Lecture 5

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Lecture5

Lecture 5: General Equilibrium

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Lecture 5: General Equilibrium

Introducing production

General equilibrium with production

Examples

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Examples

What about the producers in the economy? There are two cases.

1. There are no producers in the economy: this is what is called a pure exchange economy in which all available goods are those coming from endowments from consumers (up until now)
2. There are producers who can produce commodities in the economy (today)

Each firm j is characterized by two characteristics:

1. A production function $\boxed{f_j^l}$ for producing that good l.

f_j^l

l → BIEN FINAL.

j → FIRMA j

→ TECNOLOGÍA

The firm j has a production function of the form:

$$f_j^l(z^{j,l}) = f_j^l(z_1^{j,l}, z_2^{j,l}, \ldots, z_L^{j,l}).$$

$z_i^{j,l}$

→ INSUMO i

→ USA LA FIRMA j

→ PARA PRODUCIR l

The firm j has a production function of the form:

$$f_j^l(z^{j,l}) = f_j(z_1^{j,l}, z_2^{j,l}, \ldots, z_L^{j,l}).$$

- $z^{j,l}$ for firm j describes the vector of inputs that firm j uses in the production of good $\ell(l)$
- In other words, firm j uses $z_i^{j,l}$ units of commodity i to produce commodity l

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- Firms are owned by consumers in society
- We need to describe who owns which firm
- Ownership is taken as exogenous... → "PARAMETRO" PROBLEMA

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- For each firm j, $\theta_{ij} \in [0, 1]$
- $\sum_{i=1}^{I} \theta_{ij} = \theta_{1j} + \theta_{2j} + \cdots + \theta_{Ij} = \boxed{1}$ $\forall j$

- Firms are owned by consumers in society
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- $\sum_{i=1}^{I} \theta_{ij} = \theta_{1j} + \theta_{2j} + \cdots + \theta_{Ij} = 1$
- An implicit assumption here is that firms do not have any endowments

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AGENTES

CONSUMO

CONSUMO FIRMAS

"COSTO INSUMOS"

Definition

$((x^*, z^*), p = (p_1, \dots, p_L))$ is a competitive equilibrium if:

1. For all producers $j = 1, 2, \dots, J$, $z^{j*} = ((z_1^{j,1*}, \dots, z_L^{j,1*}), \dots, (z_1^{j,L*}, \dots, z_L^{j,L*}))$ solves:

$$\Pi_j^* := \max_{\boxed{z^j}} \boxed{\underbrace{p_\ell f_j^\ell(z_1^j, \dots, z_L^j)}_{\text{GANANCIA}} - \underbrace{\sum_{\ell'=1}^{L} p_{\ell'} z_{\ell'}^j}_{\text{COSTOS}}}.$$

(EN EQ LAS FIRMAS ESTA MAXIMIZANDO SU UTILIDAD)

2. For all consumers $i = 1, 2, \dots, I$, $x^{i*} = (x_1^{i*}, \dots, x_L^{i*})$ solves:

$$\max_{x^i} u_i(x^i)$$

GANANCIA POR SER DUEÑO FIRMAS.

$$\text{such that } \underbrace{p \cdot x^i}_{\text{GASTO}} \le \underbrace{p \cdot \omega^i}_{\text{INGRESO DE VENDER DOTACIÓN}} + \sum_{j=1}^{J} \theta_{ij} \Pi_j^*.$$

3. Markets clear: For each commodity $\ell = 1, 2, \dots, L$:

$$\underbrace{\underbrace{\sum_{i=1}^{I} x_\ell^{i*}}_{\text{DD CONSUMIDORES}} + \underbrace{\sum_{j=1}^{J} \sum_{\ell'=1}^{L} z_\ell^{j,\ell'}}_{\text{DD FIRMAS}}}_{\text{DD AGREGADA}} = \underbrace{\underbrace{\boxed{\sum_{i=1}^{I} \omega_\ell^i}}_{\text{OO CONSUMIDORES}} + \underbrace{\sum_{j=1}^{J} f_j^\ell \left(z_1^{j,\ell*}, \dots, z_L^{j,\ell*}\right)}_{\text{OO FIRMAS}}}_{\text{OO AGREGADA}}.$$

We have exactly the same basic properties as in the case of pure exchange economies

