



Lecture8-9

Lecture 8-9: Price Discrimination

Mauricio Romero

Lecture 8-9: Price Discrimination

- Introduction
- First Degree Price Discrimination
- Third Degree Price Discrimination
- Monopsony
- Double Marginalization Problem
- Profit Sharing and Double Marginalization

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- In real life, firms often have different prices for different consumers/units
- We will explore some of these now
- In a competitive market such exotic pricing schemes could never arise since $p = \text{marginal cost}$

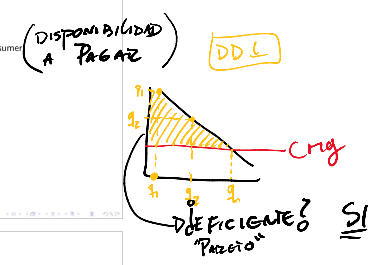
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- Suppose the firm can observe all characteristics of the consumer
- What should the firm do?



- Suppose the firm can observe all characteristics of the consumer

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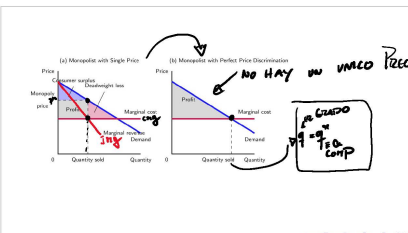
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- Demand curve illustrates the willingness to pay for the q-th unit of the product

- Suppose the firm can observe all characteristics of the consumer
- What should the firm do?
- Demand curve illustrates the willingness to pay for the q-th unit of the product
- Firm can extract all of the surplus of the consumer. How?

- Firm will price at $p(q)$ for the q-th unit and continue to produce until $p(q) = MC(q)$

- Firm will price at $p(q)$ for the q-th unit and continue to produce until $p(q) = MC(q)$
- Firm gets all of the consumer surplus as his profits:

$$\Pi = \int_0^{q^*} (p(q) - c(q))dq = \int_0^{q^*} p(q)dq - c(q^*)$$
 where q^* is the quantity at which $p(q^*) = c(q^*)$.



- Firm can do this is because it knows the exact demand curve of each consumer
- Such activity is prohibited in many countries

- Firm can do this is because it knows the exact demand curve of each consumer
- Such activity is prohibited in many countries
- Amazon tries to estimate everyone's demand curve

RAPPI Dato

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- Market is segmented (no re-selling across markets)
- Firm knows the characteristics of each market (demand curve)
- Consider the following example: Two kinds of consumers:

$$q_A(p_A) = 24 - p_A$$

$$q_B(p_B) = 24 - 2p_B$$
- constant marginal cost of production of 6

If the firm were allowed to set different prices in the different markets, then he would choose:

$$\pi_A = \max_{p_A} (24 - p_A)(p_A - 6) = (24 - p_A)(p_A - 6)$$

$$\pi_B = \max_{p_B} (24 - 2p_B)(p_B - 6) = (24 - 2p_B)(p_B - 6)$$

$q_A = 15, p_A = 9, \pi_A = 45$
 $q_B = 6, p_B = 9, \pi_B = 18$

$EC_A = \frac{1}{2} \cdot (24 - 15) \cdot 9 = 40.5$
 $EC_B = \frac{1}{2} \cdot (12 - 6) \cdot 9 = 27$

$EP_A = 9 \cdot (15 - 6) = 81$
 $EP_B = 6 \cdot (9 - 6) = 18$

$PBS_A = (18 - 9) \cdot (15 - 6) = 81$
 $PBS_B = (12 - 6) \cdot (9 - 6) = 18$

$q_A^* = 24 - 6 = 18$
 $q_B^* = 24 - 6 \cdot 2 = 12$

Total consumer surplus (CS) and profits of the firm in each market:
 $\pi_A^* = 81, \pi_B^* = 18, CS_A = 40.5, CS_B = 27$

$$q_A = 24 - p_A \rightarrow q_A = 24 - P$$

$$q_B = 24 - 2p_B \rightarrow q_B = 24 - 2P$$

$$Q = q_A + q_B = (24 - P) + (24 - 2P) = 48 - 3P$$

$$q_A = \max(0, 24 - P)$$

$$q_B = \max(0, 24 - 2P)$$

$$q_A = 0 \quad \text{si} \quad P \geq 24$$

$$q_B = 0 \quad \text{si} \quad P \geq 12$$

$$Q = \begin{cases} 48 - 3P & \text{si } P \leq 12 \\ 24 - P & \text{si } 12 < P < 24 \\ 0 & \text{si } P \geq 24 \end{cases}$$

