Lecture 1: General Equilibrium

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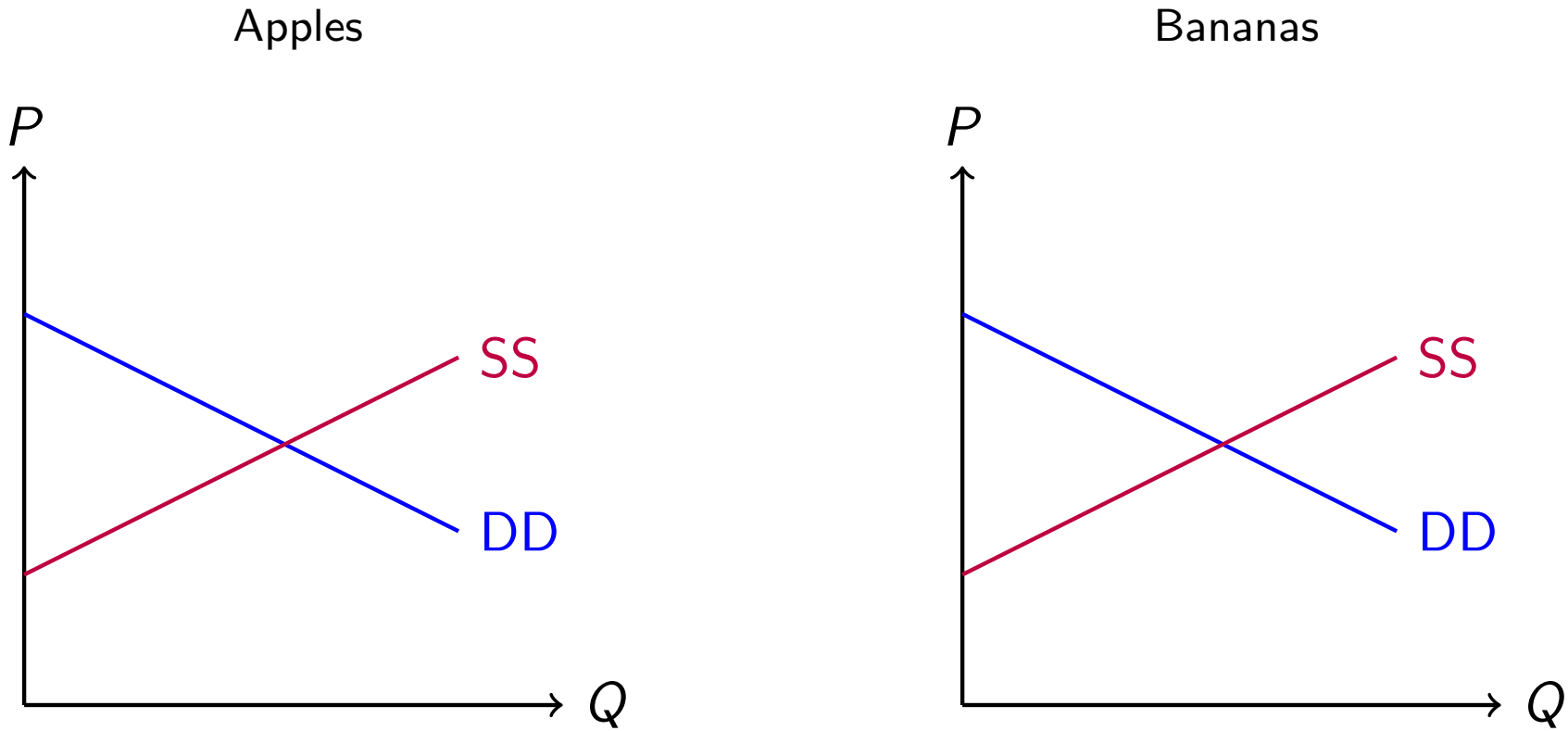
- When analyzed together, each market was viewed in isolation
- But markets are often intertwined
  - Transportation: Uber/metro/ecobici/car
  - Wages across sectors
  - Fruits
  - Beer and tacos

Example - Fruits

- Suppose that apple and bananas are substitutes



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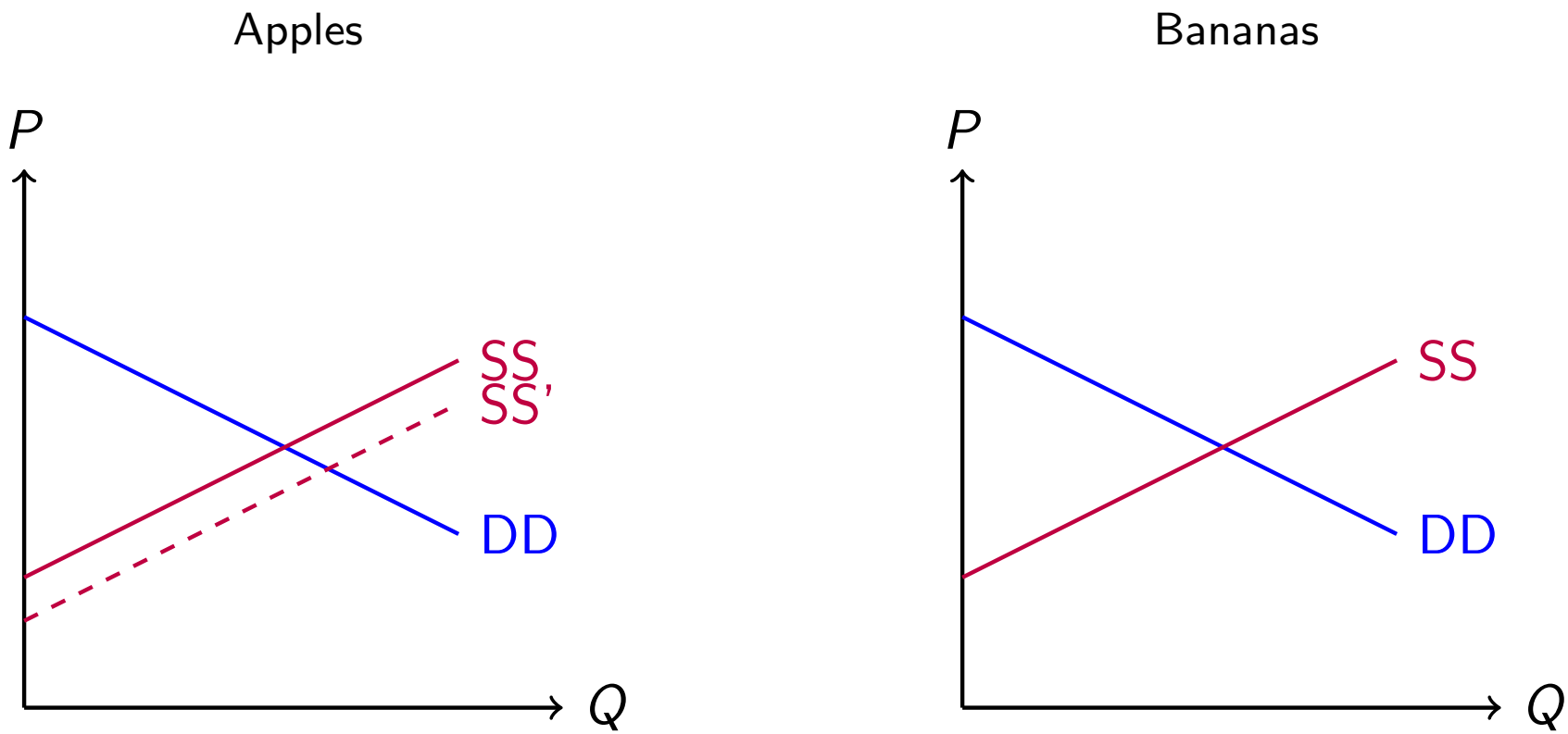

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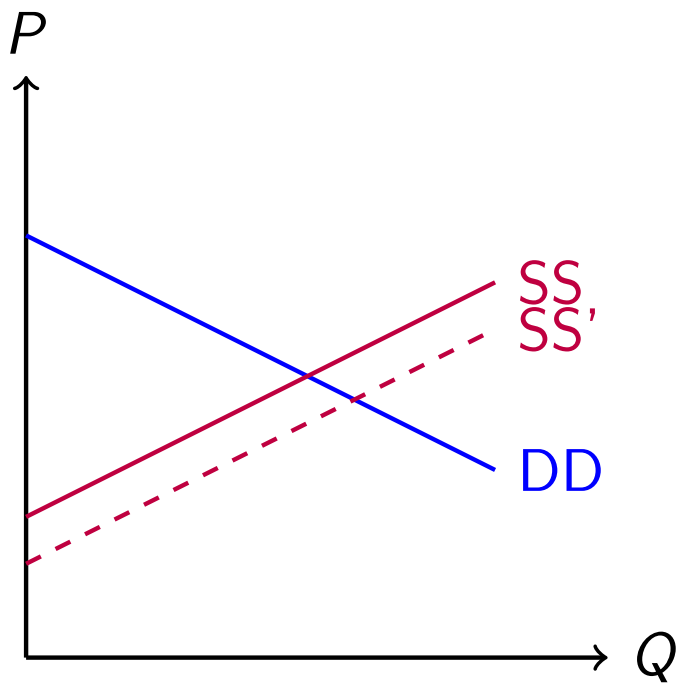
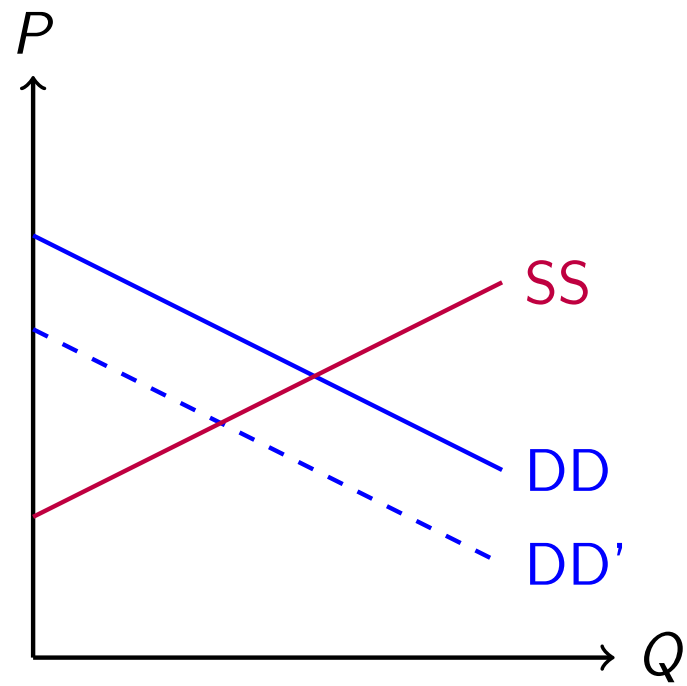
Apples



Bananas



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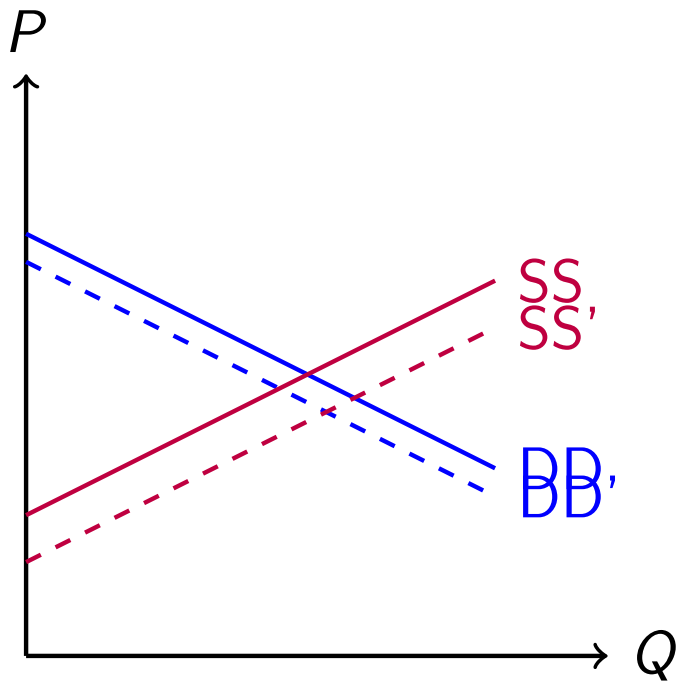

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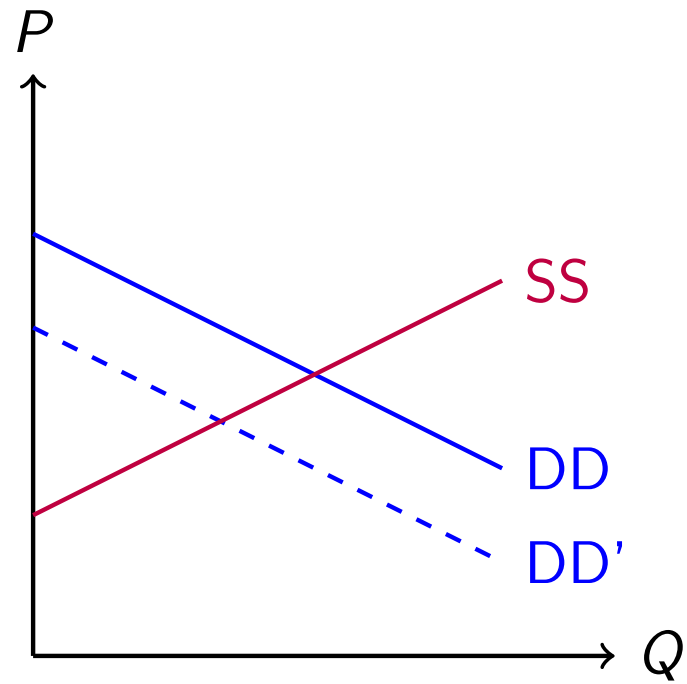

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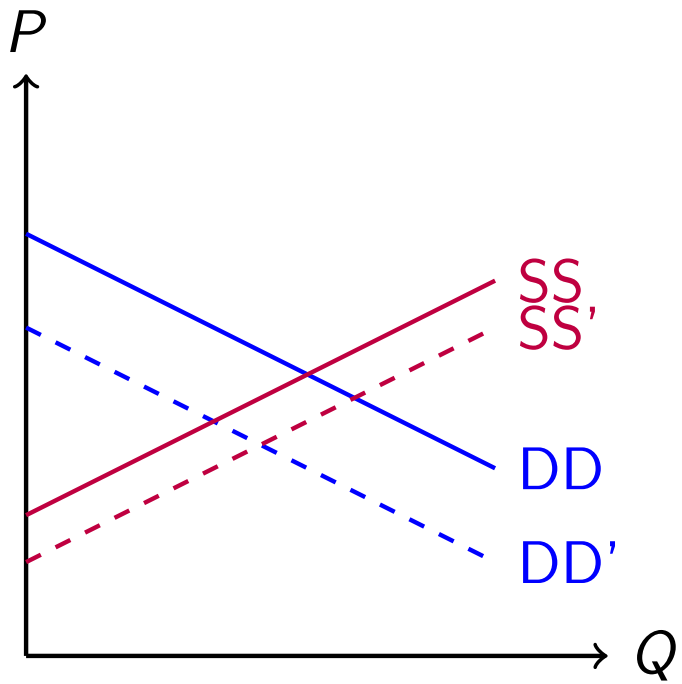

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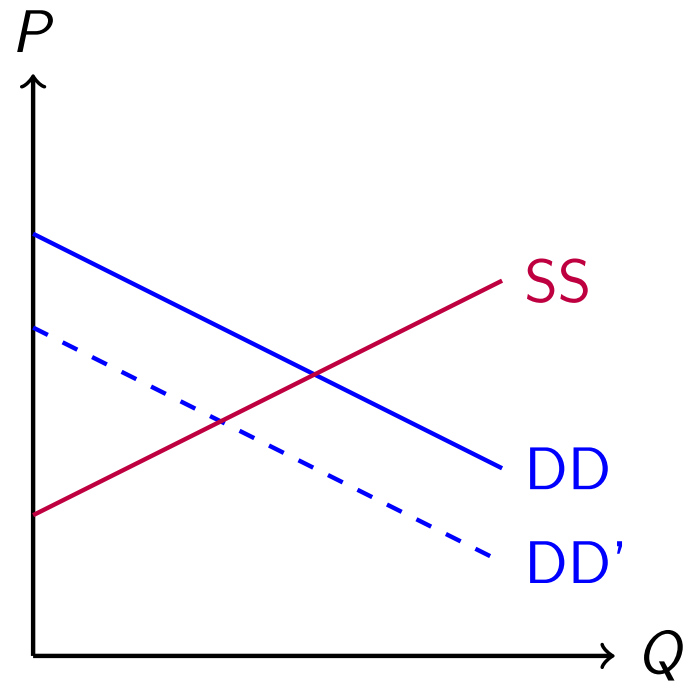

Example - Fruits

- What happens if apple and bananas are complements?

A tour down memory lane

- Léon Walras started it all (1834-1910)
  - First to use mathematical tools in economics
  - Supply and demand curves as solutions to a maximization problem
  - Started the "marginal revolution"
- Walras was ultimately after normative questions (is the market economy good?)
- But first, he tackled positive questions (is there an equilibrium? is it unique?)
- Made a lot of progress. In particular came up with "Walras Law": Sum of the values of excess demands across all markets must equal zero always

A tour down memory lane

- Vilfredo Pareto was Walras student (1848-1923)
  - Abandoned utilitarianism (i.e., utility functions)
  - Embraced "preferences"
    - Utility functions only have ordinal content
    - Comparing "utils" across individuals is meaningless
  - (Pareto) optimum/efficiency: Achieved if we can't make someone better-off without making someone worst-off

A tour down memory lane

- Francis Edgeworth (1845 – 1926)
  - Introduced indifference curves
  - Was the first to ask: Where will voluntary exchange lead to?
  - He conjecture his result was aligned with Walras' result

A tour down memory lane

- No more advances for a while (until 1950's) then
  - Kenneth Arrow
  - Gerard Debreu
  - Lionel McKenzie
- Existence
- Showed it was Pareto efficient
- Two Nobel prizes (Arrow — 1972 and Debreu — 1974)

Lecture 1: General Equilibrium

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Pure Exchange Economies

- How are goods distributed among consumers?
- What incentives are there to exchange goods? What institutions mediate the exchange?
- Is there a distribution of goods that leaves everyone satisfied and there aren't any incentives to deviate?

Pure Exchange Economies

- What are the properties of such an equilibrium?
  - Is it unique?
  - Is it stable?
  - Is it efficient?

Pure Exchange Economies

- Assume there are
  - I consumers, $\mathcal{I} = \{1, ..., I\}$
  - L goods, $\mathcal{L} = \{1, ..., L\}$
  - Each consumer i is characterized by a utility function u^i.
  - Each consumer can consume goods in $x_i \in \mathbb{R}^L_+$
  - Each consumer has an initial endowment of $w^i \in \mathbb{R}^L_+$.
  - Each consumer is characterized by the pair: $\left(u^i, w^i\right)$.
  - Assume the utility functions represent neoclassic preferences

Utility functions and neoclassic preferences

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  - If $x \succ_i y$ then $u^i(x) > u^i(y)$
  - If f is any increasing function then $f(u^i(x)) > f(u^i(y))$
  - Hence $f(u^i(\cdot))$ also represents $\succ_i$
  - $u^i(x) > u^i(y)$ means something, but $u^i(x) - u^i(y)$ does not
- Neoclassic preferences are well behaved

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  - $u^i(x) > u^i(y)$ means something, but $u^i(x) - u^i(y)$ does not
- Neoclassic preferences are well behaved
  - They can be represented by a utility function
  - They are weakly monotonic
  - They are quasi-concave

Pure Exchange Economies

Definition (Exchange economy)

A pure exchange economy is $\mathcal{E} = \left\langle \mathcal{I}, \left(u^i, w^i\right)_{i \in \mathcal{I}} \right\rangle$ where $\mathcal{I}$ is the set of agents, u^i is a representation of consumer i's preferences and w^i is consumer i's initial endowment.

- Let $w = \sum_{i=1}^{I} w^i$ be the total endowment of the economy.
- An *allocation* of resources is denoted by $x = (x^1, x^2, ..., x^I)$ where $x^i \in \mathbb{R}^L_+$.

Definition (Feasible allocation)

The set of *feasible* allocation F of an economy $\mathcal{E} = \left\langle \mathcal{I}, \left(u^i, w^i\right)_{i \in \mathcal{I}} \right\rangle$ is defined by:

$$F = \left\{ x = (x^1, x^2, ..., x^I) : x^i \in \mathbb{R}^L_+, \sum_{i=1}^{I} x^i = \sum_{i=1}^{I} w^i \right\}$$

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Pareto efficiency

Definition (Pareto efficiency)

Let $\mathcal{E}$ be an economy. A feasible allocation of resources $x = (x^1, x^2, ..., x^I)$ is Pareto efficient if there isn't another feasible allocation $\widehat{x} = (\widehat{x}^1, \widehat{x}^2, ..., \widehat{x}^I)$ such that for every agent i, $u^i(\widehat{x}^i) \geq u^i(x^i)$ and for at least one agent i^*, $u^{i^*}(\widehat{x}^{i^*}) > u^{i^*}(x^{i^*})$.

Pareto efficiency

Definition (Pareto domination)

Take two feasible allocations x and $\hat{x}$. We say that $\hat{x}$ Pareto dominates x if for all $i = 1, \ldots, I$,

$$u_i(\hat{x}_1^i, \ldots, \hat{x}_L^i) \geq u_i(x_1^i, \ldots, x_L^i)$$

and there is at least one consumer j for which

$$u_j(\hat{x}_1^j, \ldots, \hat{x}_L^j) > u_j(x_1^j, \ldots, x_L^j).$$

Thinking about Pareto efficiency

- If x is a Pareto efficient feasible allocation, does it mean that x Pareto dominates all other feasible allocations?
- If there are two allocations (x and y) is it always the case that one Pareto dominates the other?
- For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy
- Social planner should strive to achieve Pareto efficiency at the very least!

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- Social planner should strive to achieve Pareto efficiency at the very least! However, she may have other concerns such as fairness

Thinking about Pareto efficiency

- If utility is strictly increasing, then can a Pareto efficient allocation be such that $\sum_{i=1}^{I} x_j^i < \sum_{i=1}^{I} w_j^i$?

- The set of all Pareto allocations is known as the **contract curve**

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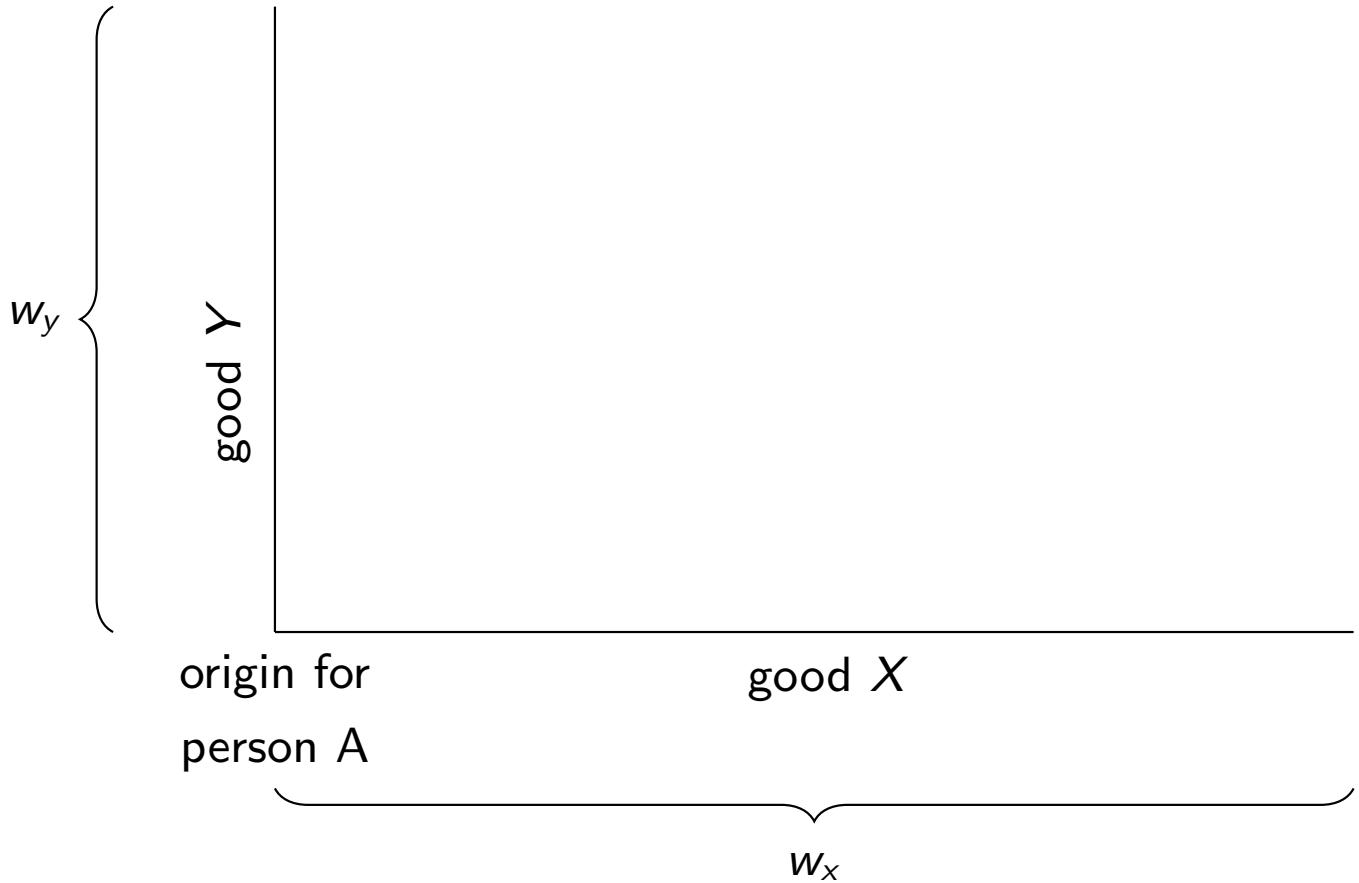
Edgeworth Box



The Edgeworth box

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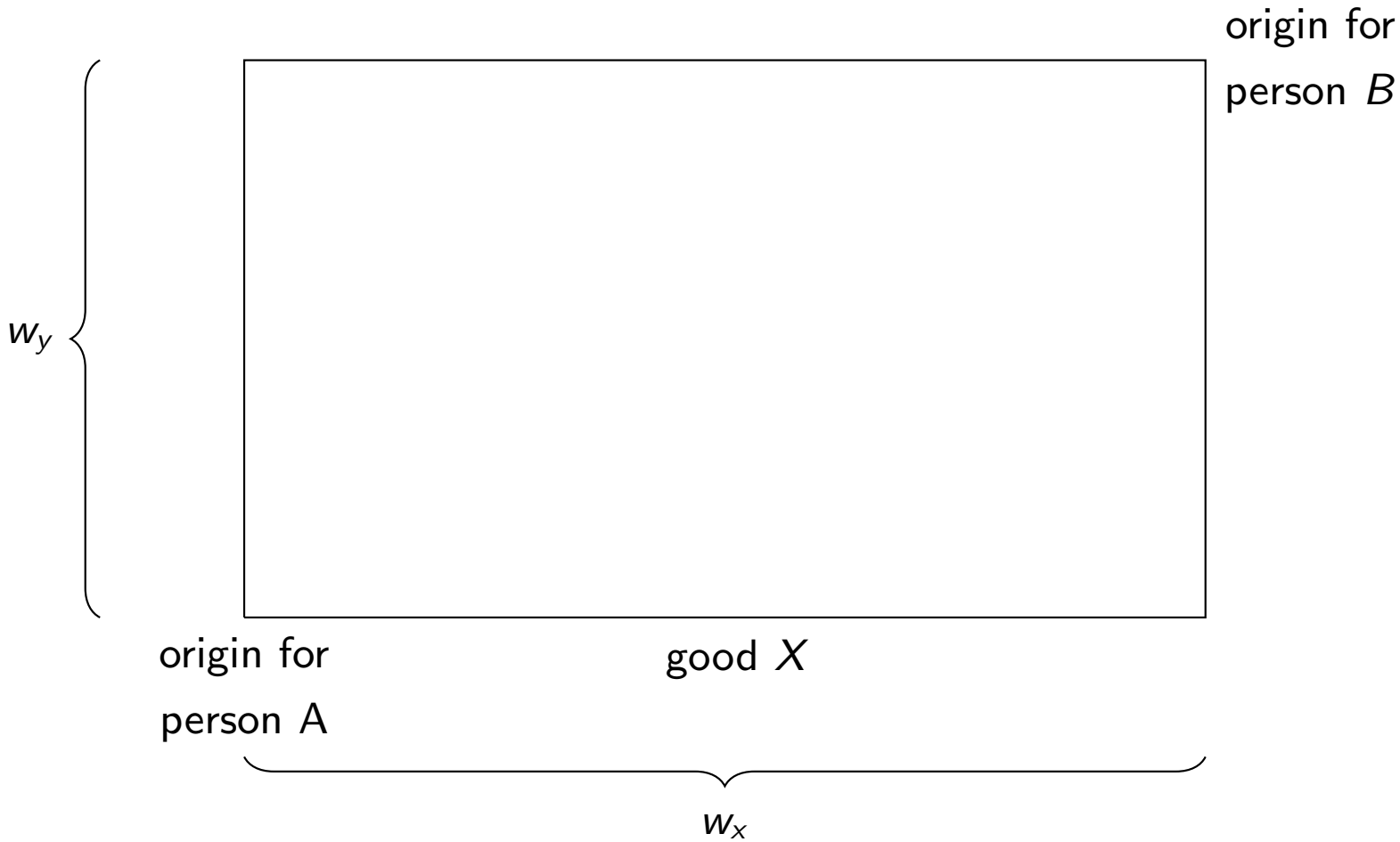

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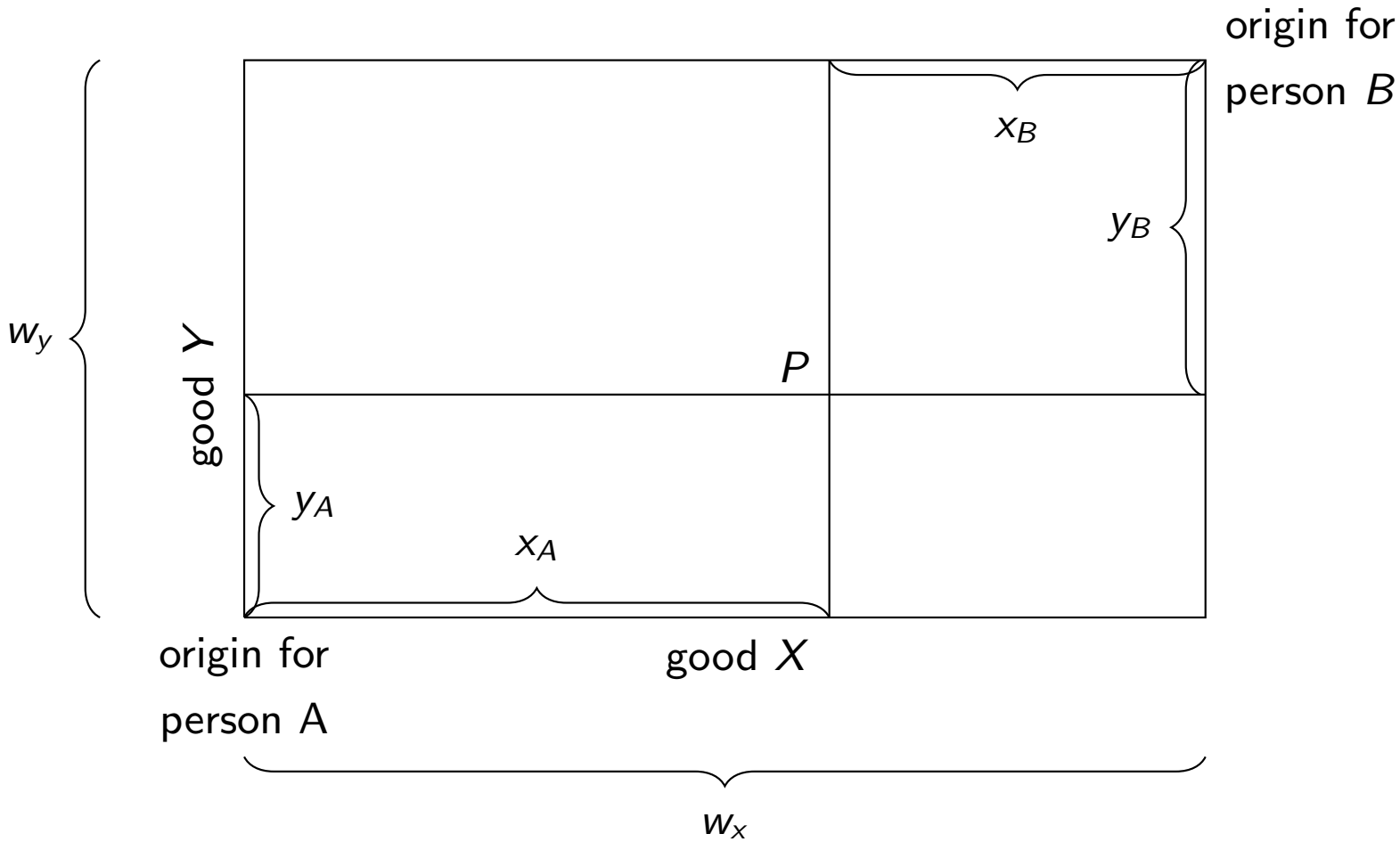

