\alpha < 2/3$ $\quad$ $\alpha = 2/3$

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$
- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$

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- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$
- Then $BR_2(\sigma_1) = \{(p, 0, 1-p), p \in [0, 1]\}$

- G dominates F (player 2)

- G dominates F (player 2)
- D dominates B (player 1)

S_2

Reduced game

S_1

	E	G
A	5, 10	3, 4
D	2, 4	8, 4

$\underline{\sigma_1} = (\alpha_A, 0, 1-\alpha_A)$

α_A Muy Cercano a 1

Quiero!

$\boxed{\sigma_1 >_1 C}$

$S_2 E: 5\alpha_A + 2(1-\alpha_A) > 4$

$S_2 G: 3\alpha_A + 8(1-\alpha_A) > 3$

$5\alpha_A + 2 - 2\alpha_A > 4$

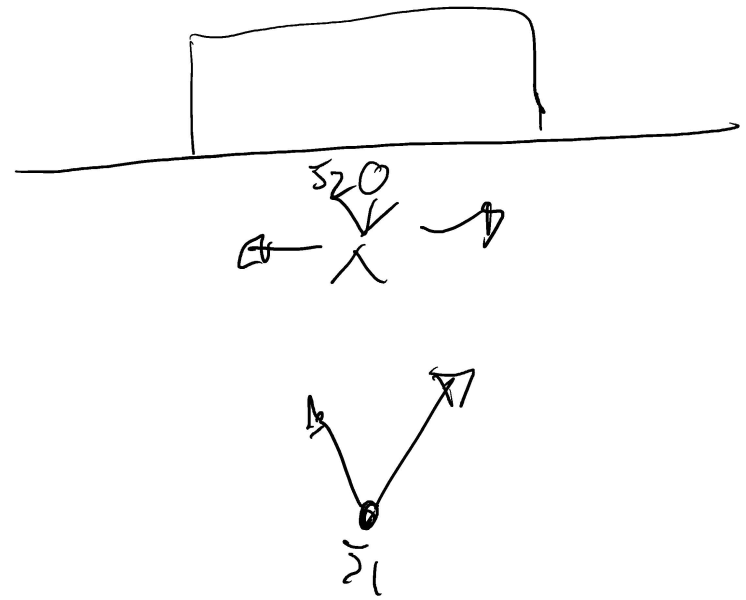
$3\alpha_A + 8 - 8\alpha_A > 3$

$3\alpha_A > 2 \to \alpha_A > 2/3$

$-5\alpha_A > -5 \to \alpha_A < 1$

Si $\alpha_A \in (2/3, 1) \Rightarrow \underline{\underline{\sigma_1 > C}}$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$



S_2

		I	D
S_1	I	-1, 1	1, -1
	D	1, -1	-1, 1

$EN = \left(\sigma_1 = (1/2, 1/2), \sigma_2 = \left(\frac{1}{2}, \frac{1}{2}\right)\right)$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\begin{aligned} \mathbb{E}U(\sigma_1, E) &> U(C, E) \\ 3p + 2 &> 4 \\ p &> \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbb{E}U(\sigma_1, G) &> \mathbb{E}U(C, G) \\ 8 - 5p &> 3 \\ p &< \frac{5}{5} = 1 \end{aligned}$$

Reduced game

		J_2 β E	$1-\beta$ G
J_1	α A	5, 10	3, 4
	$1-\alpha$ D	2, 4	8, 4

$\boxed{\text{MR } S_1}$

$\sigma_2 = (\beta, 1-\beta)$

$MR_1(\sigma_2 = (\beta, 1-\beta))$ ⇐ OBSGETIVO

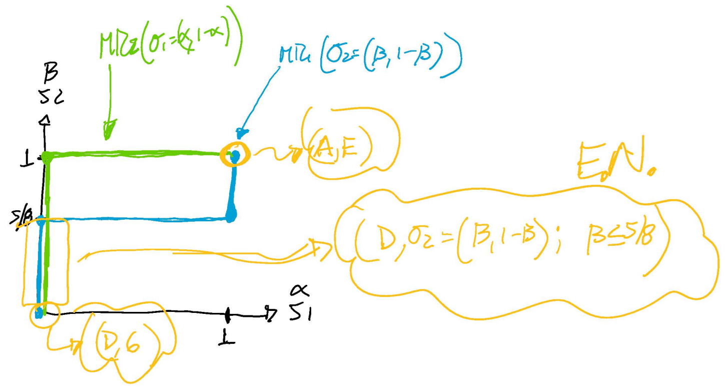
$\mathbb{E}(U_1(A, \sigma_2)) = 5\beta + 3(1-\beta) = 2\beta + 3$

$\mathbb{E}(U_1(D, \sigma_2)) = 2\beta + 8(1-\beta) = 8 - 6\beta$

$\underline{A > D}$: $2\beta + 3 > 8 - 6\beta$, $8\beta > 5$, $\beta > 5/8$

$\underline{D > A}$: $8 - 6\beta > 2\beta + 3$, $\beta < 5/8$

$\underline{D \sim A}$: $\beta = 5/8$



- Lets find $BR_1(\sigma_2 = (q, 1-q))$

$\boxed{\text{MR } S_2}$

$MR_2(\sigma_1 = (\alpha, 1-\alpha))$ ⇐ OBSGETIVO

$\mathbb{E}(U_2(\sigma_1, E)) = 10\alpha + 4(1-\alpha) = 6\alpha + 4$

$\mathbb{E}(U_2(\sigma_1, G)) = 4\alpha + 4(1-\alpha) = 4$

$\underline{E > G}$: $6\alpha + 4 > 4$, $6\alpha > 0$, $\alpha > 0$

$\cancel{\underline{G > E}: 4 > 6\alpha + 4,\ \alpha < 0}$

$\boxed{G \sim E}$: $\alpha = 0$

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- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$
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- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
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- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$

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$$BR_1(q, 1-q) = \begin{cases} \sigma_1 = (0,1) & \text{if } 0 \leq q < \frac{5}{8} \\ \sigma_1 = (1,0) & \text{if } \frac{5}{8} < q \leq 1 \\ \sigma_1 = (p, 1-p) & \text{if } \frac{5}{8} = q \end{cases}$$

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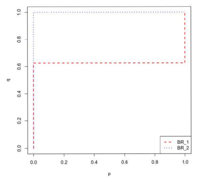
$$BR_2(p, 1-p) = \begin{cases} \sigma_2 = (1,0) & \text{if } p > 0 \\ \sigma_2 = (q, 1-q) & \text{if } p = 0 \end{cases}$$

Best responses



$NE = \{(A, E), (D, \sigma_2^q)\}$ where $\sigma_2^q = (q, 1-q)$ and $0 \leq q \leq \frac{5}{8}$