Lecture 7

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Lecture7

Lecture 7: Monopoly

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Introduction

Elasticities

Monopoly

- Firm is faced a problem like the following:

$$\max_{K,L} p_x f_x(L, K) - \underbrace{wL - rK}_{\text{Costo Insumos.}}.$$

- The firm's choice of L and K does not affect the prices p, w, r
- This is called *price-taking* behavior
- Justified if the the market is composed of many small firms

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- In many markets there is a single firm
- Since supply is completely controlled by the firm, it can use this in its favor

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- Profit maximization condition,

$$\max_{K,L} pf_x(K, L) - wL - rK.$$

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$$\max_{K,L} pf_x(K, L) - wL - rK.$$

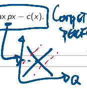
- If

$$c(x) = \min_{K,L} \overbrace{wL + rK}^{\text{Costos}} \text{ such that } f_x(K, L) = x$$

then the above is equivalent to:

$$\boxed{\max_x px - c(x).}$$

Competencia perfecta



- When firm controls supply, then:

$$\max_x \mathbf{p(x)}x - c(x)$$

Monopolio

Endogeno!

- When firm controls supply, then:

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- Consumers willingness to pay is given by the demand function

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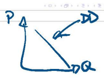
- When firm controls supply, then:

$$\max_x \boxed{\mathbf{p(x)}}x - c(x)$$

- Consumers willingness to pay is given by the demand function
- $p(x)$ is the **demand** function



- We can also represent the problem as:

$$\max_p pq(p) - c(q(p))$$

Equivalente

- $q(p)$ is the **inverse demand function**

Lecture 7: Monopoly

Introduction

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Monopoly

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Elasticities

Monopoly

Elasticities

$\Pi = pq - c(q)$

- Revenue: R(q) = p(q) q

Elasticities

- Revenue: $\boxed{R(q) = \boxed{p(q)}\, q}$
-

$$\boxed{\frac{dR}{dq}} = \boxed{p(q) + q\frac{dp}{dq}(q)} = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

$\rightarrow p\left(1 + \boxed{\frac{q}{p}\frac{\partial p}{\partial q}}\right) \rightarrow \frac{1}{\varepsilon_{q,p}}$

$\boxed{\varepsilon_{q,p} = \frac{\partial q}{\partial p} \cdot \frac{p}{q}}$

Elasticities

- Revenue: R(q) = p(q) q
-

$$\boxed{\frac{dR}{dq}} = p(q) + q\frac{dp}{dq}(q) = \underset{+}{p(q)}\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

-

$$\boxed{\frac{dR}{dq} > 0} \iff \boxed{1 > -\frac{1}{\varepsilon_{q,p}}} \iff \boxed{\varepsilon_{q,p} < -1.}$$

$1 + \frac{1}{\varepsilon_{q,p}} > 0$

$1 > -\frac{1}{\varepsilon_{q,p}}$

Elasticities

- Revenue: R(q) = p(q) q
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$$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

-

$$\boxed{\frac{dR}{dq} > 0} \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

$\boxed{\frac{\partial R}{\partial q} > 0 \text{ SSI DD ES ELASTICA}}$

Elasticities

$\varepsilon_{q,p} = \frac{\partial q}{\partial p} \cdot \frac{p}{q} = \frac{\frac{\partial q}{q}}{\frac{\partial p}{p}} = \frac{\%q}{\%p}$

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Elasticities

$$\varepsilon_{q,p} = \frac{\partial q}{\partial p} \cdot \frac{p}{q} = \frac{\frac{\partial q}{q}}{\frac{\partial p}{p}} = \frac{\% q}{\% p}$$

- If $\boxed{\varepsilon_{q,p} \in (-1, 0)}$, the demand is *inelastic*
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\varepsilon_{q,p} < -1$, then demand is *elastic*
  - An increase in price leads a big decrease in demand
  - An increase in quantity leads to a small decrease in price

$\cancel{q(p) = \alpha + bp}$

$\cancel{q(p) = \alpha p^{\kappa}}$

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?