1. When utility functions are strictly monotone, and production functions are strictly increasing, prices of each commodity and prices of each input are strictly positive
2. Walras' Law: Each consumer i spends all of his income whenever i maximizes utility
3. Walra's Law II: If the market clearing conditions hold for $\ell = 1, 2, \dots, L-1$ and $p_L > 0$ then it will also hold for market L as well.
4. If $\boxed{(x^*, z^*, p)}$ is a Walrasian equilibrium, and $\alpha > 0$, $(x^*, z^*, \alpha p)$ is also a Walrasian equilbrium.
5. The first and the second welfare theorems hold

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Robinson Crusoe

Two Factor Model

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1. Imagine the problem of Robinson Crusoe, living alone in an island. He is the only producer and the only consumer

ECON MCDO
1A → PROD
1B → CONSUMIDOR

VS

PLANIFICADOR

Suppose that the consumer (Robinson) has a utility function:

$$\boxed{u(x, L) = x^\alpha L^{1-\alpha},}$$

COCOS

OCIO

where x are coconuts. There is one firm (Robinson) that can convert labor to coconuts:

$$\boxed{f_x(L) = L^\beta}$$

TRABAJO

The endowment is $(0, \bar{L})$

COCOS

TIEMPO

$f(L_x) = L_x^\beta$

CONSUMO AGENTE

INSUMOS FIRMA (z^*)

PRECIO COCOS

PRECIO TIEMPO (W = WAGE)

A competitive equilibrium (x^*, L^*, L_x^*, p, w) satisfies the following:

1. L_x^* solves the following maximization problem:

$$\Pi^* := \max_{L_x} pL_x^\beta - wL_x.$$

2. (x^*, L^*) satisfies the following:

$$\max_{x,L} x^\alpha L^{1-\alpha} \text{ such that } px + wL \le w\bar{L} + \Pi^*.$$

$\rightarrow \underbrace{P \cdot x}_{\text{GASTO}} \le w\underbrace{(\bar{L} - L)}_{\text{TRABAJO}} + \Pi^*$

3. $\underbrace{x^*}_{DD} = \underbrace{\boxed{L_x^{*\beta}}}_{OO}$ and $\underbrace{\underbrace{L}_{DD\,C} + \underbrace{L_x}_{DD\,F}}_{DD\,AGG} = \underbrace{\bar{L}}_{\text{DOTACIÓN}\ OO\ AGG}$.

- $p_x = 0$ and $w = 0$ cannot happen in a competitive equilibrium (why?)

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- $p_x = 0$ and $w > 0$ cannot happen (why?)
- both $p_x, w > 0$ in a competitive equilibrium. We can normalize $\boxed{w = 1}$

$(P \; w)$ ES UN PRECIO DE EQ

$\left(\frac{P}{w}, 1\right)$ TAMBIEN

The problem of the firm

- We first solve the profit maximization
- This is usually a good first step because the profit enters into the demand function
- We first solve the profit maximization

For any $(p, \boxed{w = 1})$, we want to solve:

$$\boxed{\max_{L_x} pL_x^{\beta} - L_x.}$$

$$\frac{\partial \pi}{\partial L_x} = p\beta L_x^{\beta-1} - 1 = 0$$

$$L_x^{\beta-1} = \frac{1}{p\beta}$$

$$L_x^{*} = \left(\frac{1}{p\beta}\right)^{\frac{1}{\beta-1}} = (p\beta)^{\frac{1}{1-\beta}}$$

DEMANDA INSUMO

$$f(L_x) = L_x^{\beta}$$

$$L_x^{\beta} = (p\beta)^{\frac{\beta}{1-\beta}}$$

OFERTA COCOS

$$\pi^{*} = p(p\beta)^{\frac{\beta}{1-\beta}} - (p\beta)^{\frac{1}{1-\beta}}$$

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First order conditions yield:

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Therefore,

$$\Pi^{*}(p) = p\,(p\beta)^{\frac{\beta}{1-\beta}} - (p\beta)^{\frac{1}{1-\beta}} = p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right).$$

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The supply of x is then given by:

$$x^{s}(p) = L_x^{*}(p)^{\beta} = (p\beta)^{\frac{\beta}{1-\beta}}$$

The problem of the consumer

To solve for the demand curve $x^d(p), L^d(p)$, we solve:

$$\max_{x,L} x^{\alpha}L^{1-\alpha} \text{ such that } px + L \leq \bar{L} + \Pi^{*}(p).$$

