Lecture 3

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Lecture 3: General Equilibrium

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Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

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Hidden assumptions

- There is a market for each good
- Every agent can access the market without any cost
- There is a unique price for each good and all consumers know this price
- Each consumer can sell her initial endowment in the market and use the income to buy goods and services
- Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.
  - There is no centralized mechanism
  - People may not know others preferences or endowments
- There is perfect competition (i.e., everyone is a price taker)
- The only source of information agents are prices

Competitive equilibrium - Definition

Definition

A pair of an allocation and a price vector, $(x^*, p = (p_1, \ldots, p_L))$ is called a competitive equilibrium if the following conditions hold:

1. For all consumers $i = 1, 2, \ldots, I$, $x^{i*} = (x_1^{i*}, \ldots, x_L^{i*})$ solves the following maximization problem:

$$\max_{x^i} u_i(x^i)$$

$$\text{such that } p \cdot x^i \le p \cdot \omega^i = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

2. Markets clear: For each commodity $\ell = 1, 2, \ldots, L$, the following equation holds:

$$\underbrace{\sum_{i=1}^{I} x_\ell^{i*}}_{\text{DD AGREGADA}} = \underbrace{\sum_{i=1}^{I} \omega_\ell^i}_{\text{OO AGREGADA}}.$$

DDD MARSHALIANA

$x^{i*} = \text{ARG}\max_{x_i} U_i(x_i)$

s.t. $\underbrace{P \cdot x^i}_{\text{GASTO}} \le \underbrace{P \cdot w^i}_{\text{INGRESO}}$

ASIGNACIÓN RECURSOS

$P_1 x_1^i + P_2 x_2^i + \ldots + P_L x_L^i$

$P_1 w_1^i + P_2 \cdot w_2^i + \ldots + P_L w_L^i$

Competitive equilibrium - Properties

Remark

Suppose that at least one consumer has **strictly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, $p_1, p_2, \ldots, p_L > 0$.*

Remark

Suppose that at least one consumer has **weakly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, there for at least one ℓ, $p_\ell > 0$.*

Remark

If (x^*, p) is a competitive equilibrium, then (x^*, cp) for $c \in \mathbb{R}_+ +$ is also a competitive equilibrium.

TASA CAMBIO

① $x^{i*} = \text{ARG}\max_{x_i} U_i(x_i)$

s.t. $P \cdot x^i \le P \cdot w^i$ = s.t. $\cancel{c}P \cdot x^i \le \cancel{c}P \cdot w^i$

Competitive equilibrium - Walras' Law

Theorem (Walras' Law)

Suppose that consumer i has weakly monotone preferences and that $\hat{x}^i \in x^{i*}(p)$. Then

$$p \cdot \hat{x}^i = \sum_{\ell=1}^{L} p_\ell \hat{x}_\ell^i = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i = p \cdot \omega^i.$$

Theorem (Walras' Law - II)

Suppose that utility functions are **weakly** *monotonic. Suppose that $p = (p_1, \ldots, p_L)$ is such that $p_L > 0$. Take any (x^*, p) in which Condition 1 holds for each consumer $i = 1, 2, \ldots, I$ and markets clear for all commodities $\ell = 1, 2, \ldots, L-1$. Then the market clearing condition will hold for commodity L as well.*

Walras' Law - proof

- For each consumer i, we must

$$P \cdot X_i = \underbrace{\sum_{\ell=1}^{L} p_\ell x_\ell^{i*}}_{\text{GASTO}} = \underbrace{\sum_{\ell=1}^{L} p_\ell \omega_\ell^i}_{\text{INGRESO}}. = P \cdot W_i$$

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- If we sum the above across all I consumers, then we get:

$$\sum_{i=1}^{I} \sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I} \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

Walras' Law - proof

- For each consumer i, we must
$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- If we sum the above across all I consumers, then we get:
$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- Re-arranging:
$$\sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell \omega_\ell^i.$$

