Lecture 16

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Lecture16

Lecture 16: Applications of Subgame Perfect Nash Equilibrium

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Lecture 16: Applications of Subgame Perfect Nash Equilibrium

- Ultimatum Game
- Alternating offers
- Stackelberg Competition

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Ultimatum Game

Alternating offers

Stackelberg Competition

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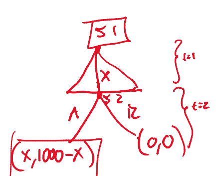
1. Player 1 makes a proposal $(x, 1000 - x)$ of how to split 1000 pesos among $(100, 900), \ldots, (800, 200), (900, 100)$
2. Player 2 accepts or rejects the proposal
3. If player 2 rejects both obtain 0. If 2 accepts, then the payoffs or the two players are determined by $(x, 1000 - x)$



t=2 J2 ACEPTA SI: $1000 - x \geq 0$

$x \leq 1000$

t=1: J1 $\boxed{x = 1{,}000 - \varepsilon}$ $\varepsilon \to 0$

COMP VEND

DAP > DAA

DAP − DAA = 1,000

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- In any SPNE, player 1 makes the proposal $(900, 100)$

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- When extreme offers like (900, 100) are made, player 2 rejects in many cases

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- When extreme offers like (900, 100) are made, player 2 rejects in many cases
- Player 2 may care about inequality or positive utility associated with "punishment" aversion

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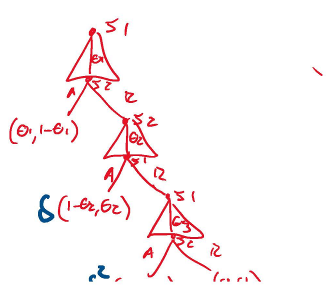
Ultimatum Game

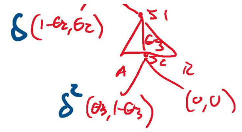
Alternating offers

Stackelberg Competition

- Two players are deciding how to split a pie of size 1

- Two players are deciding how to split a pie of size $\boxed{1}$
- The players would rather get an agreement today than tomorrow (i.e., discount factor)





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- ... and on and on for T periods

- Player 1 makes an offer θ_1
- Player 2 accepts or rejects the proposal
- If player 2 rejects, player 2 makes an offer θ_2
- If player 1 accepts or rejects the proposal
- If player 1 rejects, player 1 makes an offer θ_3
- ... and on and on for T periods
- If no offer is ever accepted, both payoffs equal zero

The discount factor is $\delta \leq 1$.
If Player 1 offer is accepted by Player 2 in round m,

$$\pi_1 = \delta^m \theta_m,$$
$$\pi_2 = \delta^m (1 - \theta_m).$$

If Player 2 offer is accepted, reverse the subscripts

- Consider first the game without discounting $\boxed{\delta = 1}$

$\boxed{T}$

J1 → ULTIMO HACER OFERTA

ULTIMO PERIODO → "JUEGO ULTIMATUM"
J1 SE QUEDA CON TODO → (1,0)

EPS: → J1 SE QUEDA CON TODO

J2 ES ULTIMO HACER OFERTA

EPS: → J2 SE QUEDA CON TODO → (0,1)

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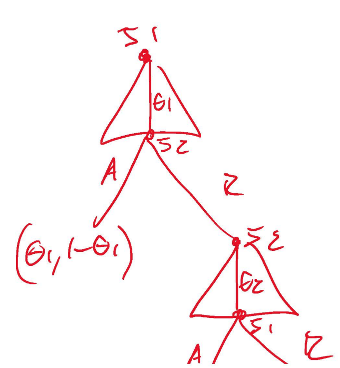
- There is a unique SPNE: The player that makes the last offer gets the whole pie
- Last-mover advantage

- In the game with discounting, the total value of the pie is 1 in the first period, δ in the second, and so forth



J1: $\theta_1 = 1 - (1-\delta)\delta \to (1 - (1-\delta)\delta,\ 1 - (1 - (1-\delta)\delta))$

J2: A $\quad 1 - \theta_1 \geq (1-\delta)\delta \to \theta_1 \leq 1 - (1-\delta)\delta$