DEL PROB. DE LA FIRMA

$$\mathcal{L} = x^{\alpha}L^{1-\alpha} + \lambda\left(\bar{L} + \pi^{*} - px - L\right)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1}L^{1-\alpha} - \lambda p = 0 \Rightarrow \alpha x^{\alpha-1}L^{1-\alpha} = \lambda p$$

$$\frac{\partial \mathcal{L}}{\partial} = (1-\alpha)x^{\alpha}L^{-\alpha} - \lambda = 0 \qquad (1-\alpha)x^{\alpha}L^{-\alpha} = \lambda$$

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$$\frac{\alpha}{1-\alpha}\frac{L}{x} = p.$$

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Substituting this back into the budget constraint, we get:

$$\frac{\alpha}{1-\alpha}L + L = \bar{L} + p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right).$$

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Then

$$x^d(p) = \frac{\alpha}{p}\left(\bar{L} + p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)\right).$$

Market Clearing

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$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1}L^{1-\alpha} - \lambda P = 0 \Rightarrow \alpha x^{\alpha-1}L^{1-\alpha} = \lambda P$$

$$\frac{\partial \mathcal{L}}{\partial L} = (1-\alpha)x^{\alpha}L^{-\alpha} - \lambda = 0 \qquad (1-\alpha)x^{\alpha}L^{-\alpha} = \lambda$$

$$\Downarrow$$

$$\frac{\alpha}{1-\alpha}\frac{L}{x} = P$$

$$\frac{\alpha}{1-\alpha}\frac{L}{P} = x$$

$$\bar{L} + \pi^* = Px + L$$

$$\bar{L} + \pi^* = \cancel{P}\left(\frac{\alpha}{1-\alpha}\frac{L}{\cancel{P}}\right) + L$$

$$\bar{L} + \pi^* = L\left(\frac{\alpha}{1-\alpha} + 1\right)$$

$$\bar{L} + \pi^* = L\left(\frac{\alpha + 1 - \alpha}{1-\alpha}\right)$$

$$\bar{L} + \pi^* = L\left(\frac{1}{1-\alpha}\right)$$

$$(1-\alpha)(\bar{L} + \pi^*) = L$$ DD OCIO

$$\frac{\alpha}{1-\alpha}\frac{L}{P} = x$$

$$\left(\frac{\alpha}{\cancel{1-\alpha}}\right)\frac{1}{P}\cancel{(1-\alpha)}(\bar{L} + \pi^*) = x$$

$$\boxed{\frac{\alpha}{P}(\bar{L} + \pi^*) = x}$$ DD COCOS

3) MCDOS VACIEN.

Cocos: $\frac{\alpha}{P}(\bar{L} + \pi^*) = (P\beta)^{\frac{\beta}{1-\beta}}$

Tiempo: $(1-\alpha)(\bar{L} + \pi^*) + (P\beta)^{\frac{1}{1-\beta}} = \bar{L}$

$$\Rightarrow \alpha\bar{L} + \alpha\pi^* = P^{\frac{\beta}{1-\beta}+1}\,\beta^{\frac{\beta}{1-\beta}}$$

$$\alpha\bar{L} + \alpha\underbrace{\left(P^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)\right)}_{\pi^*} = P^{\frac{\beta+1-\beta}{1-\beta}}\,\beta^{\frac{\beta}{1-\beta}}$$

Market Clearing

We only need to check one market clearing condition (why?)
So lets check the market clearing condition for x:

$$x^d(p) = x^s(p).$$

As a result,

$$\frac{\alpha}{p}\left(\bar{L} + p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)\right) = p^{\frac{\beta}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}.$$

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Solving this, we obtain:

$$p^* = \left(\frac{\alpha\bar{L}}{\alpha\beta^{\frac{1}{1-\beta}} + (1-\alpha)\beta^{\frac{\beta}{1-\beta}}}\right)^{1-\beta}.$$

To solve for x^*, L^*, L_x^*, we plug the price back into the demand and supply functions:

EQ MERCADO

$$\boxed{x^* = x^s(p^*) = \left(\frac{\beta^{\frac{1}{1-\beta}}\alpha\bar{L}}{\beta^{\frac{\beta}{1-\beta}}(1-\alpha) + \alpha\beta^{\frac{1}{1-\beta}}}\right)^{\beta}} \quad \text{DD Cocos}$$

$$\boxed{L^* = \bar{L} - L_x^* = \frac{1-\alpha}{1-\alpha+\alpha\beta}\bar{L}} \quad \text{DD ocio}$$

$$\boxed{L_x^* = L_x^*(p^*) = \frac{\alpha\beta}{1-\alpha+\alpha\beta}\bar{L}} \quad \text{DD TRABAJO}$$