Firm chose to set the same price in each market. Then he would maximize the following:

$$\max_{p} \left\{ \max_{p_A} (24 - p)(p - 6), \max_{p_B} (24 - p)(p - 6) + (24 - 2p)(p - 6) \right\} = \max(81, 75) = 81$$

- Price of $p^* = 15$ in both markets, which leads to only consumers in market A buying
- To summarize, the consumer surplus and profits in each market are:

$$\pi_A^* = 81, \pi_B^* = 0, CS_A = 40.5, CS_B = 0$$
- Prohibiting third degree price discrimination can exclude a whole market altogether
- Highly inefficient compared to the social welfare outcome given third degree price discrimination

Suppose that the constant marginal cost of production is now c instead of 6.

With third degree price discrimination, the firm sets the following prices:

$$\pi_A = \max_{p_A} (24 - p_A)(p_A - c) \Rightarrow p_A^* = 14, q_A^* = 10, \pi_A^* = 10 \cdot 10 = 100$$

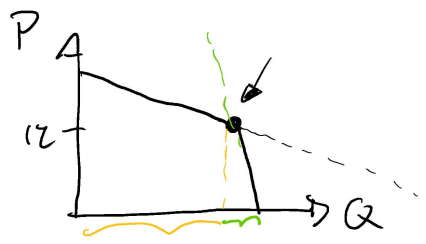
$$\pi_B = \max_{p_B} (24 - 2p_B)(p_B - c) \Rightarrow p_B^* = 8, q_B^* = 8, \pi_B^* = 8 \cdot 4 = 32$$

In this case, the profits and consumer surplus in each market is given by:

$$\pi_A^* = 100, \pi_B^* = 32, CS_A = 50, CS_B = 16, \pi^* = 132$$

SupongA $P < 12$

$$\pi = (48 - 3P)P - 6(48 - 3P)$$



Supongamos $12 < P < 24$

$$\pi = (24 - P)(P - 6)$$

$P = 12$

$$\pi = \underbrace{(12)}_q \cdot \underbrace{(12 - 6)}_{(P - c)}$$

$\pi = \max_{p_A} (24 - p_A)(p_A - 4) = \pi_A = 24$
 $\pi_B = \max_{p_B} (24 - 2p_B)(p_B - 4) = \pi_B = 8$
 $q_B = 8 \rightarrow \pi_B = 8 \cdot 4 = 32$

In this case, the profits and consumer surplus in each market is given by:
 $\pi_A = 100, \pi_B = 32, CS_A = 50, CS_B = 16, \pi = 132$

If the firm were prohibited from using third degree price discrimination, then:
 $\max_p \{ \max_{p_A} (24 - p)(p - 4), \max_{p_B} (48 - 3p)(p - 4) \}$
 $= \max(100, 108) = 108$

$p = 10$

profits in both markets and the consumer surplus in both markets:
 $\pi = 84, \pi_B = 24, CS_A = 98, CS_B = 75, \pi = 210$

- Consumers in region B are hurt but consumers in region A gain significantly leading to an increase in consumer surplus
 - The firm's joint profits are hurt but the total surplus actually increases
 - Total surplus decreases
- Third degree price discrimination is considered illegal in many countries and the European union
- It is possible to get around such allegations by claiming that the differential pricing comes from cost reasons

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- When someone or some firm is the sole buyer (monopony is the sole seller)
- Often arises in the context of firms being the sole buyers of labor

Let us study the profit maximization problem of a firm:

$\max_{K, L} p(K, L) - wL = \pi$

w is now a function of the amount of labor demanded (reflecting the power of the firm in the labor market)

$\pi = p(K, L) - w(L) \rightarrow \frac{\partial \pi}{\partial L} = \frac{w}{L} = \frac{w}{p} = \text{SALARIO REAL}$
 $\frac{\partial \pi}{\partial L} = 0 \rightarrow \frac{w}{L} = \frac{w}{p} = \text{SALARIO REAL}$