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- Suppose that the demand function is of constant elasticity κ
- $$\boxed{\frac{dq}{dp}\frac{p}{q} = \kappa < 0.}$$
- $$\boxed{\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p}} \implies \boxed{\frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.}$$

P A DIVIDIR

$$\log p^{K}$$

$$\frac{\partial \log p^{K}}{\partial p} = \frac{1}{p^{K}} \underbrace{K p^{K-1}}_{\text{DERIVADA INTERNA}} = \frac{K}{p^{K-(K-1)}} = \frac{K}{p}$$

$$\frac{\partial \log q(p)}{\partial p} = \frac{1}{q(p)} \cdot \underbrace{\frac{\partial q(p)}{\partial p}}_{\text{DERIVADA INTERNA}}$$

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?
- Suppose that the demand function is of constant elasticity κ
- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \implies \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$
- By the fundamental theorem of calculus:
  $$\log q(p) = C + \log p^{\kappa}.$$

$$e^{\log q(p)} = e^{C + \log p^{K}}$$

$$q(p) = e^{C} \cdot e^{\log(p^{K})} = \boxed{e^{C}} p^{K}$$

CONSTANTE

$$\boxed{q(p) = A p^{\boxed{K}}}$$

ELASTICIDAD

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?
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- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \implies \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$
- By the fundamental theorem of calculus:
  $$\log q(p) = C + \log p^{\kappa}.$$
- $q(p) = e^{C}p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

Elasticities

Whenever the demand function has constant elasticity κ

$q(p) = Ap^{K}$

- $q(p) = Ap^{\kappa}$ for some $A > 0$.

ELASTICIDAD

- Equivalently,
  $$\boxed{p(q) = \left(\frac{q}{A}\right)^{1/\kappa}.}$$

Lecture 7: Monopoly

- We want to study the problem: $\boxed{\max_q R(q) - c(q)} = \max_q p(q)\cdot q - c(q)$

- We want to study the problem:

$$\max_q R(q) - c(q)$$

$$\frac{\partial \pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial c}{\partial q} = 0$$

$$\underbrace{\frac{\partial R}{\partial q}}_{Img} = \underbrace{\frac{\partial c}{\partial q}}_{Cmg}$$

- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow \underbrace{p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)}_{\substack{Img \\ +}} = \boxed{\frac{dc}{dq} > 0.}$$

- We want to study the problem:

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- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = \frac{dc}{dq} > 0.$$

- This implies

$$1 + \frac{1}{\varepsilon_{q,p}} > 0 \Longleftrightarrow \boxed{\varepsilon_{q,p} < -1.}$$

$$1 + \frac{1}{\varepsilon_{q,p}} > 0$$

$$1 > -\frac{1}{\varepsilon_{q,p}}$$

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- A monopoly firm always produces at a point where demand is elastic
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits

$$R = q\,p(q)$$

$$\boxed{\frac{\partial R}{\partial q}} = p(q) + q\cdot\frac{\partial p}{\partial q} > 0$$

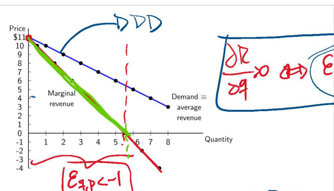
$$p\left(1 + \underbrace{\frac{q}{p}\frac{\partial p}{\partial q}}_{=\frac{1}{\varepsilon_{q,p}}}\right) > 0$$

$$\underset{+}{p}\left(1 + \frac{1}{\varepsilon_{q,p}}\right) > 0$$

$$+ \Rightarrow 1 + \frac{1}{\varepsilon_{q,p}} > 0$$

$$1 > -\frac{1}{\varepsilon_{q,p}}$$

$$\boxed{\varepsilon_{q,p} < -1}$$



$$\boxed{\frac{\partial R}{\partial q} > 0 \Leftrightarrow \varepsilon_{q,p} < -1}$$

Optimo Monopolista $\Rightarrow$ $Img = Cmg$

$$\frac{\partial R}{\partial q} = \frac{\partial c}{\partial q}$$

$$p\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = \frac{\partial c}{\partial q}$$

- We can simplify to:

$$p(q) = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq}$$

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- Since $\varepsilon_{q,p} < -1$, then

$$p = \boxed{\frac{1}{1 + \frac{1}{\varepsilon_{q,p}}}}\frac{dc}{dq} \quad \frac{dc}{dq} = Cmg.$$

$$P > Cmg$$

$$\boxed{\frac{1}{1 + \frac{1}{\varepsilon}} > 1} \rightarrow 1 > 1 + \frac{1}{\varepsilon}$$

$$0 > \frac{1}{\varepsilon}$$

- We can simplify to:

$$p(q) = \frac{1}{\ldots}\frac{dc}{\ldots}$$

$\rightarrow Cmg = 00$

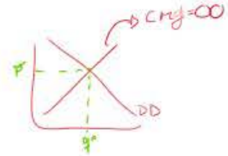
- We can simplify to:

$$p(q) = \boxed{\frac{1}{1-\frac{1}{\epsilon_{q,p}}}}\frac{dc}{dq}$$

- Since $\epsilon_{q,p} < -1$, then

$$\boxed{p} = \frac{1}{1-\frac{1}{\epsilon_{q,p}}}\frac{dc}{dq} > \boxed{\frac{dc}{dq}}$$