J2: $\theta_2 = 1 - \delta \to \delta(1 - (1-\delta),\ 1 - \delta)$

J1: A si $\delta(1 - \theta_2) \geq \delta^2 \to 1 - \theta_2 \geq \delta \to \theta_2 \leq 1 - \delta$

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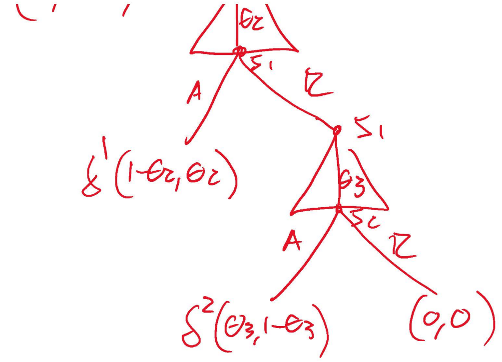
- In period $(T-1)$, Player 2 could offer Smith δ, keeping $(1-\delta)$ for himself
- Player 1 would accept (indifferent between accepting and rejecting) since the **whole pie** in the next period is worth δ

- In period $(T-2)$, Player 1 would offer Player 2 $\delta(1-\delta)$, keeping $(1-\delta(1-\delta))$ for himself



S_1 A si $\delta(1-\theta_2) \geq$ ...

$S_1 \to \theta_3 = 1 \to \delta^2(1, 0) = (\delta^2, 0)$

S_2 ACEPTA si

$\delta^2(1-\theta_3) \geq 0$

$1 \geq \theta_3$

EPS TRATO EN $t=1$

$\theta_1 = 1-(1-\delta)\delta \longrightarrow$ PAGOS $(1-(1-\delta)\delta, \ (1-\delta)\delta)$

VENTAJA ÚLTIMO $\boxed{\delta = 1} \longrightarrow$ PAGOS $(1,0)$

VENTAJA PRIMERO $\boxed{\delta \to 0} \longrightarrow$ PAGOS $\longrightarrow (1,0)$

- In period $(T-2)$, Player 1 would offer Player 2 $\delta(1-\delta)$, keeping $(1-\delta(1-\delta))$ for himself
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- Player 1 would accept...
- ...

- In equilibrium, the very first offer would be accepted, since it is chosen precisely so that the other player can do no better by waiting

Table 1 shows the progression of Player 1's shares when $\delta = 0.9$.

Table 1: Alternating Offers over Finite Time

Round	1's share	2's share	Total value	Who offers?
$T-3$	$\delta(1-\delta(1-\delta))$	$1-\delta(1-\delta(1-\delta))$	δ^{T-4}	2
$T-2$	$1-\delta(1-\delta)$	$\delta(1-\delta)$	δ^{T-3}	1
$T-1$	δ	$1-\delta$	δ^{T-2}	2
T	1	0	δ^{T-1}	1

- If $T = 3$ (i.e, 1 offers, 2 offers, 1 offers)

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- One offers $\delta(1-\delta)$, 2 accepts in period 1

- Player 1 always does a little better when he makes the offer than when Player 2 does

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- If we consider just the class of periods in which Player 1 makes the offer, Player 1's share falls

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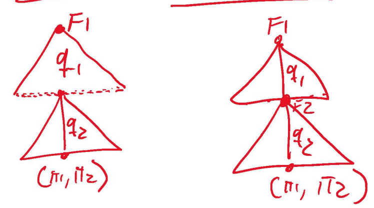
- Suppose instead that the firms move in sequence which is called a **Stackelberg competition game**
- Suppose that the inverse demand function is given by:

$$\boxed{P(q_1 + q_2).}$$

- Firms have the cost function $\boxed{c_i(q_i)}$.

COURNOT STACKELBERG



$S_1 = \{q_1 : q_1 \in \mathbb{R}_+\}.$

$S_2 = \{$ Q' HACER DEPENDIENDO DE Q' HIZO F1 $\}.$

$= \{f : \mathbb{R}_+ \to \mathbb{R}_+\}.$

$f(q_1) = q_2$

Te timing of the game is given by:

1. First Firm 1 chooses $q_1 \geq 0$
2. Second Firm 2 observes the chosen q_1 and then chooses q_2

- The game tree in this game is then depicted by an infinite tree

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- A pure strategy for firm 1 is just a choice of $q_1 \geq 0$
- A strategy for firm 2 specifies what it does after every choice of q_1
- Firm 2's strategy is a function $q_2(q_1)$ which specifies exactly what firm 2 does if q_1 is the chosen strategy of player 1

- SUGADORES ✓ (F1 y F2)
- ESTRATEGIAS ✓
- FUNCIONES PAGO

$q_2(q_1)$ q_2 es función de q_1

The utility functions for firm i when firm 1 chooses q_1 and firm 2 chooses the strategy (or function) $q_2(\cdot)$ is given by:

$$\pi_1(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_1 - c_1(q_1)$$
$$\pi_2(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_2(q_1) - c_2(q_2(q_1))$$

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- There are many Nash equilibria of this game which are a bit counterintuitive → AMENAZAS NO CREIBLES
- Cconsider the following specific game with demand function given by:

$$P(q_1 + q_2) = A - q_1 - q_2.$$

- Let the marginal costs of both firms be zero
- Then the normal form simplifies:

$$\pi_1 = u_1(q_1, q_2(\cdot)) = (\overbrace{A - q_1 - q_2(q_1)}^{P})q_1,$$
$$\pi_2 = u_2(q_1, q_2(\cdot)) = (\underbrace{A - q_1 - q_2(q_1)}_{P})q_2(q_1).$$

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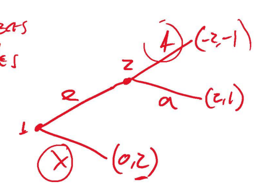
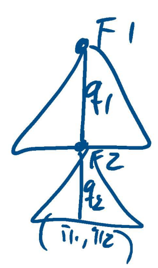
- Let $\alpha \in [0, A)$ and consider the following strategy profile:

$$q_1^* = \alpha, q_2^*(q_1) = \begin{cases} A & \text{if } q_1 \neq \alpha, \\ \frac{A-\alpha}{2} & \text{if } q_1 = \alpha. \end{cases}$$



EN: (X, f) → f ES UNA AMENAZA NO CREIBLE.
(e, a)

EPS = (e, a)



$P = A - q_1 - q_2$

$$q_2(q_1) = \begin{cases} A & \text{si } q_1 \neq 0 \\ q_2^m = \frac{A}{2} & \text{si } q_1 = 0 \end{cases}$$

→ AMENAZA NO CREIBLE

$\pi^m = (A - q_2)q_2$

$\frac{\partial \pi^m}{\partial q_2} = A - 2q_2 = 0$

$q_2^m = \frac{A}{2}$

$$P = \begin{cases} 0 & \text{si } A - q_1 - q_2 < 0 \\ A - q_1 - q_2 & \text{EN OTRO CASO} \end{cases}$$

$\pi_2(q_1, A) = \overbrace{0}^{\text{PRECIO}} \cdot A = 0$

→ $\pi_2(q_1, q_2) = (A - q_1 - q_2)q_2$

$\frac{\partial \pi_2}{\partial q_2} = A - q_1 - 2q_2 = 0$

$$\boxed{q_2 = \frac{A - q_1}{2}}$$

→ $q_2 = \frac{A - q_1}{2}$ ME DA MÁS GANANCIA

EN = $\left(q_1^* = 0,\ q_2^*(q_1) = \begin{cases} A & \text{si } q_1 \neq 0 \\ \frac{A}{2} & \text{si } q_1 = 0 \end{cases}\right)$ EQUIVALENTE (X, f)

F1 TIENE INCENTIVOS A DESVIARSE? NO

- What is an example of a Nash equilibrium of this game?
- Let $\alpha \in [0, A)$ and consider the following strategy profile:

$$q_1^* = \alpha,\ q_2^*(q_1) = \begin{cases} A & \text{if } q_1 \neq \alpha, \\ \frac{A-\alpha}{2} & \text{if } q_1 = \alpha. \end{cases}$$

- Let us check that indeed this constitutes a Nash equilibrium

- First we check the best response of player 1

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$$u_1(q_1, q_2^*(\cdot)) = \begin{cases} \left(A-\alpha-\left(\frac{A-\alpha}{2}\right)\right)\alpha > 0 & \text{if } q_1 = \alpha \\ -q_1^2 \leq 0 & \text{if } q_1 \neq \alpha. \end{cases}$$

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is solved at $q_1^* = \alpha$

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- Firm 1 is best responding to player 2's strategy.

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- Firm 2's utility function is given by:

$$u_2(q_1^*, q_2(\cdot)) = (A-\alpha-q_2(\alpha))q_2(\alpha).$$

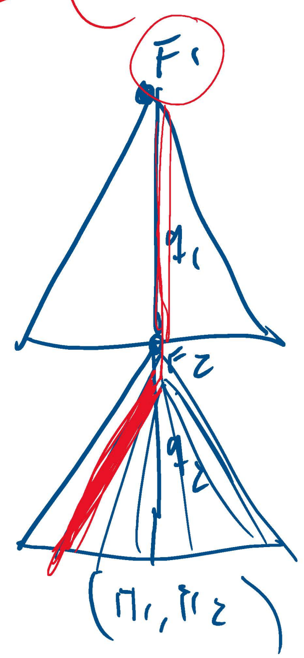
F1 TIENE INCENTIVOS A DESVIARSE? NO

F2 " " " NO!

$$\Pi_2 = (A - q_1 - q_2)q_2 = (A - q_2)q_2$$

$$\frac{\partial \Pi}{\partial q_2} = A - 2q_2 = 0 \qquad \boxed{q_2 = \frac{A}{2}}$$

EPS (AMENAZAS CREIBLES)



t = 1

t = 2

T = 2

$$\Pi_2 = (A - q_1 - q_2)q_2$$

$$\frac{\partial \Pi_2}{\partial q_2} = A - q_1 - 2q_2 = 0$$

$$\boxed{\frac{A - q_1}{2} = q_2(q_1)}$$

- Suppose that firm 1 plays the strategy q_1^*. Is firm 2 best responding?
- Firm 2's utility function is given by:

$$u_2(q_1^*, q_2(\cdot)) = (A - \alpha - q_2(\alpha))q_2(\alpha).$$

- Thus, firm 2 wants to choose the optimal strategy $q_2(\cdot)$ that maximizes the following utility:

$$\max_{q_2(\cdot)}(A - \alpha - q_2(\alpha))q_2(\alpha)$$

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$$q_2(\alpha) = \frac{A - \alpha}{2}.$$

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- In particular, q_2^* is a best response for firm 2

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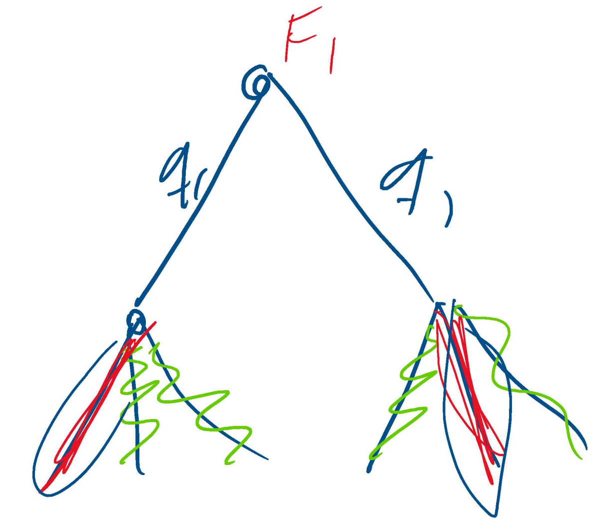
- The Nash equilibria highlighted above all lead to different predictions

$\boxed{T = 1}$



$$\Pi_1 = (A - q_1 - q_2)q_1$$

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$$\frac{\partial \Pi_1}{\partial q_1} = \frac{A - 2q_1}{2} = 0$$

$$\boxed{\frac{A}{2} = q_1^*}$$

$$\boxed{q_2 = \frac{A - \frac{A}{2}}{2} = \frac{A}{4} = q_2(q_1)}$$

$$\boxed{\underline{EPS} = \left(q_1^* = \boxed{\frac{A}{2}},\ q_2(q_1) = \frac{A - q_1}{2}\right)} \rightarrow \text{ASOCIADO "UN RESULTADO"}$$

$$q_1 = \frac{A}{2}$$

$$q_2 = \frac{A}{4}$$

¿ES $\left(q_1^* = \frac{A}{\ldots}\right.$ $\left. q_2^* = \frac{A}{\ldots}\right)$ UN EN? NO