$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} x_\ell^i = \sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \omega_\ell^i \Rightarrow 0 = \sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} x_\ell^i - \sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \omega_\ell^i$$

$$\sum_{\ell=1}^{L} p_\ell \left(\sum_{i=1}^{I} x_\ell^i - \sum_{i=1}^{I} \omega_\ell^i\right)$$

$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} (x_\ell^i - \omega_\ell^i) = 0$$

Walras' Law - proof

- For each consumer i, we must
$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- If we sum the above across all I consumers, then we get:
$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$
- Re-arranging:
$$\sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell \omega_\ell^i.$$
- Re-arranging:
$$\sum_{\ell=1}^{L} p_\ell \boxed{\sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right)} = 0.$$

$= 0$ si $\ell = 1, 2, \ldots, L-1$

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

$= 0$ PARA $\ell = 1, \ldots, L-1$

-
$$p_L \sum_{i=1}^{I}\left(x_L^{i*} - \omega_L^i\right) = 0.$$

$\cancel{p_L = 0}$ o $\sum_{i=1}^{I} x_L^i - \omega_L^i = 0$

MERCADO L SE VACIA

Walras' Law - proof

-
$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I}\left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$
-
$$p_L \sum_{i=1}^{I}\left(x_L^{i*} - \omega_L^i\right) = 0.$$
-
$$\sum_{i=1}^{I}\left(x_L^{i*} - \omega_L^i\right) = 0.$$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas /

Examples: Perfect Complements /

Examples: Perfect Substitutes /

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Cobb-Douglas

$$u_A(x,y) = x^\alpha y^{1-\alpha} \checkmark$$
$$u_B(x,y) = x^\beta y^{1-\beta}$$

Suppose

$$\boxed{\begin{aligned}\alpha &= 0.5\\ \beta &= 0.5\end{aligned}}$$
$$\omega^A = (1.5, 0.5)$$
$$\omega^B = (0.5, 1.5)$$

① DD MARSHALLIANAS

② VACIAR MERCADOS

Cobb-Douglas

Each individual solves

$$\max_{x_i, y_i} \sqrt{x^i y^i} = x^{1/2} y^{1/2}$$

$\alpha = 1/2 = \beta$

s.t.

$$p_x x^i + p_y y^i \le p_x w_x^i + p_y w_y^i$$

PLANIFICADOR PARA ENCONTRAR O.P.

$$\max_{x^A, y^A, x^B, y^B} U^A = \sqrt{x^A y^A}$$

s.t.

$$\sqrt{x^B y^B} \ge \bar{U}_B$$

$$\left.\begin{aligned} x^A + x^B &\le w_x^A + w_x^B = 2\\ y^A + y^B &\le w_y^A + w_y^B = 2\end{aligned}\right\} \text{FACTIBLE}$$

Cobb-Douglas

Each individual solves

$$max \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \le \underbrace{p_x w_x^i + p_y w_y^i}_{I}$$

We can set up a Lagrangean:

$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

$$(I - P_x X - P_y Y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2} x^{\frac{1}{2}-1} y^{1/2} - \lambda P_x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2} x^{1/2} y^{\frac{1}{2}-1} - \lambda P_y = 0$$

$$\frac{1}{2}\frac{y^{1/2}}{x^{1/2}} = \lambda P_x$$

$$\frac{1}{2}\frac{x^{1/2}}{y^{1/2}} = \lambda P_y$$

$$\frac{\cancel{\frac{1}{2}}\frac{y^{1/2}}{x^{1/2}}}{\cancel{\frac{1}{2}}\frac{x^{1/2}}{y^{1/2}}} = \frac{P_x}{P_y}$$

Cobb-Douglas

Each individual solves

$$max \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \le p_x w_x^i + p_y w_y^i$$

We can set up a Lagrangean:

$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

The FOC are:

$$\frac{Y}{\ldots} = \frac{P_x}{P_{\ldots}}$$

Cobb-Douglas

Each individual solves

$$\max \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \le p_x w_x^i + p_y w_y^i$$

We can set up a Lagrangean:

$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

The FOC are:

$$\frac{1}{2}\sqrt{\frac{y^i}{x^i}} = \lambda p_x$$
$$\frac{1}{2}\sqrt{\frac{x^i}{y^i}} = \lambda p_y$$

$$\frac{\cancel{\frac{1}{2}}\frac{y^{1/2}}{x^{1/2}}}{\cancel{\frac{1}{2}}\frac{x^{1/2}}{y^{1/2}}} = \frac{P_x}{P_y}$$

$$\frac{Y}{X} = \frac{P_x}{P_y}$$

$$Y = \frac{x P_x}{P_y}$$

$$P_x X + P_y Y = P_x W_x + P_y W_y$$

$$P_x X + \cancel{P_y}\cdot\left(\frac{x P_x}{\cancel{P_y}}\right) = P_x W_x + P_y W_y$$

$$2 P_x \cdot X = P_x W_x + P_y W_y$$

$$X^i = \frac{P_x W_x^i + P_y W_y^i}{2 P_x}$$

$$Y^i = \frac{P_x W_x^i + P_y W_y^i}{2 P_y}$$

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

$$p_x x^i + p_y y^i = p_x w_x^i + p_y w_y^i$$
$$p_x x^i + p_y x^i \frac{p_x}{p_y} = p_x w_x^i + p_y w_y^i$$
$$x^i = \frac{w_x^i p_x + w_y^i p_y}{2p_x} = \frac{I}{2P_x}$$
$$y^i = \frac{w_x^i p_x + w_y^i p_y}{2p_y} = \frac{I}{2P_y}$$

Cobb-Douglas

$$x^A = \frac{\overset{w_x^A}{1.5} p_x + \overset{w_y^A}{0.5} p_y}{2p_x}$$
$$y^A = \frac{1.5 p_x + 0.5 p_y}{2p_y}$$
$$x^B = \frac{0.5 p_x + 1.5 p_y}{2p_x}$$

$0.5 \to w_x^B$

$1.5 \to w_y^B$

$$y^B = \frac{0.5 p_x + 1.5 p_y}{2p_y}$$

Now we can use condition 2 (market clear)

Cobb-Douglas

$$x^A = \frac{1.5 p_x + 0.5 p_y}{2p_x}$$
$$y^A = \frac{1.5 p_x + 0.5 p_y}{2p_y}$$
$$\boxed{x^B = \frac{0.5 p_x + 1.5 p_y}{2p_x}}$$
$$y^B = \frac{0.5 p_x + 1.5 p_y}{2p_y}$$

Now we can use condition 2 (market clear)

DD $x^A + x^B = 2$ OO $= W_x^A + W_x^B = 1.5 + 0.5 = 2$

$$y^A + y^B = 2$$

Cobb-Douglas

DD AGREGADA

$$\underbrace{\frac{1.5 p_x + 0.5 p_y}{2p_x}}_{X^A} + \underbrace{\frac{0.5 p_x + 1.5 p_y}{2p_x}}_{X^B} = 2$$
$$\frac{p_x}{p_y} = 1$$

→ OFERTA AGREGADA

$$\frac{2P_x + 2P_y}{2P_x} = 2$$

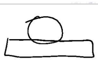
$$\frac{2P_x}{2P_x} + \frac{2P_y}{2P_x} = 2$$

$$1 + \frac{P_y}{P_x} = 2$$

$$\boxed{\frac{P_y}{P_x} = 1}$$

Cobb-Douglas

$$\frac{1.5p_x + 0.5p_y}{2p_x} + \frac{0.5p_x + 1.5p_y}{2p_x} = 2$$
$$\frac{p_x}{p_y} = 1$$
$$x^A = x^B = y^A = y^B = 1$$



$$X^{A}_{EQ} = \frac{1.5P_x + 0.5P_y}{2P_x} = \frac{1.5}{2} + \frac{0.5}{2}\cdot\left(\frac{P_y}{P_x}\right)^{1} = \frac{2}{2} = 1$$

$$X^{B}_{EQ} = 2 - X^A = 1$$

$$Y^{A}_{EQ} = \frac{1.5P_x + 0.5P_y}{2P_y} = \frac{1.5}{2}\left(\frac{P_x}{P_y}\right)^{1} + \frac{0.5}{2} = \frac{2}{2} = 1$$

$$Y^{B}_{EQ} = 2 - Y^A = 1$$

EL EQUILIBRIO COMPETITIVO ES

$$X^{EQ} = (\underset{x^A}{1}, \underset{y^A}{1}, \underset{x^B}{1}, \underset{y^B}{1})$$

$$P^{EQ} = (P_x, P_y) \text{ T.G } \frac{P_x}{P_y} = 1$$

$(1, 1)$

$(2, 2)$

(π, π)

