Lecture 3

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Lecture 3: General Equilibrium

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Lecture 3: General Equilibrium

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Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Hidden assumptions

- There is a market for each good
- Every agent can access the market without any cost
- There is a unique price for each good and all consumers know this price
- Each consumer can sell her initial endowment in the market and use the income to buy goods and services
- Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.
  - There is no centralized mechanism
  - People may not know others preferences or endowments
- There is perfect competition (i.e., everyone is a price taker)
- The only source of information agents are prices

Competitive equilibrium - Definition

Definition
A pair of an allocation and a price vector, $(x^*, p = (p_1, \ldots, p_L))$ is called a competitive equilibrium if the following conditions hold:

$\in \mathbb{R}^I \times \mathbb{R}^L$

$\in \mathbb{R}^L$

1. For all consumers $i = 1, 2, \ldots, I$, $x^{i^*} = (x_1^{i^*}, \ldots, x_L^{i^*})$ solves the following maximization problem:

$$\max_{x^i} u_i(x^i)$$

$$\text{such that } p \cdot x^i \leq p \cdot \omega^i = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

INGRESO $= P_1 \cdot w_1^i + P_2 \cdot w_2^i + \ldots + P_L \cdot w_L^i$

GASTO $= P_1 \cdot X_1^i + P_2 \cdot X_2^i + \ldots + P_L \cdot w_L^i$

2. Markets clear: For each commodity $\ell = 1, 2, \ldots, L$, the following equation holds:

$$\underbrace{\boxed{\sum_{i=1}^{I} x_\ell^{i^*}}}_{\text{DEMANDA AGREGADA } \ell} = \underbrace{\sum_{i=1}^{I} \omega_\ell^i}_{\text{OFERTA AGREGADA DE } \ell}.$$

① DEMANDA MARSHALIANA $= X^{i^*}$

② DD = OO

Competitive equilibrium - Properties

Remark
Suppose that at least one consumer has **strictly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, $p_1, p_2, \ldots, p_L > 0$.*

Remark
Suppose that at least one consumer has **weakly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, there for at least one ℓ, $\boxed{p_\ell > 0}$*

Remark
If (x^*, p) is a competitive equilibrium, then (x^*, cp) for $c \in \mathbb{R}_+ +$ is also a competitive equilibrium.

① MAX $U_i(x_i)$
s.t
$P \cdot x^i \leq P \cdot w^i$

MAX $U_i(x_i)$
s.t
$\cancel{c}P x^i \leq \cancel{c}P w^i$

$\Rightarrow$ DEMANDA MARSHALIANA ES HOMOGENEA GRADO CERO

Competitive equilibrium - Walras' Law

Theorem (Walras' Law)
Suppose that consumer i has weakly monotone preferences and that $\hat{x}^i \in x^{i^*}(p)$. Then

$$p \cdot \hat{x}^i = \sum_{\ell=1}^{L} p_\ell \hat{x}_\ell^i = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i = p \cdot \omega^i.$$

(ME GASTO TODO MI INGRESO)

Theorem (Walras' Law - II)
Suppose that utility functions are **weakly** *monotonic. Suppose that $p = (p_1, \ldots, p_L)$ is such that $\boxed{p_L > 0}$. Take any (x^*, p) in which Condition 1 holds for each consumer $i = 1, 2, \ldots, I$ and markets clear for all commodities $\ell = 1, 2, \ldots, L-1$. Then the market clearing condition will hold for commodity L as well.*

DD MARSHALIANA

DD = OO

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- If we sum the above across all I consumers, then we get:

$$\boxed{\sum_{i=1}^{I}} \sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \boxed{\sum_{i=1}^{I}} \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

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- Re-arranging:

$$\sum_{\ell=1}^{L} \sum_{i=1}^{I} p_\ell x_\ell^{i^*} = \sum_{\ell=1}^{L} \sum_{i=1}^{I} p_\ell \omega_\ell^i.$$

