Lecture11.pdf

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Lecture11....

Lecture 11: Game Theory // Preliminaries and dominance

Mauricio Romero

- We will represent games in two different ways

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- This is just a way to schematizing the game and in general it makes the analysis simpler

Normal form

The normal form consists of:

- The list of players
- The strategy space
- The pay-off functions

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There is no mention of rules or available information. Where is this hidden?

When there a few players (2 or 3) a matrix is used to represent the game in the normal form.

	s_2	s_2'
s_1	$(u_1(s_1, s_2), u_2(s_1, s_2))$	$(u_1(s_1, s_2'), u_2(s_1, s_2'))$
s_1'	$(u_1(s_1', s_2), u_2(s_1', s_2))$	$(u_1(s_1', s_2'), u_2(s_1', s_2'))$
s_1''	$(u_1(s_1'', s_2), u_2(s_1'', s_2))$	$(u_1(s_1'', s_2'), u_2(s_1'', s_2'))$

Matching-Pennies (Pares y Nones) – Simultaneous

Both players play at the same time

	1_B	2_B
1_A	(1000,-1000)	(-1000,1000)
2_A	(-1000,1000)	(1000,-1000)

Matching-Pennies (Pares y Nones) – Sequential

A plays first, then B

	$(1,1)$	$(1,2)$	$(2,1)$	$(2,2)$
1_A	(1000,-1000)	(1000,-1000)	(-1000,1000)	(-1000,1000)
2_A	(-1000,1000)	(1000,-1000)	(-1000,1000)	(1000,-1000)

Prisoner's Dilemma

There are two players $I = \{1, 2\}$ that are members of a drug cartel who are both arrested an imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack enough evidence to convict the pair on the principal charge so they must settle for a lesser charge. Simultaneously, the prosecutor offers each prisoner a deal. Each prisoner is given the opportunity to either 1) betray the other by testifying the other committed the crime or to 2) cooperate with the other prisoner and stay silent.

Prisoner's Dilemma

The strategies of player 1:

$$S_1 = \{\text{betray}_1, \text{silent}_1\}.$$

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Prisoner's Dilemma

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The strategies of player 2:

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The utility function of the players is given by:

$$u_1(b_1, b_2) = -2, u_2(b_1, b_2) = -2$$
$$u_1(b_1, s_2) = 0, u_2(b_1, s_2) = -3$$
$$u_1(s_1, b_2) = -3, u_2(s_1, b_2) = 0$$
$$u_1(s_1, s_2) = -1, u_2(s_1, s_2) = -1.$$

Prisoner's Dilemma

Prisoner's Dilemma

	s_2	b_2
s_1	$-1, -1$	$-3, 0$
b_1	$0, -3$	$-2, -2$

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  - The pay-off functions

} "REGLAS DEL JUEGO"

- The extensive form is usually accompanied by a visual representation call the "game tree"

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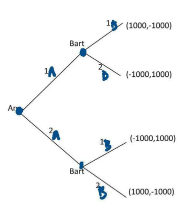
- Each node where a branch begins is a decision node, where a player needs to choose an action

- If two nodes are connected by a dotted line, it means they are in the same information set (i.e., the player is not sure in which node she is in)

Matching-Pennies (Pares y Nones) – Sequential



Pares o Nones I

Matching-Pennies (Pares y Nones) – Simultaneous

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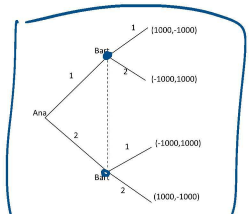
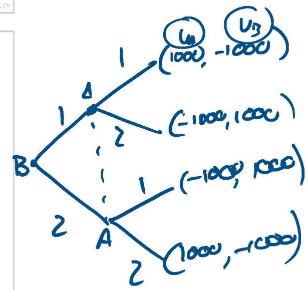


Pares o Nones II



Lecture 10: Game Theory // Preliminaries and dominance

Introduction - Continued
- Normal or extensive form
- Extensive form
- Some important remarks
- Some examples
- What's next

Static games with complete information
- Dominance of Strategies

Theorem

Every game can be represented in both forms (extensive and normal). The representation you choose will not alter the analysis, but it may be simpler to do the analysis with one form or another. A normal form game may have several extensive representations (but every extensive form has a single normal form equivalent to it); however, all of the results we will see/use are robust to the representation used.

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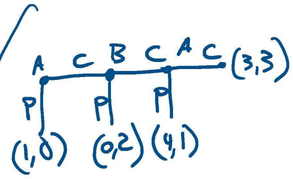
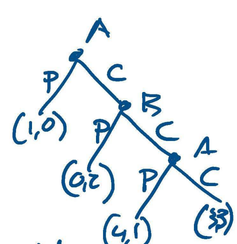
Centipede Game

Suppose there are two individuals Ana and Bernardo. Ana is given a chocolate. She can stop the game and keep the chocolate or she can continue. If she continues, Ana's chocolate is taken away and Bernardo is given two. Bernardo can then stop the game and keep two chocolates (and Ana will get zero) or can continue. If he continues, a chocolate is taken away from him and Ana is given four. Ana can stop the game and keep 4 chocolates (and Bernardo will keep one), or she can continue, in which case the game ends with three chocolates for each one.