To solve for x^*, L^*, L_x^*, we plug the price back into the demand and supply functions:

$$x^* = x^s(p^*) = \left(\frac{\beta^{\frac{1}{1-\beta}}\alpha\bar{L}}{\beta^{\frac{\beta}{1-\beta}}(1-\alpha) + \alpha\beta^{\frac{1}{1-\beta}}}\right)^{\beta}$$

$$L^* = \bar{L} - L_x^* = \frac{1-\alpha}{1-\alpha+\alpha\beta}\bar{L}$$

$$L_x^* = L_x^*(p^*) = \frac{\alpha\beta}{1-\alpha+\alpha\beta}\bar{L}$$

We can also solve for the profits of the firm in equilibrium:

$$\Pi^*(p^*) = p^{*\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right) = \frac{\alpha\bar{L}}{\alpha\beta^{\frac{1}{1-\beta}} + (1-\alpha)\beta^{\frac{\beta}{1-\beta}}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right).$$

What is the Pareto optimal allocation in this economy? Try it at home

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Examples
- Robinson Crusoe
- Two Factor Model

π^*

$$\alpha\bar{L} = p^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}} - \alpha p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)$$

$$\alpha\bar{L} = p^{\frac{1}{1-\beta}}\left(\beta^{\frac{\beta}{1-\beta}} - \alpha\left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)\right)$$

$$\frac{\alpha\bar{L}}{\beta^{\beta/1-\beta} - \alpha\left(\beta^{\beta/1-\beta} - \beta^{\frac{1}{1-\beta}}\right)} = p^{\frac{1}{1-\beta}}$$

$$\left[\frac{\alpha\bar{L}}{\beta^{\beta/1-\beta} - \alpha\left(\beta^{\beta/1-\beta} - \beta^{\frac{1}{1-\beta}}\right)}\right]^{1-\beta} = p^*$$

PRECIO EQ

⟹ PLANIFICADOR CENTRAL.

MAX $U = X^{\alpha}L^{1-\alpha}$ s.t. $\left.\begin{array}{l} X \le L_x^{\beta} \\ L + L_x \le \bar{L} \end{array}\right\}$ FACTIBLE.

⟹ OPTIMO PARETO.

Suppose that there is one consumer with a utility function:

$$\underline{u(x,y) = x^{1/2}y^{1/2}}.$$

There are two firms:

$$f_x(L_x, K_x) = L_x^{1/2}K_x^{1/2},$$
$$f_y(L_y, K_y) = L_y^{1/2}K_y^{1/2}.$$

The endowments are given by $\bar{L} = 1$, $\bar{K} = 1$, and 0 units of x and y.

What is a competitive equilibrium in this economy?

What is a competitive equilibrium in this economy? We must describe

PRECIO CAPITAL

$$(\underbrace{x^*, y^*, L_x^*, K_x^*, L_y^*, K_y^*}_{\text{ASIGNACION}}, \underbrace{p_x, p_y, r, w}_{\text{PRECIOS}}).$$

→ PRECIO TRABAJO

What is a competitive equilibrium in this economy? We must describe

$$(x^*, y^*, L_x^*, K_x^*, L_y^*, K_y^*, p_x, p_y, r, w).$$

All equilibrium prices will be strictly positive in equilibrium, hence assume $p_x = 1$

(p_x, p_y, r, w)

$\frac{1}{p_x}(p_x, p_y, r, w)$

$(1, \frac{p_y}{p_x}, \frac{r}{p_x}, \frac{w}{p_x})$

A competitive equilibrium must satisfy the following conditions:

1. Profit maximization problems: (L_x^*, K_x^*) solves:

$$\Pi_x^* := \max_{L_x, K_x} \underbrace{f_x(L_x, K_x)}_{p_x=1} - wL_x - rK_x$$

(L_y^*, K_y^*) solves:

$$\Pi_y^* := \max_{L_y, K_y} p_y f_y(L_y, K_y) - wL_y - rK_y$$

2. Utility maximization: (x^*, y^*) solves:

INGRESO $(\theta x = 1, \theta y = 1)$

$$\max_{x,y} \sqrt{xy} \text{ such that } \underbrace{x + p_y y}_{\text{COSTO CONSUMO }(p_x=1)} \le \overbrace{r\bar{K} + w\bar{L} + \Pi_x^* + \Pi_y^*}^{\text{INGRESO}}.$$

