Lecture 5

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Lecture 5: General Equilibrium

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Lecture 5: General Equilibrium

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- What about the producers in the economy? There are two cases.

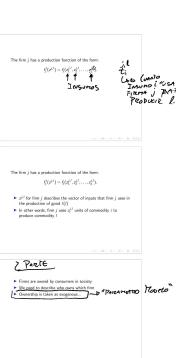
 There are <u>no reoducers</u> in the economy. this is what is called a gue exchange economy in which all available goods are those coming from endowments from consumers (up until now).

 There are producers who can produce commodities in the economy (today).

Each firm j is characterized by two characteristics: 1. A production function f(j) for producing that good L

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PARA PRODUCIE L.

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- ► Firms are owned by consumers in society
 ► We need to describe who owns which firm
 ► Ownership is taken as oognous... a more realistic model might innoher consumers choosing which firms to own
 ► IS_Owner represent the fraction of firm y that is owned by consumer.

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 ► if y will represent the fraction of firm y that is owned by consumer

 ► for each firm j. if j ∈ [0,1]

- ▶ Firms are owned by consumers in society

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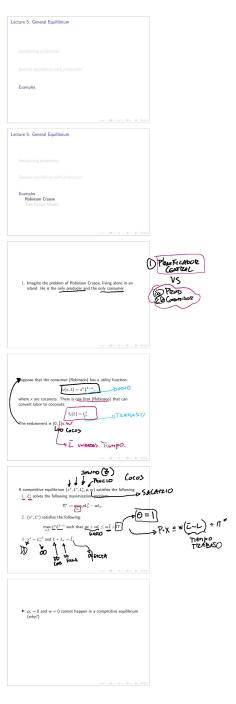
 ▶ Ownership is these as exgences. a more radiatic model might involve consumers choosing which firms to own θ_0 will express that fraction of firm j that is owned by consumer i▶ [x] each [x] or [x] is [x] in [x] owned by [x] in [x] in

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Firms are owned by consumers in society.
We need to describe who ours which firm
Ownership is faster as engenous. a more realistic model might involve consumers choosing which firms to own
θ<sub>g</sub> will respect the fraction of firm j that is owned by consumer j
For each firm j, θ<sub>g</sub> ∈ [0,1]
∑<sub>1-q</sub> θ<sub>g</sub> = θ<sub>g</sub> + θ<sub>g</sub> + θ<sub>g</sub> + · · · + θ<sub>g</sub> = 1
An implicit assumption here is that firms do not have any endowments
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                 Lecture 5: General Equilibrium
                                                   General equilibrium with production
           Lecture 5: General Equilibrium
                                                         General equilibrium with production
                      Definition Tusures Production ((\underline{x}^*, x^*) \in [0, \dots, n]) is a competitive equilibrium if.

1. fixed spoteces j = 1, 2, \dots, n,

\underline{x}^* J - (x_i^{(3)^*}, \dots, x_i^{(3)^*}), \dots, (x_i^{(d)^*}, \dots, x_i^{(d)^*}) solves
                                             \Pi_j^* := \max_{l \in \mathcal{L}} \inf_{(x_1^l, \dots, x_l^l)} \left( \sum_{i=1}^l \rho_i x_i^l \dots \sum_{i=1}^l \rho_i x_i
such that p_1x' \le p_1
                                                                                                                                                                                                                                                                             ulan α≀(×,)
           Lecture 5: General Equilibrium
                                                   Introducing production
                                                   General equilibrium with production
                                                   Examples
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\begin{array}{l} \blacktriangleright \  \, \rho_x=0 \  \, \text{and} \  \, w=0 \  \, \text{cannot happen in a competitive equilibrium} \\ \text{(why?)} \\ \blacktriangleright \  \, \rho_x>0 \  \, \text{and } w=0 \  \, \text{cannot happen (why?)} \end{array}
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 $\begin{array}{l} \blacktriangleright \ \ \, \rho_c = 0 \ \text{and} \ \, w = 0 \ \text{cannot happen in a competitive equilibrium} \\ (why?) \\ \blacktriangleright \ \, \rho_c > 0 \ \text{and} \ \, w = 0 \ \text{cannot happen} \ \, (why?) \\ \blacktriangleright \ \, \rho_c = 0 \ \text{and} \ \, w > 0 \ \text{cannot happen} \ \, (why?) \end{array}$