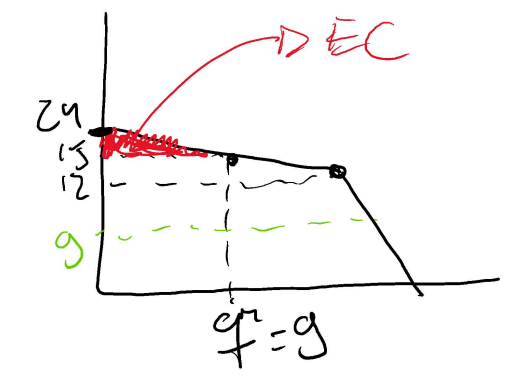
$\pi = (48 - 3p)(p - 6)$
 $\frac{\partial \pi}{\partial p} = -3(p - 6) + 1(48 - 3p) = 0$
 $-3p + 18 + 48 - 3p = 0$
 $66 = 6p$
 $p = 11$
 $\pi^* = (48 - 33)(11 - 6) = (15)(5) = 75$

$\pi^* = 15$

$EC = \frac{(24 - 15)(9)}{2} = 81/2$
 $EP = 81$

$\pi = (24 - p)(p - 6)$
 $\frac{\partial \pi}{\partial p} = -1(p - 6) + 1(24 - p) = 0$
 $-p + 6 + 24 - p = 0$
 $30 = 2p$
 $p = 15$
 $\pi^* = (24 - 15)(15 - 6) = 9 \cdot 9 = 81$

$\pi^* = 12 \cdot 6$
 $\pi^* = 72$



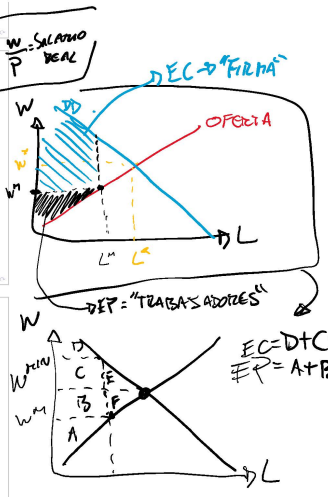
$s.t. p > 12$
 $\pi = (24 - p)(p - 4)$
 $\frac{\partial \pi}{\partial p} = -1(p - 4) + 1(24 - p) = 0$
 $-p + 4 + 24 - p = 0$
 $28 = 2p$
 $\pi^* = 14$

$s.t. p < 12$
 $\pi = (48 - 3p)(p - 4)$
 $\frac{\partial \pi}{\partial p} = -3(p - 4) + 1(48 - 3p) = 0$
 $-3p + 12 + 48 - 3p = 0$
 $60 = 6p$
 $\pi^* = 10$

The first order condition yields $\frac{\partial \pi}{\partial L} = 0 \Rightarrow \frac{w}{P} = \frac{w}{P} = \text{SALARIO REAL}$

In a competitive market $w = 0$ and so $pMPL = w$

Wages and labor below the competitive level (an argument for minimum wages and unions)



$ZQ = ZT$
 $P^* = 14$

$\pi^* = 10 \cdot 10 = 100$

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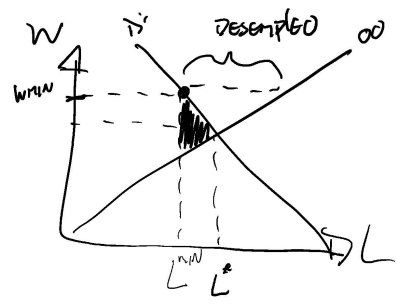
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What happens when there are multiple monopolies involved in the market?