- The firm always sets a price that is strictly above marginal cost



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- The firm always sets a price that is strictly above marginal cost
- There is a **mark-up** above marginal cost at the profit maximizing price
- The amount produced q is below the quantity where $p = MC$

$\varepsilon_{q,p} \to -\infty \Rightarrow P = Cmg$

$\varepsilon_{q,p} \to -1 \Rightarrow P = \infty$

- The above analysis already illustrates an important point against monopolies

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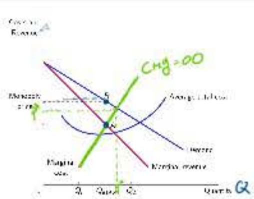
- Both consumer surplus and total surplus is less than is socially optimal

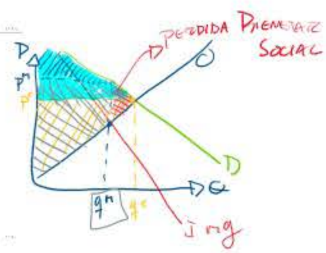
$p = Cmg$ (Comp Perfecta)

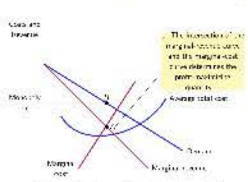
- The above analysis already illustrates an important point against monopolies

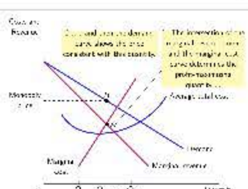
- Both consumer surplus and total surplus is less than is socially optimal

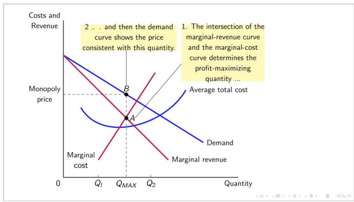
- Thus the pricing policies used by monopolies are inefficient, leading to what is called "dead-weight loss"

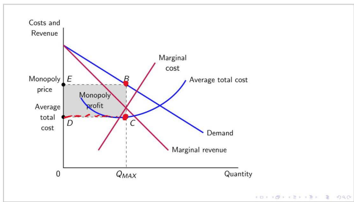


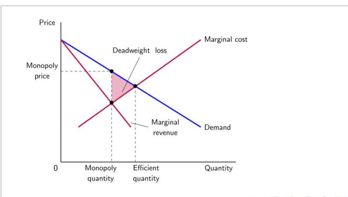












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- Demand function has constant elasticity of demand ($\boxed{q(p) = Ap^{\kappa}}$)
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- $$p = \frac{1}{1+\frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq} = \frac{1}{1+\frac{1}{\kappa}}\frac{dc}{dq}.$$
- Has a solution if and only if $\kappa < -1$

$\kappa \in (0,-1)$

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- $$\max_p pq(p) - c(q(p)).$$
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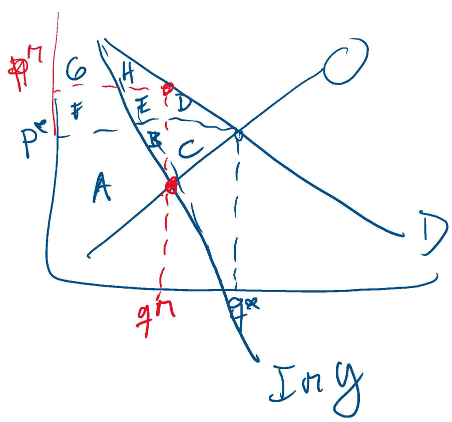

EQUILIBRIO COMPETITIVO

$EC = G + H + F + E + D$

$EP = A + B + C$

MONOPOLIO

$EC: G + H$

$EP = F + E + A + B$

$PBS = D + C$

$$\underbrace{F+E+A+B}_{\pi^M} > \underbrace{A+B+C}_{\pi^*}$$

$$\boxed{F+E>C}$$

- Demand function has constant elasticity of demand ($q(p) = Ap^{\kappa}$)
- $$\max_p pq(p) - c(q(p)).$$
- $$p = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}} \frac{dc}{dq} = \frac{1}{1 + \frac{1}{\kappa}} \boxed{\frac{dc}{dq}}.$$
- Has a solution if and only if $\kappa < -1$
- If $\kappa \geq -1$, then the firm always prefer to increase the price (no solution)
- If marginal costs are constant at c
- $$\boxed{p = \frac{c}{1 + \frac{1}{\kappa}}} \Longrightarrow q(p) = A\left(\frac{c}{1 + \frac{1}{\kappa}}\right)^{\kappa}.$$

$c(q) = c \cdot q$

$\frac{\partial c}{\partial q} = c$

$\boxed{p > Cmg}$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc.
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply.