# Lecture 14

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Lecture14

## Lecture 14: Game Theory // Nash equilibrium

Mauricio Romero

### Lecture 14: Game Theory // Nash equilibrium

Mixed strategies

Examples

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Examples

### Mixed strategies

Consider rock/paper/scissors

| | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0,0 | -1,1 | 1,-1 |
| Paper | 1,-1 | 0,0 | -1,1 |
| Scissors | -1,1 | 1,-1 | 0,0 |

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### Mixed strategies

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$S_2$

$S_1$

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- This game is entirely stochastic (ability has nothing to do with your chances of winning)
- The probability of winning with every strategy is the same
- Thus, people *tend* choose randomly which of the three options to play
- We would like the concept of Nash equilibrium to reflect this

## Mixed strategies

### Definition

A mixed strategy $\sigma_i$ is a function $\sigma_i : S_i \to [0,1]$ such that

$$\boxed{\sum_{s_i \in S_i} \sigma_i(s_i) = 1.}$$

- $\sigma_i(s_i)$ represents the probability with which player $i$ plays $s_i$

$\boxed{\sigma_1 = (1,0,0)}$

PEDRA PAPEL

$\sigma_1 = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

↳ TESOURA

$\sigma_1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\sigma_1 = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$

$\cancel{\sigma_1 = \left(\frac{1}{2}, 0, 0\right)}$

## Mixed strategies

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- $\sigma_i(s_i)$ represents the probability with which player $i$ plays $s_i$
- A **pure strategy** is simply a mixed strategy $\sigma_i$ that plays some strategy $s_i \in S_i$ with probability one
- We will denote the set of all mixed strategies of player $i$ by $\boxed{\Sigma_i}$

## Mixed strategies

- Given a mixed strategy profile $(\sigma_1, \sigma_2, \ldots, \sigma_n)$, we need a way to define how players evaluate payoffs of mixed strategy profiles

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- $$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\underbrace{\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n)}_{\text{PROB}}. = \mathbb{E}(U)$$

## Mixed strategies

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- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)

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PAPEL VS PIEDRA

- The expected utility of playing "rock" is

$$E(U_i(\underbrace{rock}_{\text{YO}}, \underbrace{\sigma_{-i}}_{\text{CONTRINCANTE}})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

PIEDRA VS TIJERA

## Mixed strategies

- Given a mixed strategy profile $(\sigma_1, \sigma_2, \ldots, \sigma_n)$, we need a way to define how players evaluate payoffs of mixed strategy profiles
- $$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n).$$
- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)
- The expected utility of playing "rock" is

$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

- If I'm randomizing over rock and scissors (i.e., $\sigma_i = (\frac{1}{2}, 0, \frac{1}{2})$) then

$$E(U_i(\sigma, \sigma_{-i})) = \underbrace{-1\frac{1}{4}}_{\text{rock vs paper}} + \underbrace{1\frac{1}{4}}_{\text{rock vs scissors}} + \underbrace{1\frac{1}{4}}_{\text{scissors vs paper}} + \underbrace{0\frac{1}{4}}_{\text{scissors vs scissors}} = \frac{1}{4}$$

## Mixed strategies

**Definition**

A (possibly mixed) strategy profile $(\sigma_1^*, \sigma_2^*, \ldots, \sigma_n)^*$ is a Nash equilibrium if and only if for every $i$,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

for all $\sigma_i \in \Sigma_i$.

## Mixed strategies

**Definition (Mixed Strategy Dominance Definition A)**

Let $\sigma_i, \sigma_i'$ be two mixed strategies of player $i$. Then $\sigma_i$ strictly dominates $\sigma_i'$ if for all mixed strategies of the opponents, $\sigma_{-i}$,

$$u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma_i', \sigma_{-i}).$$

$\boxed{\forall \sigma_i}$ CONJUNTO INFINITO

## Mixed strategies

If $\sigma_i$ is better than $\sigma_i'$ no matter what **pure strategy** opponents play, then $\sigma_i$ is also strictly better than $\sigma_i'$ no matter what **mixed strategies** opponents play

**Theorem**

*Let $\sigma_i$ and $\sigma_i'$ be two mixed strategies of player $i$. Then $\sigma_i$ strictly dominates $\sigma_i'$ if and only if for all $s_{-i} \in S_{-i}$,*