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Firm A produces factor a at no cost

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$60 - 4$
 $P^* = 10$
 $\pi^* = (18)(6) = 108$

$P^{UNICO} = 10$

- ▶ What happens when there are multiple monopolies involved in the market?
- ▶ Firm A produces factor a at no cost
- ▶ Firm b in order to supply q_b units of b must buy q_a units of a
- ▶ Firm B produces according to a cost function:

$$C(q_b) = (\rho_a + c)q_b$$

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- ▶ Firm B produces according to a cost function:

$$C(q_b) = (\rho_a + c)q_b = \rho_a q_b + \underbrace{c q_b}_{\text{Costo Marginal}}$$
- ▶ Demand equation for good b is linear:

$$q_b(\rho_b) = 100 - \rho_b$$

(Firma A
Firma B
son
Monopolistas)

- ▶ Firm B's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - \rho_a q_b - c q_b$$

- ▶ Firm B's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - \rho_a q_b - c q_b$$
- ▶ The first order condition tells us:

$$100 - 2q_b = \rho_a + c \Rightarrow \rho_a = 100 - 2q_b - c$$

$$\rho_a = 100 - 2q_a - c \quad \# \quad q_b = q_a$$

DDa.

- ▶ Firm B's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - \rho_a q_b - c q_b$$
- ▶ The first order condition tells us:

$$100 - 2q_b = \rho_a + c \Rightarrow \rho_a = 100 - 2q_b - c$$
- ▶ Since firm b is the only demander of commodity a, we have:

$$\rho_a = 100 - 2q_b - c = 100 - 2q_a - c$$

- ▶ If the price is ρ_a then the q_a that solves the above equation would be the amount demanded of good a

- ▶ If the price is ρ_a then the q_a that solves the above equation would be the amount demanded of good a
- ▶ Thus firm B's maximization problem has given us an inverse demand function for commodity a

Since firm A is also a monopolist in producing good a, we can solve firm A's maximization problem in the following way:

$$\max_q (100 - 2q_a - c) \rightarrow 0$$

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$$\max_q (100 - 2q_a - c)$$

As a result, we get:

$$100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, p_a^* = 50 - \frac{c}{2}$$

$$q_a = 9.5$$

Since firm A is also a monopolist in producing good a, we can solve firm A's maximization problem in the following way:

$$\max_q (100 - 2q_a - c)$$

As a result, we get:

$$100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, p_a^* = 50 - \frac{c}{2}$$

Firm a decides to supply the above units of a at a price $50 - c/2$

Firm B will produce $q_b^* = q_b^* = \frac{100 - c}{4}$

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Then the price is given by:

$$p_b^* = 100 - \frac{100 - c}{4} = 75 + \frac{c}{4}$$

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Then the price is given by:

$$p_b^* = 100 - \frac{100 - c}{4} = 75 + \frac{c}{4}$$

To summarize, we have:

$$\begin{aligned} p_a^* &= 50 - \frac{c}{2} & (1) \\ q_a^* &= \frac{100 - c}{4} & (2) \\ p_b^* &= 75 + \frac{c}{4} & (3) \\ q_b^* &= \frac{100 - c}{4} & (4) \end{aligned}$$

Case 1: $c = 0$

$$p_a^* = 50, q_a^* = 25, p_b^* = 75, q_b^* = 25$$

If the firms were to merge so that whatever is produced by one of the firms can be used freely by the other:

The monopolists problem becomes:

$$\max_q (100 - q)$$

The first order condition states that:

$$100 - 2q = 0 \Rightarrow q^* = 50, p^* = 50$$

Price of good b comes down from 75 to 50

Production of good b goes up from 25 to 50

This increases both the profits of the firm and the consumer surplus!

Case 2: $c = 10$

$$p_a^* = 45, q_a^* = 22.5, p_b^* = 77.5, q_b^* = 22.5$$

If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?

The monopolists problem becomes:

$$\max_q (100 - q) - 10q$$

The first order condition states that:

$$100 - 2q = 10 \Rightarrow q^* = 45, p^* = 55$$

Handwritten notes and equations:

$$\pi = \pi_a + \pi_b = q_a p_a + q_b p_b - (p_a + c) q_b$$

$$q_a p_a + q_b p_b - \frac{p_a q_b}{p_a q_a} - c q_b$$

$$q_b p_b - c q_b$$

$$q_b (100 - q_b) - c q_b \rightarrow 0$$

► The monopolists problem becomes:

$$\max_q q(100 - q) - 10q$$

► The first order condition states that:

$$100 - 2q = 10 \implies q^* = 45, p^* = 55$$

Handwritten: π, EC

► What is going on in the above examples?