$\rightarrow \sum_{\ell=1}^{L} \sum_{i=1}^{I} P_\ell X_\ell^i - \sum_{\ell=1}^{L} \sum_{i=1}^{I} P_\ell w_\ell^i = \sum_{\ell=1}^{L} \sum_{i=1}^{I} (P_\ell X_\ell^i - P_\ell w_\ell^i)$

$= \sum_{\ell=1}^{L} \sum_{i=1}^{I} P_\ell (X_\ell^i - w_\ell^i)$

$= \sum_{\ell=1}^{L} P_\ell \boxed{\sum_{i=1}^{I} (X_\ell^i - w_\ell^i)}$ $= 0$ $\ell = 1, \ldots, L-1$

$\underbrace{P_L}_{>0} \underbrace{\sum_{i=1}^{I} (X_L^i - w_L^i)}_{=0} = 0$

$$\boxed{\sum_{i=1}^{I} X_L^i = \sum_{i=1}^{I} W_L^i}$$

MERCADO "L"-ESIMO SE VACIA

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- If we sum the above across all I consumers, then we get:

$$\sum_{i=1}^{I} \sum_{\ell=1}^{L} p_\ell x_\ell^{i^*} = \sum_{i=1}^{I} \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- Re-arranging:

Walras' Law - proof

- For each consumer i, we must
$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- If we sum the above across all I consumers, then we get:
$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- Re-arranging:
$$\sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell \omega_\ell^i.$$
- Re-arranging:
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

>0 $\underbrace{\qquad}_{=0}$

$$\boxed{\sum_{i=1}^{I} x_L^i = \sum_{i=1}^{I} w_L^i}$$

MERCADO "L"-ESIMO SE VACIA

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$
-
$$p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$
-
$$p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$
-
$$\sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

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Cobb-Douglas

$$u_A(x,y) = x^\alpha y^{1-\alpha}$$
$$u_B(x,y) = x^\beta y^{1-\beta}$$

Suppose

$$\boxed{\begin{aligned}\alpha &= 0.5\\ \beta &= 0.5\end{aligned}}$$
$$\omega^A = (1.5, 0.5)$$
$$\omega^B = (0.5, 1.5)$$

(1) DD marshallianas

(2) DD AGR = OO AGR

Cobb-Douglas

Each individual solves
$$\max_{x^i, y^i} \sqrt{x^i y^i}$$
s.t.
$$\underbrace{p_x x^i + p_y y^i}_{\text{GASTO}} \le \underbrace{p_x w_x^i + p_y w_y^i}_{\text{INGRESO}}$$

$\neq$

"OPTIMOS PARETO" o PROBLEMA PLANIFICADOR

$$\max_{x^A, y^A, x^B, y^B} \underline{U_A = \sqrt{x^A y^A}} \quad \underline{\underline{\text{s.t.}}} \quad U_B = \sqrt{x^B y^B} \ge \bar{U}$$
$$\left.\begin{aligned} x^A + x^B &\le w_x^A + w_x^B \\ y^A + y^B &\le w_y^A + w_y^B \end{aligned}\right\} \text{FACTIBILIDAD}$$

Cobb-Douglas

Each individual solves
$$\max \sqrt{x^i y^i}$$
s.t.
$$p_x x^i + p_y y^i \le p_x w_x^i + p_y w_y^i$$
We can set up a Lagrangean:
$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{1}{2} x_i^{-1/2} y_i^{1/2} - \lambda P_x = 0 \longrightarrow \frac{1}{2}\frac{y_i^{1/2}}{x_i^{1/2}} = \lambda P_x$$
$$\frac{\partial \mathcal{L}}{\partial y_i} = \frac{1}{2} x^{1/2} y_i^{-1/2} - \lambda P_y = 0 \longrightarrow \frac{1}{2}\frac{x_i^{1/2}}{y_i^{1/2}} = \lambda P_y$$