3. Markets clear:

$$\underbrace{x^*}_{DD} = \underbrace{f_x(L_x^*, K_x^*)}_{OO}, \underbrace{y^*}_{DD} = \underbrace{f_y(L_y^*, K_y^*)}_{OO}, \underbrace{L_x^* + L_y^*}_{DD} = \underbrace{\bar{L}}_{OO}, \underbrace{K_x^* + K_y^*}_{DD} = \underbrace{\bar{K}}_{OO}.$$

$f_x(L_x, K_x) = L_x^{1/2} K_x^{1/2}$

$$\boxed{\begin{aligned} f_x(\lambda L_x, \lambda K_x) &= \lambda^{1/2} L_x^{1/2} \lambda^{1/2} K_x^{1/2} \\ &= \lambda L_x^{1/2} K_x^{1/2} \\ &= \lambda f(L_x, K_x) \end{aligned}}$$

$\boxed{\Pi_x > 0} \Rightarrow$ PROD. INFINITO

NO PUEDE SER EN EQ.

$\Pi(L_x, K_x)$

$$\begin{aligned} \Pi(\lambda L_x, \lambda K_x) &= p_x f(\lambda L_x, K_x) - w\lambda L_x - r\lambda K_x \\ &= p_x \lambda f(L_x, K_x) - \lambda w L_x - \lambda r K_x \\ &= \lambda \underbrace{(p_x f(L_x, K_x) - wL_x - rK_x)}_{\Pi(L_x, K_x)} \\ &= \lambda \Pi(L_x, K_x) \end{aligned}$$

$\boxed{\Pi_x < 0} \to$ PROD. 0

$\Rightarrow$ TAMPOCO PUEDE SER EQ

The problem of the firm

- We solve for profit maximization first (because Π_x^* and Π_y^* enter into the the consumer's problem)

The problem of the firm

- We solve for profit maximization first (because Π_x^* and Π_y^* enter into the the consumer's problem)
- Both firms make zero profits. Why?

$$\boxed{\begin{array}{l}\Pi x = 0 = Lx^{1/2} Kx^{1/2} - wLx - rKx \\ \Pi y = 0 = P_y Ly^{1/2} Ky^{1/2} - wLy - rKy\end{array}}$$

$$\Rightarrow \boxed{\Pi_x^* = 0} \text{ EN EQ.}$$

$$\boxed{\Pi_y^* = 0}$$

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  - This does not always happen (In the previous example, the firm made strictly positive profits)
  - this is because the production function here is of *constant returns to scale*
  - If the firm made strictly positive profits, then it could not be making maximal profits since it could double profits by multiplying all inputs by two

The problem of the firm

We solve the profit maximization of the firm that produces x

The problem of the firm

We solve the profit maximization of the firm that produces x
For any $(p_x = 1, p_y, w, r)$, we want to solve:

$$\boxed{\max_{L_x, K_x} L_x^{1/2} K_x^{1/2} - L_x w - K_x r}$$

$$\frac{\partial \Pi x}{\partial Lx} = \frac{1}{2} \frac{Kx^{1/2}}{Lx^{1/2}} - w = 0 \quad \Rightarrow \quad \frac{1}{2} \frac{Kx^{1/2}}{Lx^{1/2}} = w \quad \Rightarrow \quad \frac{Kx}{Lx} = \frac{w}{r}$$

$$\frac{\partial \Pi}{\partial Kx} = \frac{1}{2} \frac{Lx^{1/2}}{Kx^{1/2}} - r = 0 \qquad\qquad \frac{1}{2} \frac{Lx^{1/2}}{Kx^{1/2}} = r$$

