Lecture 7

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Lecture7

Lecture 7: Monopoly

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- Firm is faced a problem like the following:

$$\max_{K,L} \underbrace{p_x f_x(L,K)}_{\text{Ingreso}} - \overbrace{wL - rK}^{\text{Costos}}.$$

- The firm's choice of L and K does not affect the prices $\boxed{p}$, w, r
- This is called *price-taking* behavior Precio Aceptantes.
- Justified if the the market is composed of many small firms

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- In many markets there is a single firm
- Since supply is completely controlled by the firm, it can use this in its favor

- Profit maximization condition,

$$\max_{K,L} pf_x(K,L) - wL - rK.$$

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$$\max_{K,L} pf_x(K,L) - wL - rK.$$

- If

$$c(x) = \min_{K,L} \overbrace{wL + rK}^{\text{Costos}} \text{ such that } f_x(K,L) = x$$

funcion Costos

then the above is equivalent to:

$$\max_{x} px - c(x).$$

$\underbrace{P(x)}_{\text{DISPONIBILIDAD A PAGAR}}$

- When firm controls supply, then:

$$\max_{x} \mathbf{p(x)}x - c(x)$$

↳ DISPONIBILIDAD A PAGAR

- When firm controls supply, then:

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- Consumers willingness to pay is given by the demand function

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- Consumers willingness to pay is given by the demand function

- $p(x)$ is the **demand** function

Son EQUIVALENTES

- We can also represent the problem as:

$$\max_{p} pq(p) - c(q(p))$$

- $q(p)$ is the **inverse demand function**

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Elasticities

$\pi \to$ GANANCIAS $\to$ "PROFIT"

- Revenue: $\underbrace{R(q)}_{\text{INGRESO}} = p(q)\, q$

$$IMg = \frac{\partial R}{\partial q} = \frac{\partial P}{\partial q} \cdot q + 1 \cdot P = P\left(\frac{\partial P}{\partial q} \cdot \frac{q}{P} + 1\right) = P\left(\frac{1}{\varepsilon_{q,P}} + 1\right)$$

$\frac{\partial q}{dq} \Rightarrow \Delta$ "PEQUEÑO" EN q

$$\boxed{\varepsilon_{q,P} = \frac{\partial q}{\partial P} \cdot \frac{P}{q} = \frac{\frac{\partial q}{q}}{\frac{\partial P}{P}} = \frac{\frac{\Delta q}{q}}{\frac{\Delta P}{P}} = \frac{\Delta\% q}{\Delta\% P}}$$

Elasticities

- Revenue: $R(q) = p(q)\, q$
- $IMg = \frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$

$\boxed{IMg > 0}$ ¿? $\Rightarrow \overset{+}{\boxed{P}}\left(1 + \frac{1}{\varepsilon_{q,P}}\right) > 0$

$\to 1 + \frac{1}{\varepsilon_{q,P}} > 0 \to 1 > -\frac{1}{\varepsilon_{q,P}}$ $\to$ # NEGATIVO.

$$\boxed{\varepsilon_{q,P} < -1}$$

$$\boxed{I = P \cdot q}$$

$$\boxed{IMg > 0} \Leftrightarrow \boxed{\varepsilon_{q,P} < -1}$$

Elasticities

- Revenue: $R(q) = p(q)\, q$
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$
- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

Elasticities

- Revenue: $R(q) = p(q)\, q$
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$
- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$
- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

Elasticities

- If $\boxed{\varepsilon_{q,p} \in (-1,0)}$ the demand is *inelastic*
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\boxed{\varepsilon_{q,p} < -1}$, then demand is *elastic*
  - An increase in price leads a big decrease in demand
  - An increase in quantity leads to a small decrease in price

→ q DD NO RESPONDE MUCHO A Δ PRECIOS

q DD RESPONDE "MUCHO" A Δ PRECIOS.

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?

$$\frac{\partial q}{\partial p}\cdot\frac{P}{q} = K \rightarrow \text{ELASTICIDAD}$$

$$\boxed{q'(P)\cdot\frac{1}{q} = \frac{K}{P}} \quad \text{ECUACIÓN DIFERENCIAL}$$

$$\ln p^K = K \ln P \qquad \frac{\partial \ln p^K}{\partial P} = K\frac{1}{P}$$

$$\ln(q(P)) \qquad \frac{\partial \ln(q(P))}{\partial P} = \frac{1}{q(P)}\, q'(P)$$