Cobb-Douglas

Each individual solves
$$\max \sqrt{x^i y^i}$$
s.t.
$$p_x x^i + p_y y^i \le p_x w_x^i + p_y w_y^i$$
We can set up a Lagrangean:
$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$
The FOC are:
$$\frac{1}{2}\sqrt{\frac{y^i}{x^i}} = \lambda p_x$$
$$\frac{1}{2}\sqrt{\frac{x^i}{y^i}} = \lambda p_y$$

$$\longrightarrow \left(\frac{\cancel{\frac{1}{2}}\,\frac{y_i^{1/2}}{x_i^{1/2}}}{\cancel{\frac{1}{2}}\,\frac{x_i^{1/2}}{y_i^{1/2}}}\right) = \frac{\cancel{\lambda} P_x}{\cancel{\lambda} P_y}$$

Cobb-Douglas

Thus,
$$\boxed{\frac{y^i}{x^i} = \frac{p_x}{p_y}}$$
$$\boxed{y^i = x^i \frac{p_x}{p_y}} \longrightarrow \text{A LA RESTRICCIÓN PRESUPUESTAL}$$

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$

$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$

$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

$$\boxed{p_x x^i + p_y y^i = p_x w_x^i + p_y w_y^i}$$

$$p_x x^i + \cancel{p_y} x^i \frac{p_x}{\cancel{p_y}} = p_x w_x^i + p_y w_y^i$$

$$x^i = \frac{w_x^i p_x + w_y^i p_y}{2p_x}$$

$$y^i = \frac{w_x^i p_x + w_y^i p_y}{2p_y}$$

$2P_x \cdot X = P_x W_x + P_y W_y$

$X_i = \dfrac{P_x W_x^i + P_y W_y^i}{2P_x} = \dfrac{I}{2P_x}$

$Y_i = \dfrac{W_x^i P_x + P_y W_y^i}{2P_y} = \dfrac{I}{2P_y}$

Cobb-Douglas

$$x^A = \frac{\overbrace{1.5p_x + 0.5p_y}^{I_A}}{2p_x}$$

$$y^A = \frac{1.5p_x + 0.5p_y}{2p_y}$$

$$x^B = \frac{0.5p_x + 1.5p_y}{2p_x}$$

$$y^B = \frac{0.5p_x + 1.5p_y}{2p_y}$$

$\to I_B$

Now we can use condition 2 (market clear)

Cobb-Douglas

$$x^A = \frac{1.5p_x + 0.5p_y}{2p_x}$$

$$y^A = \frac{1.5p_x + 0.5p_y}{2p_y}$$

$$x^B = \frac{0.5p_x + 1.5p_y}{2p_x}$$

$$y^B = \frac{0.5p_x + 1.5p_y}{2p_y}$$

Now we can use condition 2 (market clear)

$$\boxed{x^A + x^B} = 2$$

$$y^A + y^B = 2$$

$= W_x^A + W_x^B = 2$ (1.5 + 0.5)

$= W_y^A + W_y^B = 2$ (0.5 + 1.5)

Cobb-Douglas

$$\underbrace{\frac{1.5p_x + 0.5p_y}{2p_x}}_{X_A} + \underbrace{\frac{0.5p_x + 1.5p_y}{2p_x}}_{X_B} = 2$$

$$\frac{p_x}{p_y} = 1$$

$\dfrac{2P_x + 2P_y}{2P_x} = 2$

$\dfrac{P_x}{P_x} + \dfrac{P_y}{P_x} = 2$

$1 + \dfrac{P_y}{P_x} = 2$

$\boxed{\dfrac{P_y}{P_x} = 1}$

Cobb-Douglas

$$\underbrace{\frac{1.5p_x + 0.5p_y}{2p_x}}_{X^A} + \frac{0.5p_x + 1.5p_y}{2p_x} = 2$$

$$\frac{p_x}{p_y} = 1$$

$$\boxed{x^A = x^B = y^A = y^B = 1}$$

$X^A = \dfrac{1.5P_x}{2P_x} + \dfrac{0.5P_y}{2P_x}$

$= \dfrac{1.5}{2} + \dfrac{0.5}{2}(1)$

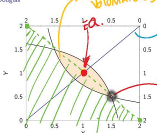
$= \dfrac{2}{2} = 1$

$X^B = 1 = (2 - X^A)$

$Y^A = \dfrac{1.5P_x + 0.5P_y}{2P_y} = \dfrac{1.5}{2}\dfrac{P_x}{P_y} + \dfrac{0.5}{2} = \dfrac{2}{2} = 1$

$Y^B = 1 = (2 - Y^A)$

Cobb-Douglas



$P_x \cdot X + P_y \cdot Y \leq 1.5P_x + 0.5P_y$

Restricción A.

$Y^A \leq \dfrac{1.5P_x - P_x X^A + 0.5P_y}{P_y}$

$Y^A \leq 1.5\dfrac{P_x}{P_y} - \dfrac{P_x}{P_y}X^A + 0.5$

$Y \leq 1.5 - X + 0.5$

$Y^A \leq 2 - X^A$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

EL EQ

$X = \underbrace{(1, 1, 1, 1)}_{\text{Asignacion}}$ (X_A, Y_A, X_B, Y_B)

$P = (P_x, P_y)$ I.G $\dfrac{P_x}{P_y} = 1$

$P = (1, 1)$ ✓

$P = (2, 2)$ ✓

$P = (3, 3)$ ✓

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

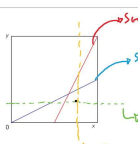
Examples: Perfect Complements ✓

Examples: Perfect Substitutes

Perfect complements

Suppose that

$$u_A(x^A, y^A) = \min(x^A, 2y^A)$$
$$u_B(x^B, y^B) = \min(2x^B, y^B)$$
$$\omega^A = (3, 1)$$
$$\omega^B = (1, 3)$$



Problema A

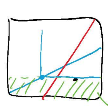
MAX $U_A = \min(x^A, 2y^A)$

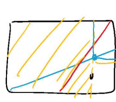
s.t. $P_x X^A + P_y Y^A \leq P_x W_x^A + P_y W_y^A$

$Y^A \leq \dfrac{P_x W_x^A + P_y W_y^A - P_x X^A}{P_y}$

$Y^A \leq \dfrac{P_x W_x}{P_y} + W_y^A - \dfrac{P_x}{P_y} X^A$

A





$P_x = 0$

ó

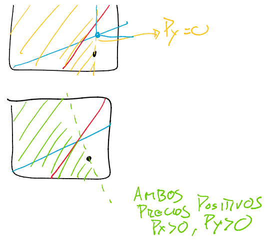
Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

↳ $P_y = 0$

$P_x = 0$
ó
$P_y = 0$

2



Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$

Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$

How does this looks in the Edgeworth box?

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

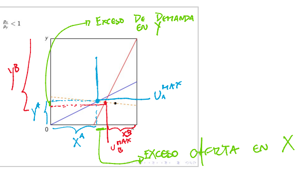
Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

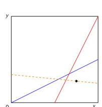
Note that, for the case $\frac{p_x}{p_y} = 1$, we have the following restriction:

$$y^A \leq 4 - x^A$$

$\frac{p_x}{p_y} < 1$

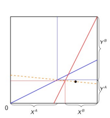


$\frac{p_x}{p_y} < 1$



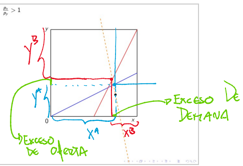
A can buy whats below the orange line, B what is above

$\frac{p_x}{p_y} < 1$

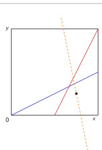


Excess demand of Y and excess supply of X

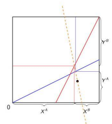
$\frac{p_x}{p_y} > 1$



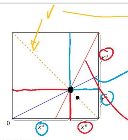
$\frac{p_x}{p_y} > 1$



$\frac{p_x}{p_y} > 1$



ES EQUILIBRIO



No excess demand or supply

$U^A = \min(x, 2y)$

$x = 2y$

$\boxed{Y^A = \frac{X^A}{2}}$

$U^B = \min(2x^B, y^B)$

$2x^B = y^B$

$2(\underbrace{4 - X^A}_{X^B}) = \underbrace{4 - Y^A}_{Y^B} \Rightarrow 8 - 2X^A = 4 - Y^A$

$4 - 2X^A = -Y^A$

$\boxed{Y^A = 2X^A - 4}$

$\frac{X^A}{2} = 2X^A - 4$

$X^A = 4X^A - 8$

$8 = 3X^A$

$\boxed{X^A = 8/3}$ $\boxed{Y^A = \frac{8}{6}}$

$\boxed{X^B = 4 - 8/3 = 4/3}$ $\boxed{Y^B = 4 - \frac{4}{3} = \frac{8}{3}}$

$Y^A = \frac{P_x}{P_y}(3) + (1) - \frac{P_x}{P_y} X^A$

$Y^A = \frac{P_x}{P_y} 3 + 1 - \frac{P_x}{P_y} X^A$

$\boxed{\frac{4}{3}} = \frac{P_x}{P_y} 3 + 1 - \frac{P_x}{P_y} \boxed{\frac{8}{3}}$

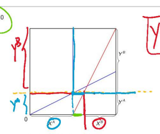
$\frac{1}{3} = \frac{P_x}{P_y} \frac{1}{3}$

$\boxed{1 = \frac{P_x}{P_y}}$

What about zero prices?

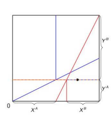
$p_x = 0$

Y ESTA EN EQ



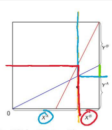
Excess supply of X? (and Y balanced?)

$p_x = 0$



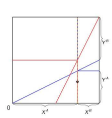
Excess supply of X? (and Y balanced?) Not really since both A and B are indifferent over a wide range that would make the market clear

$p_y = 0$



Excess supply of Y? (and X balanced?)

$p_y = 0$



Excess supply of Y? (and X balanced?) Not really since both A and B are indifferent over a wide range that would make the market clear

To sum up...

- There are multiple equilibria
- There are three price vectors associated with these equilibria
- One price vector has a unique resource allocation associated with it
- Two price vectors ($p_x = 0$ and $p_y = 0$) have *infinity* resource allocations associated with them

- $P_x = 0$, $P_y \in \mathbb{R}_+$
- $P_y = 0$, $P_x \in \mathbb{R}_+$
- $P_x / P_y = 1$

$P_x / P_y = 1$

Perfect complements

Try at home:

$$u_A(x^A, y^A) = \min(x^A, y^A)$$
$$u_B(x^B, y^B) = \min(x^B, y^B)$$
$$\omega^A = (1, 1)$$
$$\omega^B = (3, 1)$$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Perfect Substitutes

MAX $2X^A + Y^A$

s.t.

$U = 2x + y$

2 utiles/x $P_x = 2$

1 util/y $P_y = 1$

$X^A(P_x) = \begin{cases} \frac{I}{P_x} & P_x < 2 \\ [0, \frac{I}{2}] & P_x = 2 \end{cases}$

Perfect Substitutes

$$\boxed{\begin{aligned} u_A(x^A,y^A) &= 2x^A + y^A \\ u_B(x^B,y^B) &= x^B + y^B \\ \omega^A &= (1,1) \\ \omega^B &= (1,1) \end{aligned}}$$

Perfect Substitutes

$$\begin{aligned} u_A(x^A,y^A) &= 2x^A + y^A \\ u_B(x^B,y^B) &= x^B + y^B \\ \omega^A &= (1,1) \\ \omega^B &= (1,1) \end{aligned}$$

$p_x > 0$ and $p_y > 0$, why?

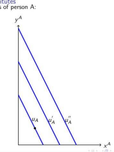
Perfect Substitutes

$$\begin{aligned} u_A(x^A,y^A) &= 2x^A + y^A \\ u_B(x^B,y^B) &= x^B + y^B \\ \omega^A &= (1,1) \\ \omega^B &= (1,1) \end{aligned}$$

$p_x > 0$ and $p_y > 0$, why? hence, normalize $p_x = 1$

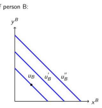
Perfect Substitutes

Peferences of person A:



Perfect Substitutes

Peferences of person B:



Perfect Substitutes

Algebraic solution

$$\max_{x^A,y^A} 2x^A + y^A$$

subject to:

$$\begin{aligned} I &= x^A + p_y y^A \\ y^A &\geq 0 \\ x^A &\geq 0 \end{aligned}$$

Perfect Substitutes

Algebraic solution

$$\max_{x^A,y^A} 2x^A + y^A$$

subject to:

$$\begin{aligned} I &= x^A + p_y y^A \\ y^A &\geq 0 \\ x^A &\geq 0 \end{aligned}$$

From the budget constraint we can obtain $y^A = \frac{I - x^A}{p_y}$, and adding the condition $y^A \geq 0$, we can conclude that $x^A \in [0, I]$.

Perfect Substitutes

Introducing y^A into the original maximization problem:

$$\max(2 - \frac{1}{p_y})x^A + \frac{I}{p_y} \quad s.t. \; x^A \in [0, I]$$

Which is a maximization of a straight line with slope $\left(2 - \frac{1}{p_y}\right)$ over an interval.

Perfect Substitutes

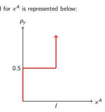
The demand for goods of individual A is

$$X^A = \begin{cases} 0 & \text{if } p_y < 0.5 \\ [0, I] & \text{if } p_y = 0.5 \\ I & \text{if } p_y > 0.5 \end{cases}$$

$$Y^A = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 0.5 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 0.5 \\ 0 & \text{if } p_y > 0.5 \end{cases}$$

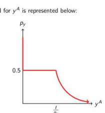
Perfect Substitutes

The demand for x^A is represented below:



Perfect Substitutes

The demand for y^A is represented below:



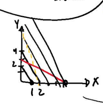
MAX $2X^A + Y^A$
s.t.
$P_X X + P_Y Y \leq \underbrace{P_x + P_y}_{\text{INGRESO}}$

$\to U = 2X + Y$

~~LAGRANGE~~

$\to Y \leq \frac{P_x + P_y - P_x X}{P_y}$

$Y \leq \frac{P_x}{P_y} + 1 - \frac{P_x}{P_y} \cdot X$



2 UTILES/X $\quad P_x = 2$
1 UTIL/1Y $\quad P_y = 1$

$$X^*(P_x) = \begin{cases} \frac{I}{P_x} & P_x < 2 \\ [0, \frac{I}{P_x}] & P_x = 2 \\ 0 & P_x > 2 \end{cases}$$

$$Y^*(P_x) = \begin{cases} 0 & P_x < 2 \\ [0, I] & P_x = 2 \\ I & P_x > 2 \end{cases}$$

$\to I/P_y = I$

$P_y = 1$

$U_A(X_A, Y_A) = 2X_A + Y_A$

$X \Rightarrow \frac{2 \text{ UTILES}}{P_X}$

$Y \Rightarrow \frac{1 \text{ UTIL}}{P_Y}$

TODO X

$\frac{2 \text{ UTILES}}{P_X} > \frac{1 \text{ UTIL}}{P_Y}$

$2 > \frac{P_X}{P_Y}$

$2 > P_Y$

TODO Y

$\frac{2U}{P_X} < \frac{1U}{P_Y}$

$2 < \frac{P_X}{P_Y}$

$2 < P_X$

INDIF (X,Y)

$\frac{2U}{P_X} = \frac{1U}{P_Y}$

$2 = \frac{P_X}{P_Y}$

$2 = P_X$

$U_B = X^B + Y^B$

$X \Rightarrow \frac{1U}{P_X}$

$Y \Rightarrow \frac{1U}{P_Y}$

TODO X

$\frac{1U}{P_X} > \frac{1U}{P_Y}$

$1 > \frac{P_X}{P_Y}$

TODO Y

$\frac{1U}{P_X} < \frac{1U}{P_Y}$

$1 < \frac{P_X}{P_Y}$

INDIF

$\frac{1U}{P_X} = \frac{1U}{P_Y}$

$1 = \frac{P_X}{P_Y}$

$$X^B(P_X) = \begin{cases} I/P_X & P_X < 1 \\ [0, I/P_X] & P_X = 1 \\ 0 & P_X > 1 \end{cases}$$

$$Y^B(P_X) = \begin{cases} 0 & P_X < 1 \\ [0, I] & P_X = 1 \\ I & P_X > 1 \end{cases}$$