The problem of the firm

We solve the profit maximization of the firm that produces x

For any $(p_x = 1, p_y, w, r)$, we want to solve:

$$\max_{L_x, K_x} L_x^{1/2} K_x^{1/2} - L_x w - K_x r$$

First order conditions yield:

$$\boxed{\frac{K_x}{L_x} = \frac{w}{r}} \Rightarrow \boxed{K_x = \frac{w}{r} L_x}$$

$$L_x^{1/2} K_x^{1/2} - L_x w - K_x r = 0$$

The problem of the firm

We solve the profit maximization of the firm that produces x

For any $(p_x = 1, p_y, w, r)$, we want to solve:

$$\max_{L_x, K_x} L_x^{1/2} K_x^{1/2} - L_x w - K_x r$$

First order conditions yield:

$$\frac{K_x}{L_x} = \frac{w}{r}$$

Therefore,

$$\underbrace{\left(\frac{w}{r}\right)^{1/2} L_x^{1/2}}_{K_x} L_x^{1/2} - L_x w - \overset{K_x}{\frac{w}{r} L_x} r = 0$$

$$\left(\frac{w}{r}\right)^{1/2} - 2w = 0$$

$$\frac{1}{2} = w^{1/2} r^{1/2}$$

$$\frac{1}{4} = wr$$

$$\left(\frac{w}{r}\right)^{1/2} L_x - L_x w - w L_x = 0$$

$$\cancel{L_x}\left(\left(\frac{w}{r}\right)^{1/2} - 2w\right) = 0$$

$$\frac{w^{1/2}}{r^{1/2}} = 2w$$

$$1 = \frac{2w\,r^{1/2}}{w^{1/2}}$$

$$1 = 2w^{1/2} r^{1/2}$$

$$\frac{1}{2} = w^{1/2} r^{1/2}$$

$$\boxed{\frac{1}{4} = wr}$$

The problem of the firm

We solve the profit maximization of the firm that produces y

$$\Pi_y = P_y L_y^{1/2} K_y^{1/2} - wL_y - rL_y$$

$$\frac{\partial \Pi_y}{\partial L_y} = \frac{P_y \frac{1}{2} K_y^{1/2}}{L_y^{1/2}} - w = 0$$

$$\frac{\partial \Pi_y}{\partial K_y} = \frac{\frac{1}{2} P_y L_y^{1/2}}{K_y^{1/2}} - r = 0$$

$$\boxed{\frac{K_y}{L_y} = \frac{w}{r}} \qquad \boxed{\frac{K_x}{L_y} = \frac{w}{r}}$$

$$K_y = L_y \frac{w}{r}$$

The problem of the firm

We solve the profit maximization of the firm that produces y

$$\max_{L_y, K_y} p_y L_y^{1/2} K_y^{1/2} - L_y w - K_y r$$

$$\frac{K_y}{L_y} = \frac{K_x}{L_x}$$

The problem of the firm

We solve the profit maximization of the firm that produces y

$$\max_{L_y, K_y} p_y L_y^{1/2} K_y^{1/2} - L_y w - K_y r$$

First order conditions yield:

$$\frac{K_y}{L_y} = \frac{w}{r}$$

DD = OO

$$K_x + K_y = 1$$
$$L_x + L_y = 1$$

The problem of the firm

Therefore,

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$$\boxed{\frac{K_y}{L_y} = \frac{K_x}{L_x}}$$

$$\frac{1 - K_x}{1 - L_x} = \frac{K_x}{L_x}$$

$$L_x - K_x L_x = K_x - K_x L_x$$

$$\boxed{L_x = K_x}$$

$$w = r$$

$$\boxed{\frac{1}{4} = wr}$$

$$\boxed{\frac{L_x}{K_x} = \frac{w}{r} \qquad 1 = \frac{w}{r}}$$

$$\boxed{r = w}$$

$$\frac{1}{4} = w^2$$

$$\boxed{\frac{1}{2} = w = r}$$

We also know that $p_y = 1$. Why?

$$\Pi_y = P_y K_y^{1/2} L_y^{1/2} - wL_y - rK_y = 0$$

$$= P_y K_y - wK_y - rK_y = 0$$

$$\cancel{K_y}(P_y - w - r) = 0$$

$$\boxed{\frac{K_y}{L_y} = 1 = \frac{K_x}{L_x} = \frac{K_x}{K_x}}$$

$$\Rightarrow K_y = L_y$$

$$P_y = w + r$$

$$\boxed{P_y = \frac{1}{2} + \frac{1}{2} = 1}$$

The problem of the firm

- We cannot solve for the supply function because the firm obtains zero profit regardless of how much it produces
- But we already know the prices!

The problem of the consumer

$$\max_{x,y} \sqrt{xy} \text{ such that } x+y \leq \boxed{1}.$$

The solution to this gives:

$$x^* = y^* = \frac{1}{2}.$$

market clearing

By market clearing we must have:

$$\frac{1}{2} = L_x^* = K_x^* = L_y^* = K_y^*,$$

$$\max_{x,y} \; x^{1/2}y^{1/2} \quad \text{s.a} \quad \cancel{P_x}^{\to 1} \cdot x + \cancel{P_y}^{\to 1} y \leq w\bar{L} + r\bar{K} + \cancel{\Pi_x}^{\to 0} + \cancel{\Pi_y}^{\to 0}$$

$$x + y \leq \tfrac{1}{2}(1) + \tfrac{1}{2}(1) = 1$$

$$\mathcal{L} = x^{1/2}y^{1/2} + \lambda(1 - x - y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2}\frac{y^{1/2}}{x^{1/2}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2}\frac{x^{1/2}}{y^{1/2}} - \lambda = 0$$

$$\Rightarrow \frac{y}{x} = 1 \qquad \boxed{y = x}$$

$$1 = x + y$$

$$\Rightarrow \boxed{x = 1/2, \; y = 1/2}$$

EQ

$$\left(\begin{array}{l} x^* = \frac{1}{2}, \; y^* = \frac{1}{2}, \; K_x^* = \frac{1}{2} \\ K_y^* = \frac{1}{2}, \; L_x^* = \frac{1}{2}, \; L_y^* = 1/2 \\ P_x = 1, \; P_y = 1, \; w = \frac{1}{2}, \; r = 1/2 \end{array} \right)$$

X

$$\frac{1}{2} = K_x^{1/2} L_x^{1/2}$$

$$K_x = L_x$$

$$\frac{1}{2} = K_x^{1/2} K_x^{1/2} = K_x$$

$$\boxed{K_x^* = 1/2 = L_x^*}$$

Y

$$\frac{1}{2} = K_y^{1/2} L_y^{1/2}$$

$$\frac{1}{2} = K_y^{1/2} K_y^{1/2}$$

$$\boxed{\frac{1}{2} = K_y^* = L_y^*}$$

OPTIMOS DE PARETO

$$\max_{x,y,L_x,L_y,K_x,K_y} \; x^{1/2} y^{1/2} \quad \text{s.a.} \quad \left| \begin{array}{l} K_x + K_y = 1 \\ L_x + L_y = 1 \\ x = L_x^{1/2} K_x^{1/2} \\ y = L_y^{1/2} K_y^{1/2} \end{array} \right. \Rightarrow \begin{array}{l} x = L_x^{1/2} K_x^{1/2} \\ y = (1 - L_x)^{1/2} (1 - K_x)^{1/2} \end{array}$$

$$\max_{L_x, K_x} \; \underbrace{\left(L_x^{1/2} K_x^{1/2}\right)}_{x}{}^{1/2} \underbrace{\left((1-L_x)^{1/2}(1-K_x)^{1/2}\right)}_{y}{}^{1/2} = L_x^{1/4} K_x^{1/4} (1-L_x)^{1/4} (1-K_x)^{1/4}$$

$$\ldots (1-L_x)^{-3/4}(-1)\Big)\Big) = 0$$

$$\max_{L_x, K_x} \left(\underbrace{L_x \quad K_x}_{X}\right) / \left(\underbrace{(1-L_x) \quad (\cdots)}_{Y}\right)$$

$$\frac{\partial}{\partial X} =$$

$$\frac{\partial}{\partial L_x} = \cancel{K_x^{1/4}(1-K_x)^{1/4}}\left(\cancel{\tfrac{1}{4}} L_x^{-3/4}(1-L_x)^{1/4} + L_x^{1/4}\cancel{\tfrac{1}{4}}(1-L_x)^{-3/4}(-1)\right) = 0$$

$$\frac{(1-L_x)^{1/4}}{L_x^{3/4}} - \frac{L_x^{1/4}}{(1-L_x)^{3/4}} = 0$$

$$\frac{(1-L_x)^{1/4}}{L_x^{3/4}} = \frac{L_x^{1/4}}{(1-L_x)^{3/4}}$$

$$1 - L_x = L_x$$

$$\boxed{L_x = 1/2} \Rightarrow \boxed{L_y = 1/2}$$

$$\frac{\partial}{\partial K_x} \Rightarrow \boxed{K_x = 1/2} \Rightarrow \boxed{K_y = 1/2}$$

$$X = 1/2 = L_x^{1/2} K_x^{1/2}$$

$$Y = 1/2$$