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?
- Suppose that the demand function is of constant elasticity κ

$$\frac{\partial \ln(q(P))}{\partial P} = \frac{\partial \ln P^K}{\partial P} \rightarrow \int \frac{\partial \ln(q(P))}{\partial P}\,dP = \int \frac{\partial \ln P^K}{\partial P}\,dP$$

$$\ln(q(P)) = \ln P^K + C \qquad \frac{\partial}{\partial P}$$

$$e^{\ln(q(P))} = e^{\ln P^K + C}$$

$$q(P) = e^{\ln P^K}\boxed{e^C} \rightarrow \text{CONSTANTE "A"}$$

$$\boxed{q(P) = P^K A}$$

$$\left(\frac{q}{A}\right)^{1/K} = P(q)$$

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?
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-
$$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$

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$$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
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$$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$

Elasticities

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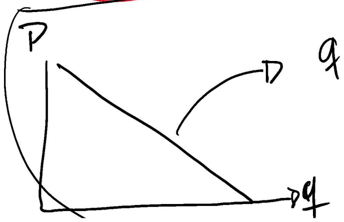
$$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$
- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$



$$q = \alpha + \beta P$$

$$\rightarrow \varepsilon_{q,P} = \frac{\partial q}{\partial P}\cdot\frac{P}{q} = \beta\cdot\frac{P}{\alpha+\beta P}$$

Elasticities

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- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$
- $q(p) = e^{C}p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

Elasticities

Whenever the demand function has constant elasticity κ

- $q(p)Ap^{\kappa}$ for some $A > 0$.
- Equivalently,
$$\boxed{p(q) = \left(\frac{q}{A}\right)^{1/\kappa}.}$$

Lecture 7: Monopoly

Introduction

Elasticities

Monopoly

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Introduction

Elasticities

Monopoly

- We want to study the problem:

$$\max_q R(q) - c(q)$$

$R(q) = P(q) \cdot q$

$$\frac{\partial \pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial c}{\partial q} = 0$$

$$\Rightarrow \frac{\partial R}{\partial q} = \frac{\partial c}{\partial q}$$

IMg = CMg

- We want to study the problem:

$$\max_q R(q) - c(q)$$

- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow \boxed{p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)} = \boxed{\frac{dc}{dq} > 0.}$$

IMg = CMg

SIEMPRE (CMg > 0)

ENTONCES IMg > 0

$\boxed{\varepsilon_{q,p} < -1}$

- We want to study the problem:

$$\max_q R(q) - c(q)$$

- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \implies p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = \frac{dc}{dq} > 0.$$

- This implies

$$1 + \frac{1}{\varepsilon_{q,p}} > 0 \iff \varepsilon_{q,p} < -1.$$

El Prob. Monopolista solamente tiene solución si DD es elástica.

- $$1 + \frac{1}{\varepsilon_{q,p}} > 0 \iff \varepsilon_{q,p} < -1.$$

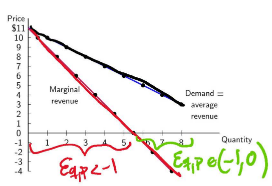
- A monopoly firm always produces at a point where demand is elastic

"Ejercicio hipotético"

si $\underline{\varepsilon_{q,p} \in (-1,0)} \rightarrow IMg < 0 \Rightarrow \downarrow q$ $\rightarrow \uparrow$ Ingresos, $\rightarrow \downarrow$ Costos $\Rightarrow \uparrow \pi$

- $$1 + \frac{1}{\varepsilon_{q,p}} > 0 \iff \varepsilon_{q,p} < -1.$$

- A monopoly firm always produces at a point where demand is elastic
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits



- We can simplify to: $$\boxed{p(q) = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}} \frac{dc}{dq}.}$$

$$\boxed{P\left(1 + \frac{1}{\varepsilon}\right)} = \frac{\partial C}{\partial q}$$

$$\frac{1}{1 + \frac{1}{\varepsilon_{q,p} < -1}} \in (-1,0)$$ $\in (0,1)$ > 1

- We can simplify to:

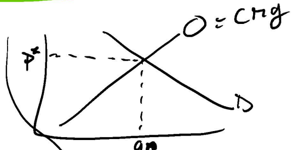
$$p(q) = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}} \frac{dc}{dq}.$$

- Since $\varepsilon_{q,p} < -1$, then

$$p(q) = p = \boxed{\frac{1}{1 + \frac{1}{\varepsilon_{q,p}}}} \frac{dc}{dq} > \frac{dc}{dq} = CMg$$ $\rightarrow > 1$

$\Rightarrow P > CMg$

$\rightarrow$ Elasticidad DD determina diferencia P y CMg



$$\boxed{P^* = CMg(q^*)}$$

$$\boxed{\begin{array}{l} \varepsilon \to -\infty \Rightarrow P = CMg \\ \varepsilon \to -1 \Rightarrow P = \infty \end{array}}$$

- We can simplify to:

$$p(q) = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}} \frac{dc}{dq}.$$

$$\Rightarrow \frac{1}{\varepsilon} = \frac{\%\Delta P}{\%\Delta q}$$

P

- We can simplify to:
$$p(q) = \frac{1}{1+\frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq}.$$
- Since $\varepsilon_{q,p} < -1$, then
$$p = \frac{1}{1+\frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq} > \frac{dc}{dq}.$$
- The firm always sets a price that is strictly above marginal cost

→ "MARK-UP"

$\varepsilon \to -\infty \Rightarrow P = Cmg$
$\varepsilon \to -1 \Rightarrow P = \infty$

EN UN MONOPOLIO
⇒ ENTRE + ELASTICA
LA DD "MAS PARECIDO"
A COMPETENCIA PERFECTA.

$\varepsilon_{q,P}$ DD

$$\frac{\Delta q\%}{\Delta p\%} \rightarrow \frac{\Delta \% q}{\Delta P\%} = \frac{\partial q}{\partial P}\cdot\frac{P}{q}$$

DQ

- We can simplify to:
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- There is a **mark-up** above marginal cost at the profit maximizing price

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- The amount produced q is below the quantity where $p = MC$.

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- Both consumer surplus and total surplus is less than is socially optimal

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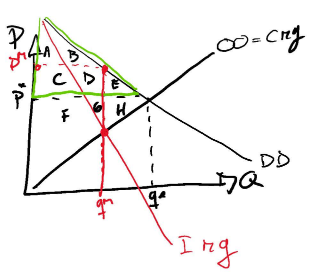
- Both consumer surplus and total surplus is less than is socially optimal
- Thus the pricing policies used by monopolies are inefficient, leading to what is called "dead-weight loss"



Comp. Perfecta
EC = A+B+C+D+E
EP = F+G+H

MONOP.
EC = A+B
EP = C+D+F+G
PBS = E+H

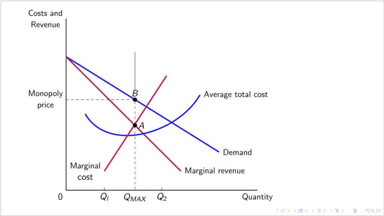
Tiene q' ser
C+D+F+G > F+G+H
C+D > H

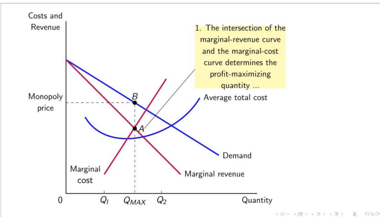
Costs and Revenue

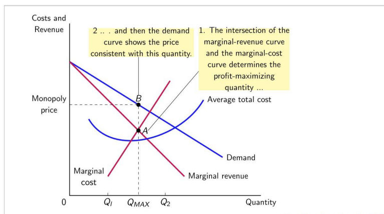
Img

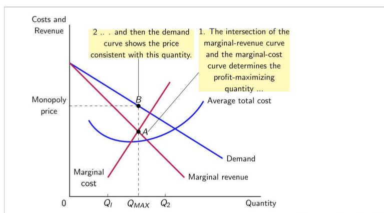
EP = C+D+F+G

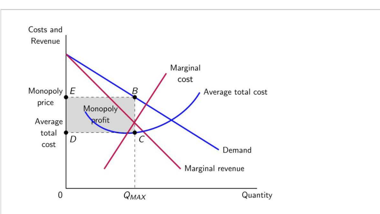
PBS = E+H

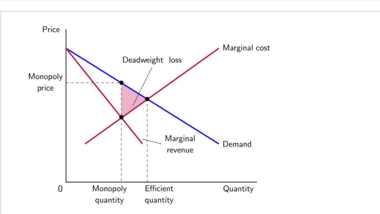












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- Has a solution if and only if $\kappa < -1$
- If $\kappa \geq -1$, then the firm always prefer to increase the price (no solution)
- If marginal costs are constant at c
- $$p = \frac{c}{1 + \frac{1}{\kappa}} \Longrightarrow q(p) = A \left(\frac{c}{1 + \frac{1}{\kappa}} \right)^{\kappa}.$$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc.
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply.

VACUNA

$\varepsilon_{q,p}$ BAJA

$$\frac{1}{\varepsilon_{q,p}} = \frac{\%\Delta P}{\%\Delta q} = \varepsilon_{P,q}$$