▶ $p_s = 0$ and w = 0 cannot happen in a competitive equilibrium (why?) ▶ $p_s > 0$ and w = 0 cannot happen (why?) ▶ $p_s = 0$ and w > 0 cannot happen (why?) ▶ $p_s = 0$ and w > 0 cannot happen (why?) ▶ $p_s = 0$ and $p_s = 0$ cannot happen (why?) ▶ $p_s = 0$ and $p_s = 0$ cannot happen (why?) (Prin) Son DE EQ (Prin) TANDIEN

- The problem of the firm $\label{eq:problem} We first solve the profit maximization <math display="block"> \mbox{\bf This is usually a good first step because the profit enters into the demand function <math display="block"> \mbox{\bf We first is solve the profit maximization}$ For any (p, w=1), we want to solve:

 $\max_{L_x} \rho L_x^{\beta} - L_x$.



- The problem of the firm

 We first solve the profit maximization

 This is usually a good first step because the profit enters into
 the demand function

 We first show the profit maximization
 For any (p, w = 1), we want to solve:

 $\max_{L_x} p L_x^{\beta} - L_x$.

First order conditions yield: $L_s^*(\rho) = (\rho \beta)^{\frac{1}{1-\beta}}$.

 $\Pi^*(\rho) = \rho (\rho \beta)^{\frac{S}{1-\beta}} - (\rho \beta)^{\frac{1}{1-\beta}} = \rho^{\frac{1}{1-\beta}} \left(\beta^{\frac{S}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right).$

The problem of the firm We first solve the profit maximization This is usually a good first step because the profit enters into the demand function ► We first solve the profit maximization For any (p, w = 1), we want to solve: $\max_{L_x} pL_x^{\beta} - L_x$. First order conditions yield: $L_{s}^{*}(\rho) = (\rho\beta)^{\frac{1}{1-\beta}}$. $\Pi^*(p) = p(p\beta)^{\frac{\beta}{1-\beta}} - (p\beta)^{\frac{1}{1-\beta}} = p^{\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{2-\beta}} - \beta^{\frac{1}{1-\beta}}\right).$

The supply of x is then given by:

is then given by: $x^{s}(p) = L_{x}^{s}(p)^{\beta} = (p\beta)^{\frac{\beta}{1-\beta}}$

The problem of the consumer

To solve for the demand curve $x^d(p)$, $L^d(p)$, we solve:

The problem of the consumer To solve for the demand curve $x^d(\rho), L^d(\rho)$, we solve: $\max_{x,L} x^{\alpha} L^{1-\alpha} \text{ such that } px + L \leq \overline{L} + \Pi^*(p).$

By the first order condition, we get:

$$\frac{\alpha}{1-\alpha}\frac{L}{x}=p.$$

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$$\frac{\alpha}{1-\alpha}\frac{L}{x}=p.$$

Substituting this back into the budget constraint, we get:

$$\frac{\alpha}{1-\alpha}L+L=\bar{L}+p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\alpha}{1-\beta}}-\beta^{\frac{1}{1-\beta}}\right).$$

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 $L^{\theta}(p) = (1 - \alpha) \left(\tilde{L} + \rho^{\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right) \right).$

The problem of the consumer To solve for the demand curve $x^d(\rho), L^d(\rho)$, we solve: $\max_{\mathbf{x},L} \mathbf{x}^{\alpha} L^{1-\alpha} \text{ such that } p\mathbf{x} + L \leq \overline{L} + \Pi^*(p).$

By the first order condition, we get:

$$\frac{\alpha}{1-\alpha}\frac{L}{x}=p.$$

$$\frac{\alpha}{1-\alpha}L+L=\tilde{L}+p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}}-\beta^{\frac{1}{1-\beta}}\right).$$

Thus,
$$L^d(p) = (1 - \alpha) \left(\overline{L} + p^{\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{2-\beta}} - \beta^{\frac{1}{1-\beta}} \right) \right).$$

$$\times^d(p) = \frac{\alpha}{p} \left(\overline{L} + p^{\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{2-\beta}} - \beta^{\frac{1}{1-\beta}} \right) \right).$$

Market Clearing

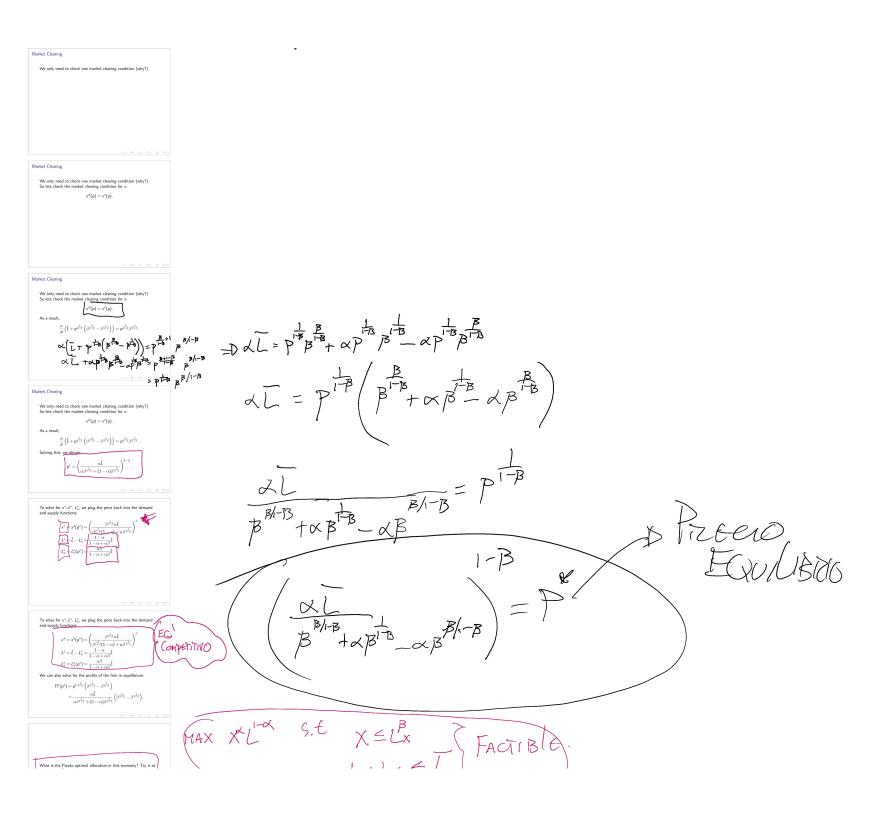
We only need to check one market clearing condition (why?)

I + Tit = L (Fat 100)

TX =X

正+11 = p(() + L [+11 = & +1

I+ H = PX+L





MAX XLIX S.t X \(\times L\) FACTIBLE.

Lecture 5: General Equilibrium

Robinson Crusoe Two Factor Model

Suppose that there is one consumer with a utility function: $u(x,y) = x^{1/2}y^{1/2}.$

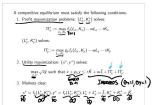
 $f_x(L_x, K_x) = L_x^{1/2} K_x^{1/2}$, $f_y(L_y, K_y) = L_y^{1/2} K_y^{1/2}$.

The endowments are given by $\underline{I}=1,\ \underline{R}=1,\ \text{and }0$ units of x and

What is a competitive equilibrium in this economy?

What is a competitive equilibrium in this economy? We must describe $\underbrace{[(\kappa^*, y^*, L_n^*, K_n^*, L_p^*, K_p^*, p_n, p_p, r, w)_{-}}_{All \ equilibrium \ cross will \ be \ strictly \ positive in equilibrium, hence assume \underbrace{p_n = 1}_{D_n = 1}$

(Px,Py,r,w) ⇒ 大(Px,Py,r,w)-(日常常常)



The problem of the firm

We solve for profit maximization first (because Π_x and Π_y enter into the the consumer's problem)

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- We solve for profit maximization first (because Π^{*}_x and Π^{*}_y enter into the the consumer's problem)
 Both firms make zero profits. Why?

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 Both firms make zero profits. Why?
 This does not always happen (In the previous example, the firm made strictly positive profits)

The problem of the firm

- We solve for profit maximization first (bucause Π^{*}_k and Π^{*}_j enter into the the consumer's problem)
 Both firms make zero porfix. Why?
 ➤ This does not always happen (in the previous example, the firm mudes text/pty positive profits)
 ➤ this is because the production function here is of constant returns to scale

The problem of the firm

- ▶ We solve for profit maximization first (because Π_s* and Π_y* enter into the the consumer's problem)
 ▶ Both firms make zero porfits. Why?
 ▶ This does not always happen (in the previous example, the firm make strictly positive porfits)
 For this does not always always first firs

II(Lx, kx) = Lx 1/2 kx - wlx - rkx

JII(20, hkx) = \(\frac{1/2}{2} \frac{1}{2} \frac{1}{2 E) [[(Lx, Kx) 11x70 =7 DD Lx=00 =0 lov Eq. Triel =Dykin solling any-TITLO -D LX=0 -D NO 63 POSIBLE THEO EN EQ TO SUMPLE OF TRANSA (ON THIS OF CHECKINGS A SCICLA) Tred





The problem of the firm

We solve the profit maximization of the firm that produces x for any $(p_x = 1, p_x, w_x)$, we want to solve $\max_{k \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} \frac{|x_k|^2}{|x_k|^2} \sum_{k \in \mathbb{Z}} \frac{|x_k|^2}{|x_k|^2} \sum_{k \in \mathbb{Z}} \frac{1}{|x_k|^2} \sum_{k \in \mathbb{Z}} \frac{1}{|x$

The problem of the firm

We solve the profit maximization of the firm that produces
$$x$$
 for any $(x_1 - l_1, y_1, y_2)$, we cannot be ables.

$$\frac{1}{2} = \frac{1}{2} L_x^{1/2} R_x^{1/2} - L_x w - R_x r$$

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The problem of the firm

We solve the profit maximization of the firm that produces x for any $(x_1 - l_1, y_2, w_1)$, we want to solve.

For any $(x_2 - l_1, y_2, w_1)$, we want to solve.

First order conditions yield:

$$\frac{1}{L_x} = \frac{w}{r}$$
First order conditions yield:

$$\frac{1}{L_x} = \frac{w}{r}$$

The problem of the firm
$$\begin{aligned} \text{We show the point maximization of the firm that produces } x \\ \text{For any } (x_0^{-1} - \frac{1}{k^2} - \frac{1}{k^2}$$

The problem of the firm

We solve the profit maximization of the firm that produces
$$y$$

$$\lim_{R \to \infty} \rho_{x} J_{x}^{(2)} R_{x}^{(2)} - L_{y} w - K_{y} r$$

$$\lim_{R \to \infty} \rho_{x} J_{x}^{(2)} R_{x}^{(2)} - L_{y} w - K_{y} r$$

$$\lim_{R \to \infty} P_{y} \int_{\mathbb{R}^{N}} \frac{1}{k^{2}} K_{y}^{(2)} K_{y}^{(2)} - \frac{1}{k^{2}} K_{y}^{(2)} K_{y}^{(2)} - \frac{1}{k^{2}} K_{y}^{(2)} K_{y}^{(2)}$$

The problem of the firm
$$We solve the profit maximization of the firm that produces y
$$\max_{p,q} \rho_p L_p^{1/2} K_p^{1/2} - L_p w - K_p r$$$$

$$\frac{kx}{lx} = \frac{ky}{ly}$$

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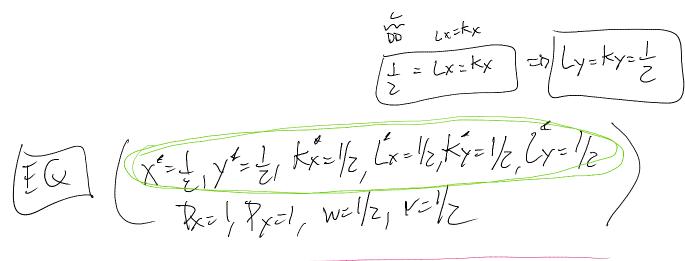
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The problem of the firm
                \frac{K_y}{L_y} = \frac{w}{r}
                                          Ty=0= Py Ly ky - wLy-kyr=0
The problem of the firm
  Therefore
                                    That xy s.t P_xX+P_yY \leq wL+pK+P_x+P_y

s.t x+y \leq \frac{1}{2}+\frac{1}{2}=1
The problem of the consumer
           \max_{x} \sqrt{xy} such that x + y \le 1.
                                                  J= 12 + > (1-X-X)
 The solution to this gives:
               x^* = y^* = \frac{1}{2}.
                                                \frac{8x}{3} = \frac{5x_{1/2}}{1} - y = 0
\frac{8x}{\lambda} = \frac{5x_{1/2}}{\lambda_{1/2}} - y = 0
\frac{x}{\lambda} = 1
market clearing
                                                34 > 7 x1/2 - 7 = 0
           \frac{1}{2} = L_x^* = K_x^* = L_y^* = K_y^*,
```



$$y \in \{y(Ly, ky) = Ly\}$$

$$\{x + Ly \in L = 1\}$$

$$\{x + ky \in R = 1\}$$

MAX VXY S.E X= Lx Kx Y= (1-1x) (1-kx)

1/2 (1/2) (1-Lx) 1/2 (1-tx) 1/2 (1-tx) 1/2 (1-tx) 1/2