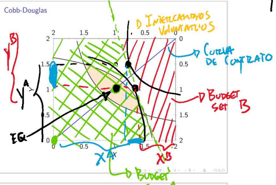
$$X^A = \frac{1.5}{2} + \frac{0.5}{2}\left(\frac{P_y}{P_x}\right)^{1/2} = \frac{1.5}{2} + \frac{0.5}{4} = \frac{3.5}{4}$$

$$X^B = \frac{0.5}{2} + \frac{1.5}{2}\left(\frac{P_y}{P_x}\right)^{1/2} = \frac{0.5}{2} + \frac{1.5}{4} = \frac{2.5}{4}$$

$$\frac{P_x}{P_y} = 2$$

$$X^A + X^B = \frac{6}{4} = \frac{3}{2} < 2 = \text{OFERTA}$$

Cobb-Douglas

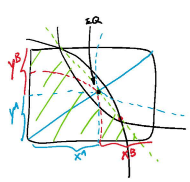


$$P_x \cdot X^A + P_y Y^A \le P_x W_x^A + P_y W_y^A$$

$$Y^A \le \frac{P_x W_x^A + P_y W_y^A - P_x \cdot X}{P_y}$$

$$Y^A \le \underbrace{\frac{P_x}{P_y} W_x^A + W_y^A}_{\text{INTERCEPTO}} \underbrace{- \frac{P_x}{P_y}}_{\text{PENDIENTE}} \cdot X$$

"="



Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Lecture 3: General Equilibrium

Perfect complements

Suppose that

$$u_A(x^A, y^A) = \min(x^A, 2y^A)$$
$$u_B(x^B, y^B) = \min(2x^B, y^B)$$
$$\omega^A = (3, 1)$$
$$\omega^B = (1, 3)$$

MAX $U_A = \min(x^A, 2y^A)$

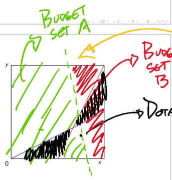
s.a

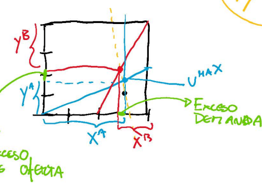
$3P_x + P_y \geq P_x X^A + P_y Y^A$

$$Y^A \leq \frac{3P_x + P_y - P_x \cdot X^A}{P_y}$$

$$Y^A \leq 3\frac{P_x}{P_y} + 1 - \frac{P_x}{P_y} \cdot X^A$$

$\frac{P_x \uparrow}{P_y \downarrow}$

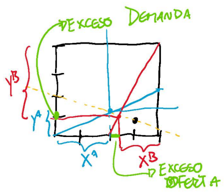




Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$



$\frac{P_x \downarrow}{P_y \uparrow}$

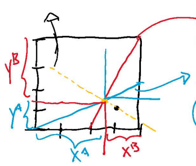
Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$



$2X^B = Y^B$ $(U_B = \min(2X^B, Y^B))$

$2(\underbrace{4 - X^A}_{X^B}) = \underbrace{4 - Y^A}_{Y^B} \Rightarrow 8 - 2X^A = 4 - Y^A$

$4 - 2X^A = -Y^A$

$\boxed{2X^A - 4 = Y^A}$

$X = 2Y$ $(U_A = \min(X^A, 2Y^A))$

$Y^A = \frac{X^A}{2}$

$\frac{X^A}{2} = 2X^A - 4$

$X^A = 4X^A - 8$

$8 = 3X^A$

$\boxed{\frac{8}{3} = X^A}$

$X^B = 4 - \frac{8}{3}$

$\boxed{X^B = \frac{4}{3}}$

$Y^A = \frac{8/3}{2} = \frac{8}{6}$

$\boxed{Y^A = 4/3}$

$Y^B = 4 - 4/3$

$\boxed{Y^B = 8/3}$

Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$

How does this looks in the Edgeworth box?

$$Y^A \leq \frac{3P_x}{P_y} + 1 - \frac{P_x}{P_y} X^A$$

$$\frac{4}{3} = \frac{3P_x}{P_y} + 1 - \frac{P_x}{P_y}\left(\frac{8}{3}\right)$$

$$\left(\frac{4}{3} - 1\right) = \frac{P_x}{P_y}\left(3 - \frac{8}{3}\right)$$

$$\frac{1}{3} = \frac{P_x}{P_y}\left(\frac{1}{3}\right)$$

$$1 = \frac{P_x}{P_y}$$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

(P_x, P_y) T.G $\frac{P_x}{P_y} = 1$

$(1, 1)$

$(2, 2)$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

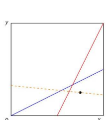
Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

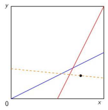
Note that, for the case $\frac{p_x}{p_y} = 1$, we have the following restriction:

$$y^A \leq 4 - x^A$$

$\frac{p_x}{p_y} < 1$

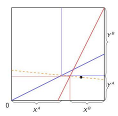


$\frac{p_x}{p_y} < 1$



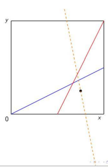
A can buy whats below the orange line, *B* what is above

$\frac{p_x}{p_y} < 1$

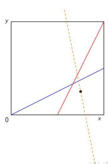


Excess demand of Y and excess supply of X

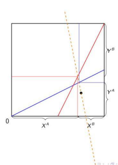
$\frac{p_x}{p_y} > 1$



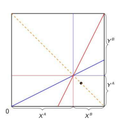
$\frac{p_x}{p_y} > 1$



$\frac{p_x}{p_y} > 1$

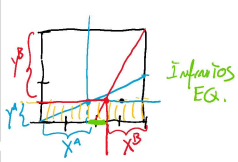


$\frac{p_x}{p_y} = 1$

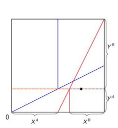


No excess demand or supply

What about zero prices?

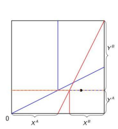


$\boxed{p_x = 0}$



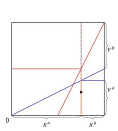
Excess supply of X? (and Y balanced?)

$p_x = 0$



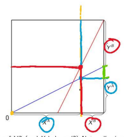
Excess supply of X? (and Y balanced?) Not really since both *A* and *B* are indifferent over a wide range that would make the market clear

$p_y = 0$



Excess supply of Y? (and X balanced?)

$p_y = 0$



Excess supply of Y? (and X balanced?) Not really since both A and B are indifferent over a wide range that would make the market clear

To sum up...

- There are multiple equilibria
- There are three price vectors associated with these equilibria
- One price vector has a unique resource allocation associated with it
- Two price vectors ($p_x = 0$ and $p_y = 0$) have *infinity* resource allocations associated with them

1) $P_x/P_y = 1$
2) $P_x = 0$ ($P_y \in \mathbb{R}_+$)
3) $P_y = 0$ ($P_x \in \mathbb{R}_+$)

$P_x/P_y = 1 \Rightarrow (1,1), (2,2), (c,c)$ $c \in \mathbb{R}_+$

Perfect complements

Try at home:

$$u_A(x^A, y^A) = \min(x^A, y^A)$$
$$u_B(x^B, y^B) = \min(x^B, y^B)$$
$$\omega^A = (1,1)$$
$$\omega^B = (3,1)$$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

Perfect Substitutes

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$u_B(x^B, y^B) = x^B + y^B$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

1) $\max_{x^A, y^A} U_A = 2x^A + y^A$

s.t

$P_x x^A + P_y y^A \leq P_x \cdot 1 + P_y \cdot 1$

$\mathcal{L} = 2x^A + y^A + \lambda(I - P_x x - P_y y)$

$\frac{\partial \mathcal{L}}{\partial x} = 2 - \lambda P_x$

$\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda P_y$

2 utiles/x

1 util/y

$P_x = \left(\begin{array}{c}\text{el precio}\\ \text{relativo}\end{array}\right)$

1 → Todo de X
2 → Indif entre X y Y
4 → Todo en Y

$P_y = 1$

$(P_x, P_y) = EQ$

$\left(\frac{P_x}{P_y}, 1\right) = EQ$

$U_A = 2x^A + y^A$

Gasta X	Gasta Y
$\frac{2 \text{ utiles}}{P_x}$	$\frac{1 \text{ util}}{P_y}$

Todo X	Todo Y	Indiff (x, y)
$\frac{2U}{P_x} > \frac{1U}{P_y}$	$\frac{2U}{P_x} < \frac{1U}{P_y}$	$2 = \frac{P_x}{P_y}$
$2 > \frac{P_x}{P_y}$	$2 < \frac{P_x}{P_y}$	$2 = P_x$
$2 > P_x$	$2 < P_x$	

$\boxed{P_y = 1}$

$$X^A(P_x) = \begin{cases} I/P_x & P_x < 2 \\ [0, I/P_x] & P_x = 2 \\ 0 & P_x > 2 \end{cases}$$

Perfect Substitutes

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$u_B(x^B, y^B) = x^B + y^B$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

$p_x > 0$ and $p_y > 0$, why?

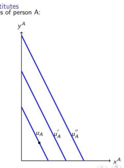
Perfect Substitutes

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$u_B(x^B, y^B) = x^B + y^B$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

$p_x > 0$ and $p_y > 0$, why? hence, normalize $p_x = 1$

Perfect Substitutes

Peferences of person A:



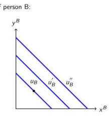
B) $U_B = x^B + y^B$

Gasta X	Gasta Y	
$\frac{1U}{P_x}$	$\frac{1U}{P_y}$	
Todo X	**Todo Y**	**Indif**
$\frac{1U}{P_x} > \frac{1U}{\cancel{P_y} \to 1}$	$\frac{1U}{P_x} < \frac{1U}{\cancel{P_y} \to 1}$	$\frac{1U}{P_x} = \frac{1U}{P_x}$
$1 > P_x$	$1 < P_x$	$1 = P_x$

$$X^B(P_x) = \begin{cases} I/P_x & P_x < 1 \\ (0, I/P_x) & P_x = 1 \end{cases}$$

Perfect Substitutes

Peferences of person B:



Perfect Substitutes

Algebraic solution

$$\max_{x^A, y^A} 2x^A + y^A$$

subject to:

Perfect Substitutes

Algebraic solution

$$\max_{x^A,y^A} 2x^A + y^A$$

subject to:

$$I = x^A + p_y y^A$$
$$y^A \geq 0$$
$$x^A \geq 0$$

Perfect Substitutes

Algebraic solution

$$\max_{x^A,y^A} 2x^A + y^A$$

subject to:

$$I = x^A + p_y y^A$$
$$y^A \geq 0$$
$$x^A \geq 0$$

From the budget constraint we can obtain $y^A = \frac{I-x^A}{p_y}$, and adding the condition $y^A \geq 0$, we can conclude that $x^A \in [0, I]$.

Perfect Substitutes

Introducing y^A into the original maximization problem:

$$\max(2 - \frac{1}{p_y})x^A + \frac{I}{p_y} \quad s.t.\ x^A \in [0, I]$$

Which is a maximization of a straight line with slope $\left(2 - \frac{1}{p_y}\right)$ over an interval.