- because the first firm is a monopolist, it charges a mark up above marginal cost for its intermediate good
- This then distorts the marginal cost of firm B up additionally
- This then leads an even larger mark up on top of this additional marginal cost that affects the price of good b
- Essentially a markup on product a indirectly leads to an even larger markup on the final product b
- This is called the **double marginalization problem**

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► Double marginalization can lead to inefficiently high prices and inefficiently low levels of production

► By merging, both profits of the firm and consumer surplus may simultaneously go up

► Difficult to tell if two firms are merging to solve a double marginalization problem or if they are simply merging to create a monopoly

► What are some potential ways to solve this problem without merges?

► One possible way might be to engage in profit sharing

► Firms agree to share profits according to the following rule

► Prices charged for good a are zero

► In exchange, the profits of firm B are shared via a split of α going to firm A and $(1 - \alpha)$ going to firm B

► Firms agree to share profits according to the following rule

► Prices charged for good a are zero

► In exchange, the profits of firm B are shared via a split of α going to firm A and $(1 - \alpha)$ going to firm B

► Firm A's decision is trivial. He simply produces $q_a = q_b$

► Firm B chooses to maximize:

$$\max_q (1 - \alpha)(100 - q)q - \alpha q = (1 - \alpha) \max_q (100 - q)q - \alpha q$$

Term inside the parentheses is just the monopoly profits if the two firms merged:

$$(1 - \alpha) \max_q \Pi^M(q)$$

Handwritten: π^M (FIRMAS FUSIONADAS)

Handwritten: $q^* = q^*$ (FUSIONADAS)

► The firms will produce at the monopoly quantities which we found were strictly greater than if the two firms produced completely separately without any such agreement

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◀ ▶ ⏪ ⏩ 🔍 🗑️

- ▶ The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement
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- ▶ For any $\alpha \in (0, 1)$, we get an increase in consumer surplus and total profits

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- ▶ Really, for any α ?

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- ▶ The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement
- ▶ The price will be the monopoly price
- ▶ For any $\alpha \in (0, 1)$, we get an increase in consumer surplus and total profits
- ▶ Really, for any α ? $\alpha \cdot \pi$ $1 - \alpha$ $\alpha \pi^A = \pi^A$
- ▶ Such arrangements can break down easily. Profits are hard to verify.

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- ▶ Profits are usually difficult to verify. However, revenues are much easier to check.

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- ▶ Firms enter into an arrangement where the revenues are shared according to α (firm A) and $(1 - \alpha)$ (firm B) split

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- ▶ Profits are usually difficult to verify. However, revenues are much easier to check.
- ▶ Firms enter into an arrangement where the revenues are shared according to α (firm A) and $(1 - \alpha)$ (firm B) split
- ▶ Suppose that $\alpha = 1/2$ and $c = 10$. Then firm 2 maximizes:

$$\max_q \frac{1}{2}q(100 - q) - 10q.$$

◀ ▶ ⏪ ⏩ 🔍 🗑️

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$$\max_q \frac{1}{2}q(100-q) - 10q$$
- The first order condition gives:

$$\frac{1}{2}MR(q) = MC = 10 \Rightarrow MR(q) = 2MC = 20$$

Handwritten note: Inf: CH

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- Firms enter into an arrangement where the revenues are shared according to α (firm A) and $1-\alpha$ (firm B) split
- Suppose that $\alpha = 1/2$ and $c = 10$. Then firm 2 maximizes:

$$\max_q \frac{1}{2}q(100-q) - 10q$$
- The first order condition gives:

$$\frac{1}{2}MR(q) = MC = 10 \Rightarrow MR(q) = 2MC = 20$$
- Firm will produce below monopoly profits since it will produce at a point where $MR = 2MC$ instead of $MR = MC$

Handwritten notes:
 $\alpha \rightarrow 1/2$
 $\alpha \cdot Y_b = \frac{c}{\alpha}$
 OPERAZIONE
 PER
 APPLICARE

Solving, we get:

$$100 - 2q = 20 \Rightarrow p^* = 60, q^* = 40 < q^m = 45$$

Handwritten note: Pb

Solving, we get:

$$100 - 2q = 20 \Rightarrow p^* = 60 > p^m = 55, q^* = 40 < q^m = 45$$

This does solve the double marginalization problem slightly:

$$p_0^* = 77.5 > p^* = 60, q_0^* = 22.5 < q^* = 40$$