Repaso Parcial 1B

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$$u(x, y) = x^{1/3} y^{2/3}.$$

The production functions of the firms display decreasing returns to scale:

$$x = f(k_x, l_x) = k_x^{1/3} l_x^{1/3}, \qquad y = g(k_y, l_y) = k_y^{1/3} l_y^{1/3}.$$

Let (p, q) be the respective prices of the two consumption goods, r the price of capital, and w the price of labor. Since we know that competitive equilibrium prices have one degree of freedom, we normalize by setting $q = 1$ (ie, we take the second consumption good as *numéraire*).

In order to derive the competitive equilibrium we will proceed as follows:

PASO 1

1. Obtain the demand functions of the consumer for the two consumption goods, as functions of the consumer's wealth (which will have to be precised later).
2. For each firm, obtain its demand for inputs and its output supply, as functions of all prices and the exogenous variables $\bar{k}$ and $\bar{l}$. Next, derive the firm's profits.
3. ~~Derive the consumer's wealth as function of the prices and the parameters.~~
4. Apply the supply equals demand condition to three of the markets (since we know by Walras' Law that the fourth one will automatically be in equilibrium). This will yield the equilibrium prices as functions of the parameters, and from them we obtain the competitive equilibrium allocation also as a function of the parameters.

$$f_x(K_x, l_x) = K_x^{1/3} l_x^{1/3}$$

1) FIRMAS

$$\max_{K_x, l_x} \Pi_x = \left(\boxed{K_x^{1/3} l_x^{1/3}}\right) P - r K_x - w l_x$$

$$\frac{\partial \Pi}{\partial K_x} = \frac{1}{3} K_x^{-2/3} l_x^{1/3} \cdot P - r = 0$$

$$\frac{\partial \Pi}{\partial l_x} = \frac{1}{3} l_x^{-2/3} K_x^{1/3} \cdot P - w = 0$$

$$\Rightarrow \frac{\cancel{\frac{1}{3}} K_x^{-2/3} l_x^{1/3} \cancel{P}}{\cancel{\frac{1}{3}} l_x^{-2/3} K_x^{1/3} \cancel{P}} = \frac{r}{w}$$

$$\frac{l_x}{K_x} = \frac{r}{w}$$

$$\boxed{l_x = K_x \frac{r}{w}}$$

$$\frac{1}{3} K_x^{-2/3} l_x^{1/3} \cdot P - r = 0$$

$$\frac{1}{3} K_x^{-2/3} K_x^{1/3} \left(\frac{r}{w}\right)^{1/3} P - r = 0$$

$$\frac{1}{3} K_x^{-1/3} \left(\frac{r}{w}\right)^{1/3} P = r$$

$$\frac{1}{3} \left(\frac{r}{w}\right)^{1/3} \frac{P}{r} = K_x^{1/3}$$

$$\frac{1}{27} \left(\frac{r}{w}\right) \frac{P^3}{r^3} = K_x$$

$$\boxed{\frac{P^3}{27 w r^2} = K_x^*}$$

$$l_x = K_x \frac{r}{w} = \frac{P^3}{27 w r^2} \cdot \frac{r}{w}$$

$$\boxed{l_x = \frac{P^3}{27 w^2 r}}$$

$$\Pi_y = \left(K_y^{1/3} l_y^{1/3}\right) q - r K_y - w l_y$$

$$\boxed{K_y = \frac{q^3}{27 w r^2}} \quad \boxed{l_y = \frac{q^3}{27 w^2 r}} \quad \text{PERO } q = 1$$

$$\boxed{K_y = \frac{1}{27 w r^2}} \quad \boxed{l_y = \frac{1}{27 w^2 r}}$$

$$\Pi_x = \left(K_x^{1/3} l_x^{1/3}\right) P - w l_x - r K_x$$

$$\Pi_x = P \left(\frac{P^3}{27 w r^2}\right)^{1/3} \left(\frac{P^3}{27 w^2 r}\right)^{1/3} - w \frac{P^3}{27 w^2 r} - r \frac{P^3}{27 w r^2}$$

$$\Pi_x = \frac{P^3}{9 w r} - \frac{P^3}{27 w r} - \frac{P^3}{27 w r}$$

$$\Pi_x = \frac{3P^3 - P^3 - P^3}{27 w r}$$

$$\boxed{\Pi_x = \frac{P^3}{27 w r}} \quad \boxed{\Pi_y = \frac{q^3}{27 w r} = \frac{1}{27 w r}}$$

$$X^o = K_x^{1/3} l_x^{1/3} = \left(\frac{P^3}{27 w r^2}\right)^{1/3} \left(\frac{P^3}{27 w^2 r}\right)^{1/3}$$

$$\boxed{X^o = \frac{P^2}{9 w r}} \quad \boxed{Y^o = \frac{q^2}{9 w r} = \frac{1}{9 w r}}$$

2) MAX CONSUMIDOR

$$\text{MAX} \quad X^{1/3} Y^{2/3} \quad \text{s.a.} \quad \underbrace{P_x X + P_y Y}_{\text{GASTO}} \leq \underbrace{\bar{l} w + \bar{k} r + \Pi_x + \Pi_y}_{\text{INGRESO}}$$

$$\mathcal{L} = X^{1/3} Y^{2/3} + \lambda\left(\bar{l} w + \bar{k} r + \Pi_x + \Pi_y - P_x X - P_y Y\right)$$

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{3} X^{-2/3} Y^{2/3} - \lambda P_x = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{2}{3} X^{1/3} Y^{-1/3} - \lambda P_y = 0$$

$$\Rightarrow \frac{\frac{1}{3} X^{-2/3} Y^{2/3}}{\frac{2}{3} X^{1/3} Y^{-1/3}} = \frac{P_x}{P_y}$$

$$= \frac{Y}{2X} = \frac{P_x}{P_y}$$

$$\Rightarrow \boxed{Y = 2X \frac{P_x}{P_y}}$$

$$\bar{l} w + \bar{k} r + \Pi_x + \Pi_y = P_x X + \cancel{P_y}\left(2X \frac{P_x}{\cancel{P_y}}\right)$$

$$\bar{l} w + \bar{k} r + \Pi_x + \Pi_y = 3 P_x X$$

$$X^D = \frac{\bar{l} w + \bar{k} r + \Pi_x + \Pi_y}{}$$

$$P_x = P$$

$$X^D = \frac{\bar{l}w + \bar{k}r + \Pi x + \Pi y}{3P_x} \qquad P_x = P$$

$$Y^D = \frac{2}{3}\,\frac{\bar{l}w + \bar{k}r + \Pi x + \Pi y}{P_y} \qquad P_y = q$$

3) MCDOS VACIEN!

$$l_x + l_y = \bar{l} \;\Rightarrow\; \frac{p^3}{27w^2r} + \frac{1}{27w^2r} = \bar{l}$$

$$K_x + K_y = \bar{K} \;\Rightarrow\; \frac{p^3}{27wr^2} + \frac{1}{27wr^2} = \bar{K}$$

$$\boxed{X^D = X^O \qquad Y^D = Y^O}$$

$$\Rightarrow \frac{p^3+1}{27w^2r} = \bar{l} \Rightarrow$$

$$\frac{p^3+1}{27wr^2} = \bar{K}$$

$$\frac{l_x}{K_x} = \frac{w}{r} = \frac{l_y}{K_y} \Rightarrow \frac{l_x}{K_x} = \frac{\bar{l} - l_x}{\bar{K} - K_x} = l_x\bar{K} - \cancel{l_x K_x} = \bar{l}K_x - \cancel{K_x l_x}$$

por vaciado mcdo

$$= l_x\bar{K} = \bar{l}K_x$$

$$\frac{l_x}{K_x} = \frac{\bar{l}}{\bar{K}}$$

$$\frac{\dfrac{p^3}{27w^2r}}{\dfrac{p^3}{27wr^2}} = \frac{\bar{l}}{\bar{K}} \Rightarrow \boxed{\frac{r}{w} = \frac{\bar{l}}{\bar{K}}}$$

$$\frac{\bar{l}w + \bar{k}r + \overbrace{\frac{p^3}{27wr}}^{\Pi x} + \overbrace{\frac{1}{27wr}}^{\Pi y}}{3P} = \frac{p^2}{9wr}$$

$$\bar{l}w + \bar{k}r + \frac{p^3+1}{27wr} = \frac{p^3}{9wr}$$

$$\bar{l}w + \bar{k}r + \frac{1}{27wr} = \frac{8p^3}{27wr}$$

$$\frac{27wr(\bar{l}w + \bar{k}r) + 1}{8} = p^3$$

$$\frac{p^3+1}{27wr^2} = \bar{K} \rightarrow p^3 = 27\bar{K}wr^2 - 1$$

$$\frac{27wr(\bar{l}w + \bar{K}r) + 1}{8} = 27\bar{K}wr^2 - 1$$

$$\frac{r}{w} = \frac{\bar{l}}{\bar{K}} \Rightarrow r = w\frac{\bar{l}}{\bar{K}}$$

$$\frac{27w\left(w\frac{\bar{l}}{\bar{K}}\right)\left(\bar{l}w + \bar{K}\left(w\frac{\bar{l}}{\bar{K}}\right)\right) + 1}{8} = 27\bar{K}w\left(w\frac{\bar{l}}{\bar{K}}\right)^2 - 1$$

$$\frac{27w^2\frac{\ell}{K}(2\ell w)+1}{8} = 27Kw^3\frac{\ell^2}{K^2} - 1$$

$$54\frac{w^3\ell^2}{K} + 1 = 27(8)\frac{w^3\ell^2}{K} - 8$$

$$9 = 162\frac{w^3\ell^2}{K}.$$

$$\frac{9\bar{K}}{162\bar{\ell}^2} = w^3$$

$$\boxed{w = \left(\frac{9\bar{K}}{162\bar{\ell}^2}\right)^{1/3}} \qquad \boxed{r = w\frac{\bar{\ell}}{K} = \left(\frac{9\bar{K}}{162\ell^2}\right)^{1/3}\frac{\bar{\ell}}{K}.}$$

$$P = 27Kwr^2 - 1$$

$$\boxed{P = 27\bar{K}\left(\frac{9K}{162\ell^2}\right)^{1/3}\left(\frac{9K}{162\ell^2}\right)^{2/3}\left(\frac{\ell}{K}\right)^2 - 1}$$

Reemplazando en $K_x^*, \ell_x^*, K_y^*, \ell_y^*, X^D, Y^D$

Tenemos la asig. de eq.

1. (30 puntos) Considere un monopolio con función de costos totales $CT(q) = q^2$. Enfrenta una demanda inversa $p(q) = 100 - q$.

$\rightarrow CMg = 2q$

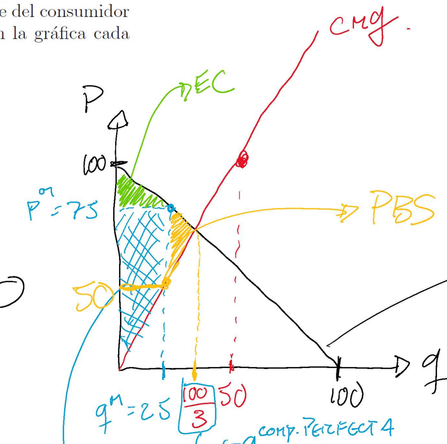
(a) (10 puntos) Encuentre la cantidad y el precio al que vendería el monopolista, así como las ganancias del monopolista (excedente del productor), el excedente del consumidor y el costo en bienestar social. Grafique este mercado mostrando en la gráfica cada una de las variables que se piden.

$$\text{MAX } \Pi = \underbrace{(100-q)}_{P}q - q^2$$

$$\frac{\partial \Pi}{\partial q} = 100 - 2q - 2q = 0$$

$$100 - 4q = 0$$

$$\boxed{q^m = 25}$$



$CMg = DD$

$2q = 100 - q$

$3q = 100 \Longrightarrow q = \frac{100}{3}$

$\rightarrow 100 - q = P$

$q = 100 - P$

$$\rightarrow EP = \pi^* = (100-25)25 - 25^2$$

$$= 25(25) - 25^2$$

$$\boxed{q^m = 25}$$
$$\boxed{p^m = 75}$$

$$EP = \pi^* = (100-25)25 - 25^2$$
$$= 75(25) - 25^2$$
$$\boxed{\pi^* = 1250}$$

$$\boxed{EC = \frac{25(25)}{2} = \frac{625}{2}}$$

$$\boxed{PBS = \frac{25\left(\frac{100}{3} - 25\right)}{2} = 104.125}$$

(b) (5 puntos) Suponga que el gobierno pone un impuesto/subsidio de t pesos por unidad vendida (si es positivo es impuesto, si es negativo es un subsidio) de forma que ahora el precio que recibe el monopolista si vende q unidades es $p(q,t) = 100 - q - t$ (el precio que paga el consumidor sigue siendo $p(q) = 100 - q$). Encuentre la cantidad que produciría el monopolista en función de t.

$$\pi^t = \underbrace{(100 - q - t)}_{p} q - q^2$$

$$\frac{\partial \pi}{\partial q} = 100 - 2q - t - 2q = 0$$

$$100 - t = 4q$$

$$\boxed{\frac{100-t}{4} = q^m}$$

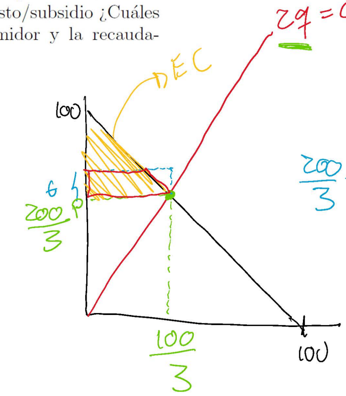
(c) (10 puntos) Encuentre el nivel de t que haría que el monopolista produzca la cantidad de competencia perfecta. Si el gobierno establece este impuesto/subsidio ¿Cuáles serían las ganancias del monopolista, el excedente del consumidor y la recaudación/costo del impuesto/subsidio?

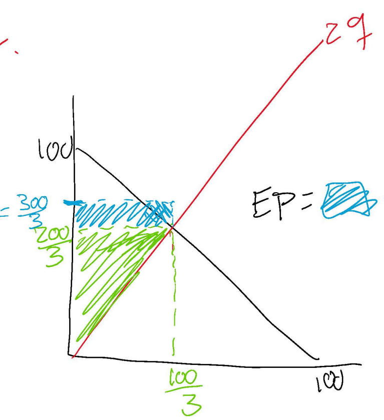
$$q^m = \frac{100-t}{4} = \frac{100}{3}$$

$$400 = 300 - 3t$$

$$3t = -100$$

$$\boxed{t = \frac{-100}{3}}$$



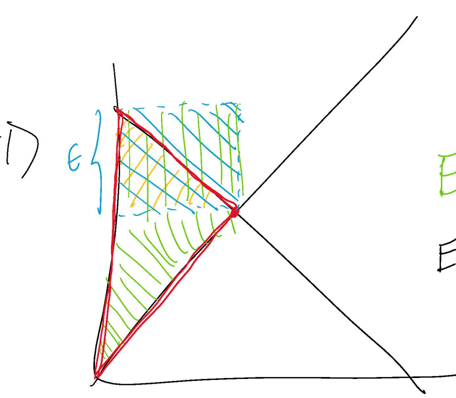


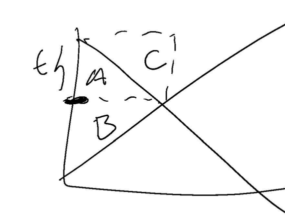
Costo Subsidio =

$$\boxed{EC = \frac{\left(100 - \frac{200}{3}\right)\left(\frac{100}{3}\right)}{2} = 555{,}44}$$

$$EP = \frac{\frac{100}{3}\left(\frac{200}{3}\right)}{2} + \frac{100}{3}\left(\frac{100}{3}\right)$$

$\Rightarrow$





EP = A + B + C

EC = A

CS: = A + C

ES = + − = A + B + C + A −

ES = = EQ. Competitivo.

$(A+C) = A+B$

$EC = \frac{(100 - \frac{200}{3})(\frac{[illegible]}{3})}{2} = 555,44$

$= \frac{3(\frac{[illegible]}{3})}{2} + \frac{[illegible]}{3}(3)$

$= 1111.11 + 1111.11$

Costo Sub: -1111.11

(d) (5 puntos) ¿Desde el punto de vista social conviene que el gobierno ponga este impuesto? Justifique su respuesta.