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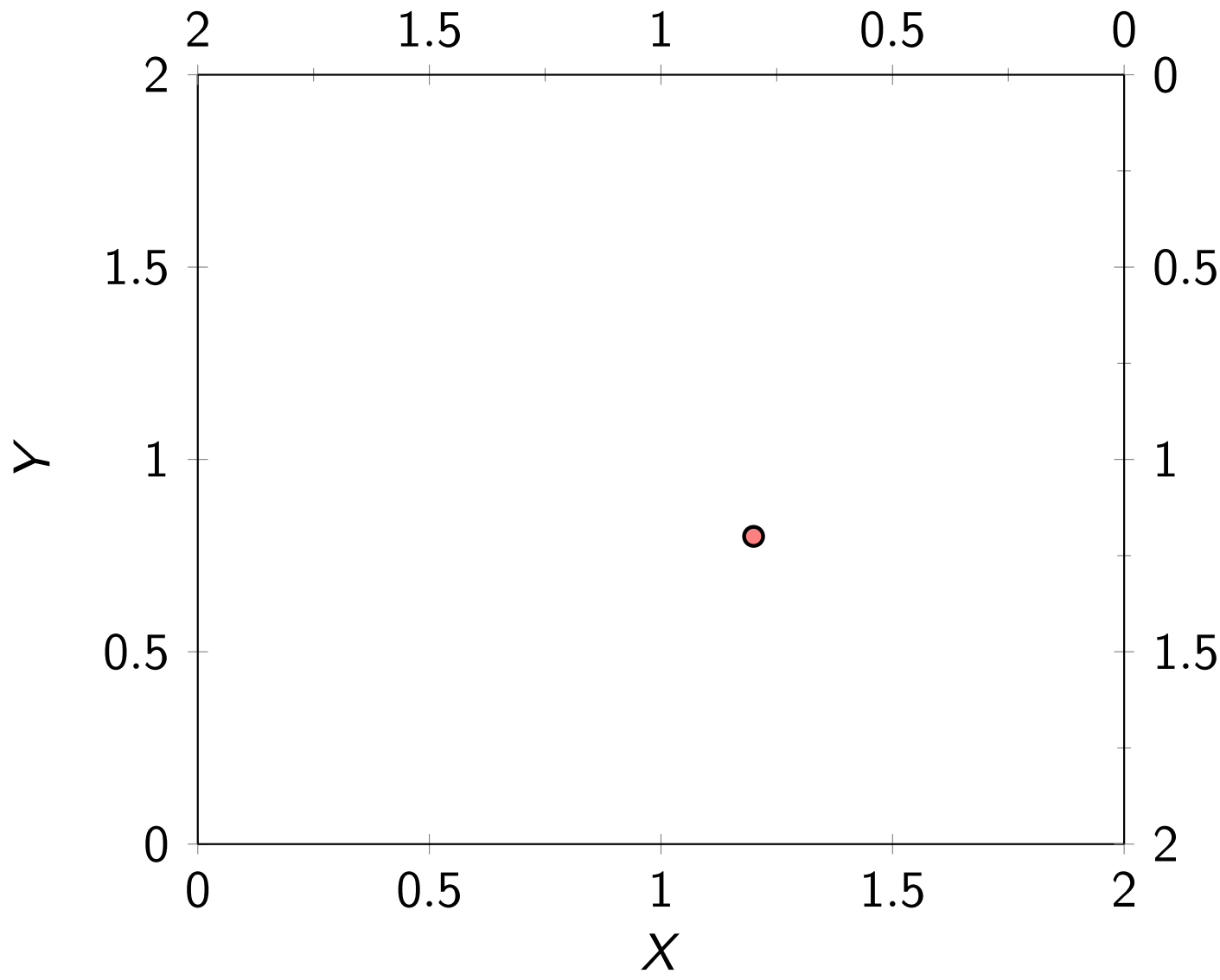

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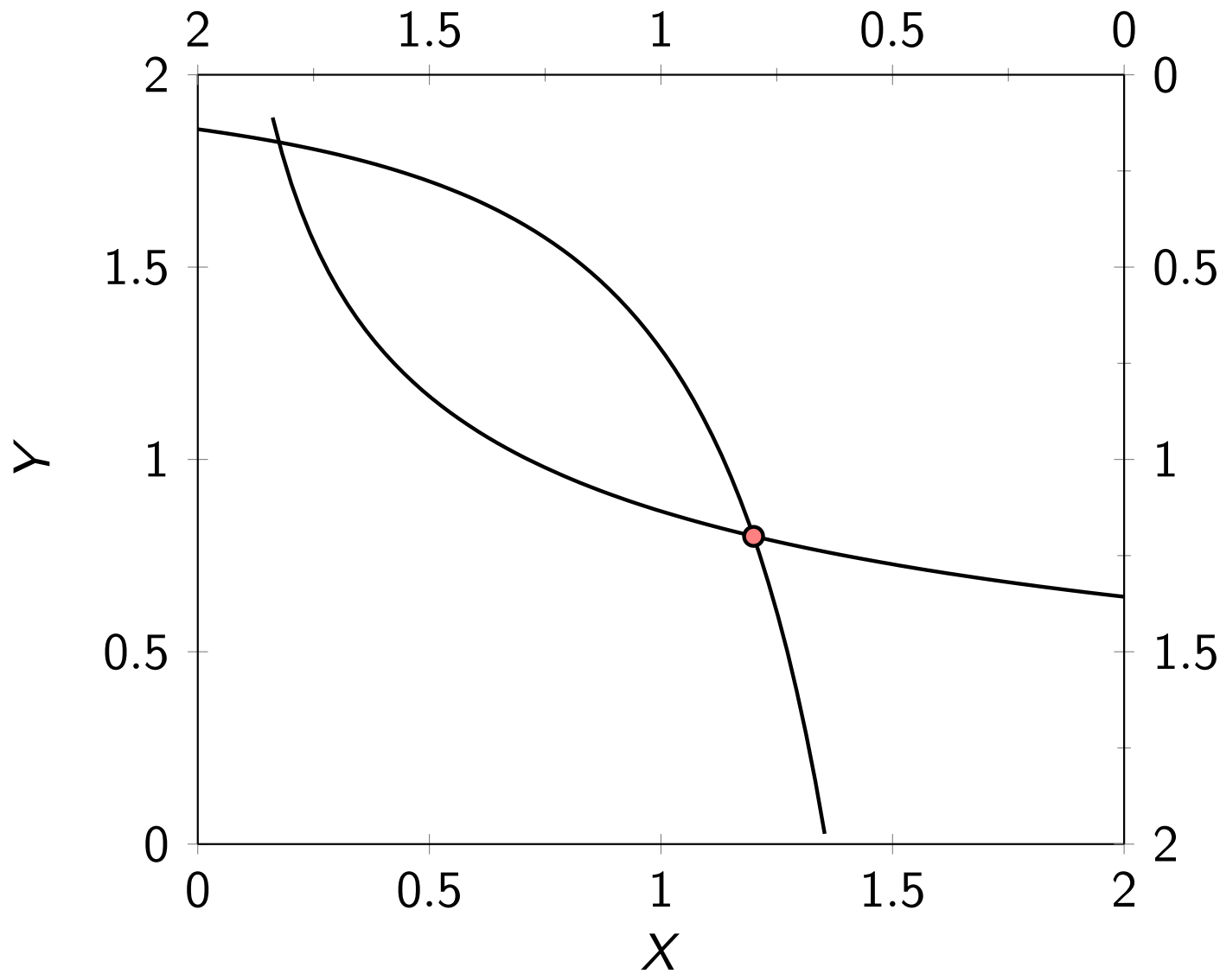

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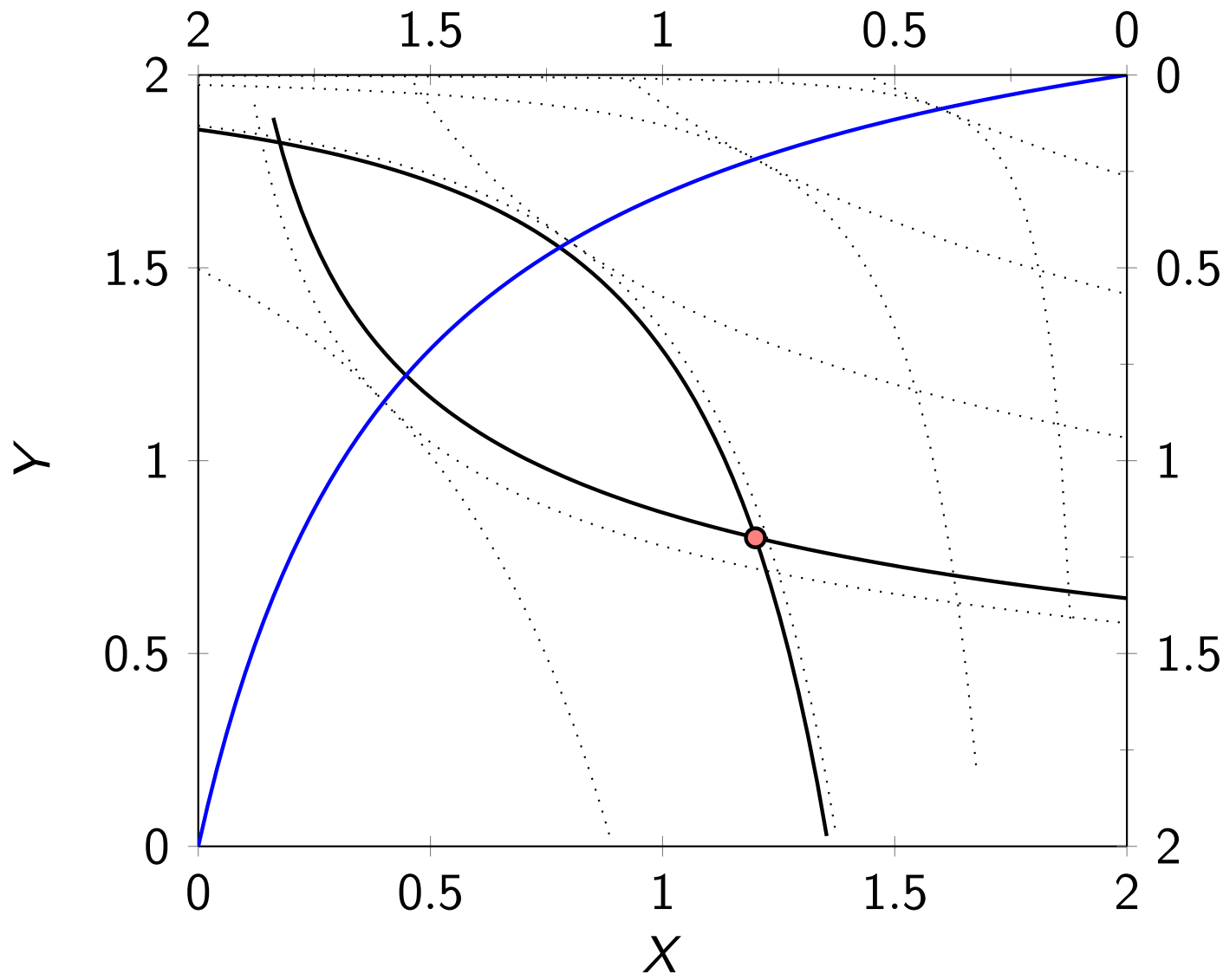

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