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- In particular, in the Nash equilibrium corresponding to $\alpha = 0$, the equilibrium outcome is for firm 1 to choose a quantity of 0 and firm 2 setting a price of $A/2$
- This would be the same outcome if firm 2 were the monopolist in this market

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---

- Consider the equilibrium in which $\alpha = 0$
- This equilibrium is highly counterintuitive because firm 2 obtains monopoly profits
- The reason is that essentially firm 2 is playing a strategy that involves **non-credible threats**

YES $\left(q_1 = \frac{A}{2},\ q_2 = \frac{A}{4}\right)$ an NE? NO

$$\Pi_1 = \left(A - q_1 - \frac{A}{4}\right) q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = A - 2q_1 - \frac{A}{4} = 0$$

$$\frac{3A}{4} = 2q_1$$

$$\boxed{\frac{3A}{8} = q_1^*}$$

- Consider the equilibrium in which $\alpha = 0$
- This equilibrium is highly counterintuitive because firm 2 obtains monopoly profits
- The reason is that essentially firm 2 is playing a strategy that involves **non-credible threats**
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- If firm 1 were to hypothetically choose $q_1 > 0$, then firm 2 would obtain negative profits if it indeed follows through with $q_2^*(q_1)$.

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- Lets continue with the setting in which marginal costs are zero and the demand function is given by $A - q_1 - q_2$

- We always start with the smallest/last subgames which correspond to the decisions of firm 2 after firm 1's choice of q_1 has been made

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- So, player 2 solves:

$$\max_{q_2(\cdot)} (A - q_1 - q_2(q_1))q_2(q_1).$$

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- In this case, the first order condition implies:

$$q_2^*(q_1) = \frac{A - q_1}{2}.$$

- Thus, in any SPNE, player 2 must play the following strategy:

$$q_2^*(q_1) = \begin{cases} \frac{A-q_1}{2} & \text{if } q_1 \leq A \\ 0 & \text{if } q_1 > A. \end{cases}$$

- Then player 1's utility function given that player 2 plays q_2^* is given by:

$$u_1(q_1, q_2^*(\cdot)) = q_1(A - q_1 - q_2^*(q_1)) = \begin{cases} q_1(A - q_1) & \text{if } q_1 > A, \\ q_1 \frac{A-q_1}{2} & \text{if } q_1 \leq A. \end{cases}$$

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- Firm 1 will never choose $q_1 > A$ since then it obtains negative profits

- Thus, firm 1 maximizes:

$$\max_{q_1 \in [0,A]} q_1 \frac{A - q_1}{2}.$$

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- The **equilibrium outcome** is for firm 1 to choose $A/2$ and firm 2 to choose $A/4$

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- Lets solve for the set of SPNE (which is the same as NE) in the Cournot game with the same demand function and same costs

- In this case, (q_1^*, q_2^*) is a NE if and only if

$$q_1^* \in BR_1(q_2^*), q_2^* \in BR_2(q_1^*).$$

- For $q_1^* \in BR_1(q_2^*)$, we need q_1^* to solve the following maximization problem:

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- As a result, solving these two equations, we get:

$$\boxed{q_1^* = q_2^* = \frac{A}{3}.}$$

In the Cournot game, note that firms' payoffs are:

$$\pi_1^c = \boxed{\frac{A^2}{9}}, \pi_2^c = \boxed{\frac{A^2}{9}}.$$

As we already saw, this was not Pareto efficient since each firm is getting a payoff that is strictly less than $1/2$ of the monopoly profits.

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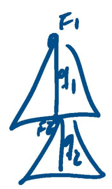
- Thus, the firms' payoffs in the SPNE is:

$$\pi_1^s = \frac{1}{4}A \cdot \frac{A}{2} = \boxed{\frac{A^2}{8}}, \pi_2^s = \frac{1}{4}A \cdot \frac{A}{4} = \boxed{\frac{A^2}{16}}.$$



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- This is intuitive since firm 1 always has the option of choosing the Cournot quantity $q_1 = A/3$, in which case firm 2 will indeed choose $q_2^*(q_1) = A/3$ giving a payoff of $A^2/9$
- But by choosing something optimal, firm 1 will be able to do even better