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

## Proof- Part 1

- Since $S_{-i} \subseteq \Sigma_{-i}$, if $\sigma_i$ strictly dominates $\sigma_i'$

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- Since $S_{-i} \subseteq \Sigma_{-i}$, if $\sigma_i$ strictly dominates $\sigma_i'$
- Then for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

## Proof - Part 2

- To prove the other direction, suppose that for all $s_{-i} \in S_{-i}$,

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- For any $\sigma_{-i}$,

$$\begin{aligned} u_i(\sigma_i, \sigma_{-i}) &= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) \end{aligned}$$

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- So

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i', s_{-i}) = u_i(\sigma_i', \sigma_{-i})$$

## Mixed strategies

**Definition (Mixed Strategy Dominance Definition B)**
Let $\sigma_i, \sigma_i'$ be two mixed strategies of player $i$. Then $\sigma_i$ strictly dominates $\sigma_i'$ if for all pure strategies of the opponents, $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

$\boxed{\forall s_{-i}}$

Conjunto finito.

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## Battle of the sexes

$S_2$

$S_1$

| | G | P |
|---|---|---|
| G | 2,1 | 0,0 |
| P | 0,0 | 1,2 |

$\lambda \quad 1-\lambda$

PURAS

$EN = \{(G,G), (P,P)\}$

MR $S_1$

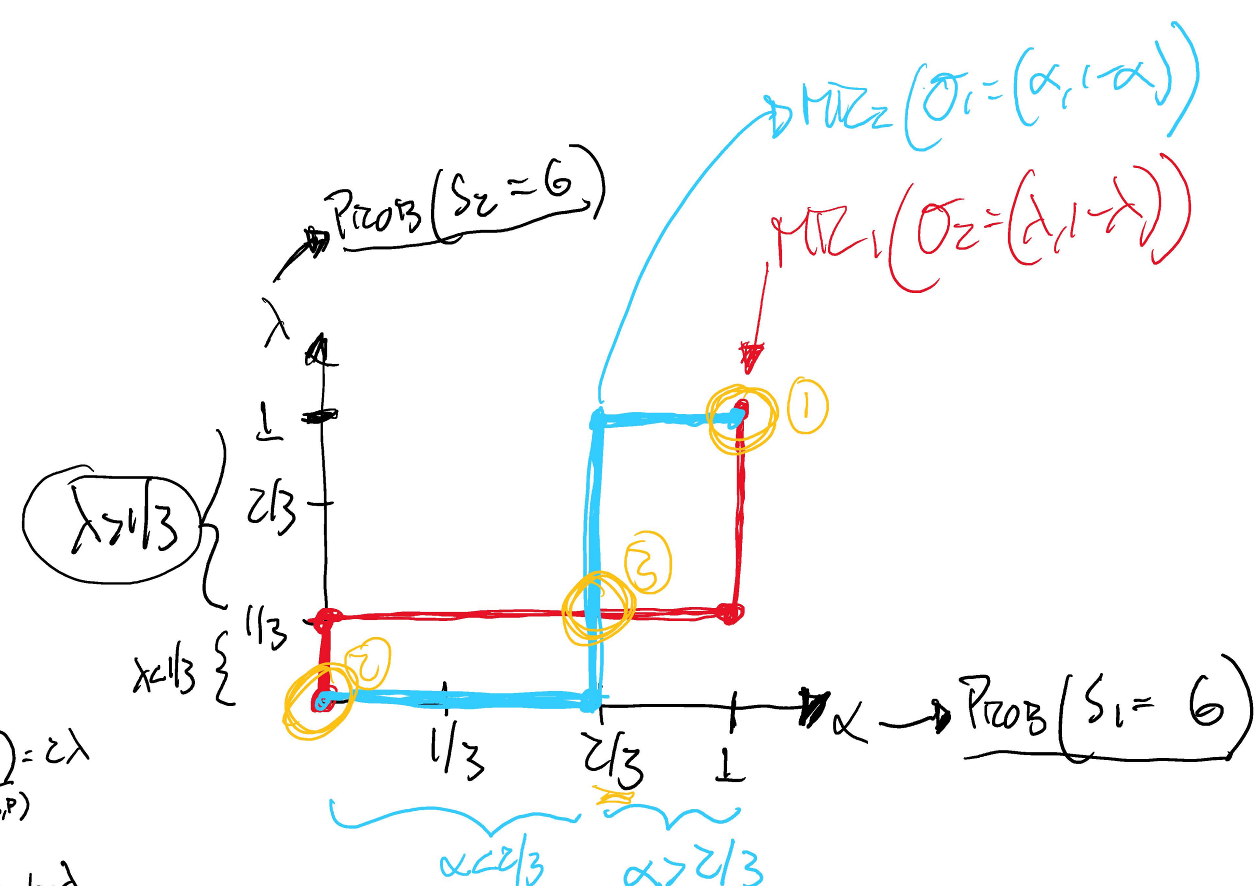
$\sigma_2 = (\lambda, 1-\lambda)$

$\boxed{\text{Prob}(S_2=G)}$ ... Prob$(S_2=P)$

$U_1(G, \sigma_2) = \underbrace{2}_{U(G,G)} \cdot \underbrace{\lambda}_{P(G,G)} + \underbrace{0}_{U(G,P)} \cdot \underbrace{(1-\lambda)}_{P(G,P)} = 2\lambda$

$U_1(P, \sigma_2) = 0 \cdot \lambda + 1 \cdot (1-\lambda) = 1-\lambda$

$\boxed{G > P}$ $\boxed{P > G}$ $\boxed{P \sim G}$




$EN = \begin{cases} (1) \Rightarrow (G,G) \\ (2) \Rightarrow (P,P) \\ (3) \Rightarrow \left(\sigma_1 = \left(\frac{2}{3}, \frac{1}{3}\right), \sigma_2 = \left(\frac{1}{3}, \frac{2}{3}\right)\right) \end{cases}$

ESTRATEGIAS PURAS

ESTRATEGIAS MIXTAS

$\alpha$ $\lambda$

Battle of the sexes

| | G | P |
|---|---|---|
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- There are two pure strategy equilibria $(G, G)$ and $(P, P)$

Battle of the sexes

$s_2$

$s_1$


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- There are two pure strategy equilibria $(G, G)$ and $(P, P)$


- We now look for Nash equilibria that involve randomizationby the players

Battle of the sexes

- Let $\lambda$ be the probability with which player 1 chooses $G$ and $q$ be the probability with which player 2 plays $G$

Battle of the sexes

- Let $\lambda$ be the probability with which player 1 chooses $G$ and $q$ be the probability with which player 2 plays $G$

- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$

Battle of the sexes

- Let $\lambda$ be the probability with which player 1 chooses $G$ and $q$ be the probability with which player 2 plays $G$
- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$

- **Case 1:** If $q > 1/3$, then $2q > 2/3 > 1-q$ and therefore, the best response is $\lambda = 1$

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- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$

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$\boxed{G > P}$

$2\lambda > 1-\lambda$

$3\lambda > 1$

$\lambda > 1/3$

$\boxed{P > G}$

$1-\lambda > 2\lambda$

$1 > 3\lambda$

$\lambda < 1/3$

$\boxed{P \sim G}$

$1-\lambda = 2\lambda$

$\lambda = 1/3$

$$MR_1(\sigma_2 = (\lambda, 1-\lambda)) = \begin{cases} G & s_1 \quad \lambda > 1/3 \\ \boxed{G \sim P \quad s_1 \quad \lambda = 1/3} & \rightarrow \sigma_1 = (\alpha, 1-\alpha), \ \alpha \in [0,1] \\ P & s_1 \quad \lambda < 1/3 \end{cases}$$

$MR_2(\sigma_1 = (\alpha, 1-\alpha))$

$\alpha \rightarrow \text{Prob}(s_1 = G)$

MR $s_2$

$$U_2(\sigma_1, G) = 1\alpha + 0(1-\alpha) = \alpha$$

$$U_2(\sigma_1, P) = 0 \cdot \alpha + 2(1-\alpha) = 2 - 2\alpha$$

$\boxed{G > P}$

$\alpha > 2 - 2\alpha$

$3\alpha > 2$

$\alpha > 2/3$

$\boxed{P > G}$

$\alpha < 2/3$

$\boxed{P \sim G}$

$\alpha = 1/3$

$s_1$

$\sigma$

$s_2$

1

## Battle of the sexes

- Let $\lambda$ be the probability with which player 1 chooses $G$ and $q$ be the probability with which player 2 plays $G$
- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$
- **Case 1:** If $q > 1/3$, then $2q > 2/3 > 1-q$ and therefore, the best response is $\lambda = 1$
- **Case 2:** if $q = 1/3$, then $2q = 2/3 = 1-q$ and therefore, the best response is $\lambda \in [0,1]$
- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$
- Thus, the best response function is given by:

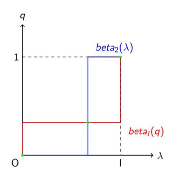
$$BR_1(q) = \begin{cases} 1 & \text{if } q > 1/3 \\ [0,1] & \text{if } q = 1/3 \\ 0 & \text{if } q < 1/3. \end{cases}$$

## Battle of the sexes

Similarly we can calculate the best response function for player 2 and we get:

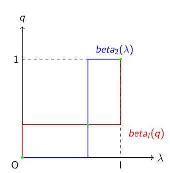
$$BR_2(\lambda) = \begin{cases} 1 & \text{if } \lambda > 2/3 \\ [0,1] & \text{if } \lambda = 2/3 \\ 0 & \text{if } \lambda < 2/3. \end{cases}$$

## Battle of the sexes




- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$

## Battle of the sexes

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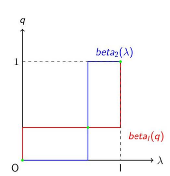
## Battle of the sexes




- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$
- First two are the pure strategy NE we had found before
- Last is a strictly mixed NE: both players randomize

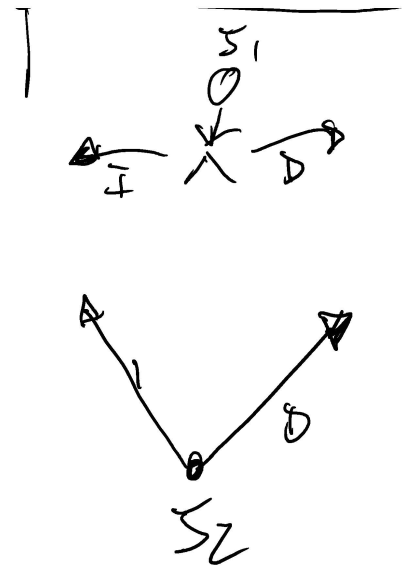
Consider the following game

$S_2$

| | | | $\beta_E$ | $\beta_F$ | $1-\beta_E-\beta_F$ |
|---|---|---|---|---|---|
| | | | E | [illegible] | G |
| $S_1$ | $\alpha_1$ | A | 5, 10 | [illegible] | 3, 4 |
| | $\alpha_2$ | [illegible] | [illegible] | [illegible] | [illegible] |
| | $\alpha_3$ | C | 4, 2 | [illegible] | 3, 8 |
| | $1-\alpha_1-\alpha_2-\alpha_3$ | D | 2, 4 | [illegible] | 8, 4 |

$G >_2 F$
$D >_1 B$

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$




| | | $S_2$ | |
|---|---|---|---|
| | | I | D |
| $S_1$ | I | 1, −1 | −1, 1 |
| | D | −1, 1 | 1, −1 |

$$\boxed{EN = \left(\sigma_1 = \left(\tfrac{1}{2}, \tfrac{1}{2}\right), \sigma_2 = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right)}$$

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$
- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$


- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$
- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$

- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$


- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$
- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$
- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$

- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$


- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$
- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$
- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$
- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$

- Then $BR_2(\sigma_1) = \{(p, 0, 1-p), p \in [0, 1]\}$

- $G$ dominates $F$ (player 2)


- $G$ dominates $F$ (player 2)


- $D$ dominates $B$ (player 1)

$J_2$

$\beta$ $1-\beta$

Reduced game

$J_1$

| | | E | G |
|---|---|---|---|
| $\alpha_A$ | A | 5, 10 | 3, 4 |
| $\alpha_C$ | [illegible] | [illegible] | [illegible] |
| $1-\alpha_A-\alpha_C$ | D | 2, 4 | 8, 4 |