Perfect Substitutes

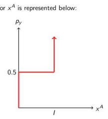
The demand for goods of individual A is

$$X^A = \begin{cases} 0 & \text{if } p_y < 0.5 \\ [0, I] & \text{if } p_y = 0.5 \\ I & \text{if } p_y > 0.5 \end{cases}$$

$$Y^A = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 0.5 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 0.5 \\ 0 & \text{if } p_y > 0.5 \end{cases}$$

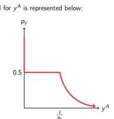
Perfect Substitutes

The demand for x^A is represented below:



Perfect Substitutes

The demand for y^A is represented below:



Perfect Substitutes

Algebraic solution
For person B the solution is analogous, but we have the following maximization problem: Introducing y^A into the original maximization problem:

$$\max(1 - \frac{1}{p_y})x^B + \frac{I}{p_y} \quad s.t.\ x^B \in [0, I]$$

Which is a maximization of a straight line with slope $\left(1 - \frac{1}{p_y}\right)$ over an interval.

Perfect Substitutes

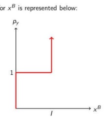
The demand for goods of individual B is

$$X^B = \begin{cases} 0 & \text{if } p_y < 1 \\ [0, I] & \text{if } p_y = 1 \\ I & \text{if } p_y > 1 \end{cases}$$

$$Y^B = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 1 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 1 \\ 0 & \text{if } p_y > 1 \end{cases}$$

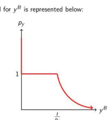
Perfect Substitutes

The demand for x^B is represented below:



Perfect Substitutes

The demand for y^B is represented below:



$$X^B(P_x) = \begin{cases} I/P_x & P_x < 1 \\ \boxed{(0, I/P_x) \quad P_x = 1} \\ 0 & P_x > 1 \end{cases}$$

PASO 2 ¿CUANDO DD = OO?

$Px = (0,1)$ | 1 | $(1,2)$ | 2 | $(2,\infty)$

$Px \in (0,1)$

$\underbrace{DD^A + DD^B}_{DD\ AGG}$ $\quad \underbrace{2}_{OO\ AGG}$

$\frac{I^A}{Px} + \frac{I^B}{Px}$

$\frac{1 \cdot Px + 1\cancel{Py}^{1}}{Px} + \frac{1Px + 1\cancel{Py}^{1}}{Px}$

$\frac{2Px + 2}{Px}$

$\underbrace{2 + \frac{2}{Px}}_{DD\ AGG} > \underbrace{2}_{OO\ AGG}$

NO EQ

$Px = 1$

$\frac{I^A}{Px} + DD^B \quad \underbrace{2}_{OO\ AGG}$

$\frac{Px + Py}{Px} + DD^B$

$2 + \underbrace{DD^B}$ $\quad 2$

$L_B = 0$
PARA Q
DD = OO

$\underline{EQ}$
$(Px = 1, Py = 1)$
$X^A = 2$
$X^B = 0$
$Y^A = 0$
$Y^B = 2$

$Px \in (1,2)$

$\frac{I^A}{Px} + 0$

$\frac{Px + \cancel{Py}^{1}}{Px}$

$1 + \frac{1}{Px} < \underbrace{2}_{OO\ AGG}$

NO EQ!

$Px = 2$

$\underbrace{DD^A} + 0$

$\frac{I^A}{Px} = \frac{Px + \cancel{Py}^{1}}{Px}$

$1 + \frac{1}{Px}$

$1 + \frac{1}{2}$

$\frac{3}{2} < 2 \quad$ OO AGG

NO EQ!

$Px \in (2,\infty)$

$0 + 0$

$0 < 2$

NO EQ!

Perfect Substitutes

When is the market for good X balanced (how about good y?)

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

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- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
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- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$

Perfect Substitutes

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- Try $p_y < 0.5$
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- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
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Perfect Substitutes

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- Try $p_y < 0.5$
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- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
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- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
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- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$
- Try $p_y > 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
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Perfect Substitutes

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- One possible equilibrium $(X^A = 2, X^B = 0, Y^A = 0, Y^B = 2)$
- Try $p_y > 1$
- $X^A = I$ and $X^B = I$
- $X^A = I = 1 + p_y$ and $X^B = I = 1 + p_y$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B = 2 + 2p_y > 2$