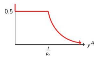
$\to I/P_Y = I$

PASO 2

P_X: (0,1) NO EQ	1	(1,2)	2	$(2,\infty)$
$\underbrace{\underbrace{\frac{I^A}{P_X}}_{DD_A} + \underbrace{\frac{I^B}{P_X}}_{DD_B}}_{\text{DD AGREGADA}}$ vs. $\underbrace{2}_{\text{OO AGREG.}}$; $\frac{P_X + P_Y}{P_X} + \frac{P_X + P_Y}{P_X}$; $\frac{2P_X + 2}{P_X}$; $\underbrace{2 + \frac{2}{P_X}}_{DD} > \underbrace{2}_{OO}$	$\frac{I^A}{P_X} + DD_B$ vs. 2; $\frac{P_X + P_Y}{P_X} + DD_B$ vs. 2; $\frac{2}{1} + DD_B$ vs. 2; $2 + DD_B$ vs. 2; $\Rightarrow DD_B = 0$ PARA Q' OO = DD; **EQ** $(P_X = 1, P_Y = 1)$: $X^A = 2$, $X^B = 0$, $Y^A = 0$, $Y^B = 2 = \frac{P_X + P_Y}{P_Y} = \frac{2}{1}$	2 vs. $\frac{I^A}{P_X} + 0$; $\frac{P_X + P_Y}{P_X}$; $1 + \frac{P_Y}{P_X}$; $\boxed{1 + \frac{1}{P_X} < 2}$ No EQ	$DD^A + 0$; $\frac{I^A}{P_X}$; $\frac{P_X + P_Y}{P_X}$; $\frac{2+1}{2}$; $\frac{3}{2} < 2$; NO EQ	$\underbrace{0 + 0}_{DD} < \underbrace{2}_{OO}$; NO EQ

$U^A = X^A + Y^A$

$$X^A = \begin{cases} I^A/P_X & P_X < 1 \\ [0, I/P_X] & P_X = 1 \end{cases}$$



Perfect Substitutes

Algebraic solution
For person B the solution is analogous, but we have the following maximization problem: Introducing y^A into the original maximization problem:

$$\max(1 - \frac{1}{p_y})x^B + \frac{I}{p_y} \quad s.t.\ x^B \in [0, I]$$

Which is a maximization of a straight line with slope $\left(1 - \frac{1}{p_y}\right)$ over an interval.

$$U^A = X^A + Y^A$$
$$U^B = X^B + Y^B$$

$$X^A = \begin{cases} I^A/P_x & P_x < 1 \\ [0, I/P_x] & P_x = 1 \\ 0 & P_x > 1 \end{cases}$$

$$X^B = \begin{cases} I^B/P_x & P_x < 1 \\ [0, I/P_x] & P_x = 1 \\ 0 & P_x > 1 \end{cases}$$

Perfect Substitutes

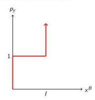
The demand for goods of individual B is

$$X^B = \begin{cases} 0 & \text{if } p_y < 1 \\ [0, I] & \text{if } p_y = 1 \\ I & \text{if } p_y > 1 \end{cases}$$

$$Y^B = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 1 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 1 \\ 0 & \text{if } p_y > 1 \end{cases}$$

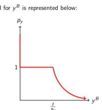
Perfect Substitutes

The demand for x^B is represented below:



Perfect Substitutes

The demand for y^B is represented below:



Perfect Substitutes

When is the market for good X balanced (how about good y?)

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$
- Try $p_y > 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$
- Try $p_y > 1$
- $X^A = I$ and $X^B = I$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$
- Try $p_y > 1$
- $X^A = I$ and $X^B = I$
- $X^A = I = 1 + p_y$ and $X^B = I = 1 + p_y$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B = 2 + 2p_y > 2$