$\boxed{A \geq C}$

$\boxed{\sigma_1 > C}$ Busquemos $\sigma_1$ q' Domine a G

$\boxed{\sigma_1 = (\alpha, 0, 1-\alpha)}$

$\mathbb{E}(U_1(\sigma_1, E)) = 5\alpha + 2(1-\alpha) > 4 = U_1(C, E)$

$\mathbb{E}(U_1(\sigma_1, G)) = 3\alpha + 8(1-\alpha) > 3 = U_1(C, G)$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates $C$
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\mathbb{E}U(\sigma_1, E) \quad > \quad U(C, E)$$

$5\alpha + 2 - 2\alpha > 4 \Rightarrow 3\alpha > 2$ $\alpha > 2/3$

$3\alpha + 8 - 8\alpha > 3$ $-5\alpha > -5$ $\alpha < 1$

$\boxed{\alpha \in (2/3, 1)}$ ✓

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates $C$
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\begin{aligned} \mathbb{E}U(\sigma_1, E) &> U(C, E) \\ 3p + 2 &> 4 \\ p &> \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbb{E}U(\sigma_1, G) &> \mathbb{E}U(C, G) \\ 8 - 5p &> 3 \\ p &< \frac{5}{5} = 1 \end{aligned}$$

$5\alpha + 2 - 2\alpha > 4 \Rightarrow 3\alpha > 2$ } $\alpha > 2/3$

$3\alpha + 8 - 8\alpha > 3$ $\quad -5\alpha > -5$ } $\alpha < 1$

$\boxed{\alpha \in (2/3, 1)}$ ✓

Reduced game

| | | $\beta$ E | $1-\beta$ G |
|---|---|---|---|
| $\alpha$ | A | 5, 10 | 3, 4 |
| $1-\alpha$ | D | 2, 4 | 8, 4 |

$\underline{MR_1(\sigma_2 = (\beta, 1-\beta))}$

$E(U_1(A, \sigma_2)) = 5\beta + (1-\beta)3 = 2\beta + 3$

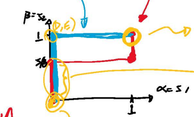
$E(U_1(D, \sigma_2)) = 2\beta + 8(1-\beta) = 8 - 6\beta$

$\underline{A > D}$: $2\beta + 3 > 8 - 6\beta$, $8\beta > 5$, $\underline{\beta > 5/8}$

$\underline{D > A}$: $2\beta + 3 < 8 - 6\beta$, $\vdots$, $\beta < 5/8$

$\underline{D \sim A}$: $\underline{\underline{\beta = 5/8}}$

$MR_2(\sigma_1)$ $\quad$ $MR_1(\sigma_2)$




$EN = (A, E)$

$EN = (D, \sigma_2 = (\beta, 1-\beta)) : \beta < 5/8$ $\rightarrow$ $\underline{\underline{\text{INFINITOS}}}$

$EN = (\underline{D}, \underline{G})$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$

$\underline{\underline{MR_2(\sigma_1 = (\alpha, 1-\alpha))}}$

$E(U_2(\sigma_1, E)) = 10\alpha + 4(1-\alpha) = 6\alpha + 4$

$E(U_2(\sigma_1, G)) = 4\alpha + 4(1-\alpha) = 4$

$\underline{E > G}$: $6\alpha + 4 > 4$, $6\alpha > 0$, $\alpha > 0$

~~$\underline{G > E}$: $\alpha < 0$~~

$\underline{G \sim E}$: $\underline{\underline{\alpha = 0}}$

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- Thus

$$BR_1(q, 1-q) = \begin{cases} \sigma_1 = (0,1) & \text{if } 0 \leq q < \frac{5}{8} \\ \sigma_1 = (1,0) & \text{if } \frac{5}{8} < q \leq 1 \\ \sigma_1 = (p, 1-p) & \text{if } \frac{5}{8} = q \end{cases}$$

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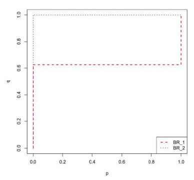
$$BR_2(p, 1-p) = \begin{cases} \sigma_2 = (1,0) & \text{if } p > 0 \\ \sigma_2 = (q, 1-q) & \text{if } p = 0 \end{cases}$$

## Best responses




$NE = \{(A, E), (D, \sigma_2^q)\}$ where $\sigma_2^q = (q, 1-q)$ and $0 \leq q \leq \frac{5}{8}$