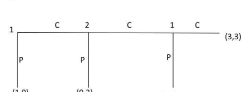
$S_A = \{(P,P); (P,C); (C,P); (C,C)\}$ (1er nodo, 3er nodo)

$S_B = \{P, C\}$

	P	C
(PP	1,0	1,0
	1,0	1,0

Centipede Game

The extensive form is



(1,0) (0,2) (4,1)

PP	1,0	1,0
PC	1,0	1,0
CP	0,2	4,1
CC	0,2	3,3

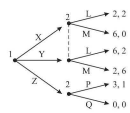
A

Centipede Game

The normal form is

	C	P
C,C	3,3	0,2
C,P	4,1	0,2
P,C	1,0	1,0
P,P	1,0	1,0

Consider the following game in extensive form:



$S_1 = \{X, Y, Z\}$

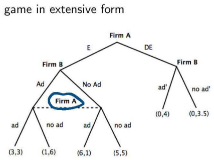
$S_2 = \{(L,P); (LQ); (MP); (MQ)\}$

S_1	LP	LQ	MP	MQ
X	2,2	2,2	6,0	6,0
Y	6,2	6,2	2,6	2,6
Z	3,1	0,0	3,1	0,0

The normal form is:

1 \ 2	LP	LQ	MP	MQ
X	2, 2	2, 2	6, 0	6, 0
Y	6, 2	6, 2	2, 6	2, 6
Z	3, 1	0, 0	3, 1	0, 0

Consider the following game in extensive form



Normal →

$S_A = \{EA, EN, DA, DN\}$

$S_B = \{AA', NA', AN', NN'\}$

Firma A	AA'	NA'	AN'	NN'
EA	3,3	6,1	3,3	6,1
EN	1,6	5,5	1,6	5,5
DA	0,4	0,4	0,3.5	0,3.5
DN	0,4	0,4	0,3.5	0,3.5

The normal form is:

	Ad, ad'	$Ad, no\ ad'$	$No\ Ad, ad'$	$No\ Ad, no\ ad'$
(E, ad)	3,3	3,3	6,1	6,1
$(E, no\ ad)$	1,6	1,6	5,5	5,5
(DE, ad)	0, 4	0,3.5	0,4	0,3.5
$(DE, no\ ad)$	0, 4	0,3.5	0,4	0,3.5

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- This is a concept equivalent to general equilibrium, where given market prices, everyone is optimizing, markets empty, and therefore no one has incentives to deviate, but nobody told us how we got there .. . (the Walrasian auctioneer?)

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Introduction - Continued

Static games with complete information

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Static games with complete information

- Games where all players move simultaneously and only once
- If players move sequentially, but can not observe what other people played, it's equivalent to a static game
- Only consider games of complete information (all players know the objective functions of their opponents)
- These are very restrictive conditions but they will allow us to present very important concepts that will be easy to extend to more complex games
- As each player faces one contingency, the strategies are identical to the actions.

Lecture 10: Game Theory // Preliminaries and dominance

Dominance

- Intuitively if a strategy s_i always results in a greater utility than s_i', regardless of the strategy followed by the other players then the strategy s_i' should never be chosen by individual i

Dominance

s_i **strictly dominates** s_i' if no matter what the opponent does, s_i gives a better payoff to i than s_i'

Definition

Let s_i, s_i' be two pure strategies. Then we say that s_i strictly dominates s_i' if for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

Dominance

A pure strategy s_i is **strictly dominant** if s_i strictly dominates every other strategy s_i'

Definition

Let s_i be a pure strategy of player i. Then s_i is strictly dominant if for all $s_i' \neq s_i$, s_i strictly dominates s_i'.

Dominance

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Dominance

- Intuitively if a strategy s_i always results in a greater utility than s_i', regardless of the strategy followed by the other players then the strategy s_i' should never be chosen by individual i
- We can eliminate any strategy that is strictly dominated

Dominance in the prisoners dilemma

	[illegible]	NC
[illegible]	[illegible]	[illegible]
NC	[illegible]	2,2

→ (NC, NC)

- NC dominates C for both individuals

Dominance in the prisoners dilemma

	C	NC
C	5,5	0,10
NC	10,0	2,2

- NC dominates C for both individuals
- (NC, NC) is not a Pareto Optimum.

max U(x)
x
s.a.
px < p.w

Dominance in the prisoners dilemma

	C	NC
C	5,5	0,10
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- *NC* dominates *C* for both individuals
- (NC, NC) is not a Pareto Optimum.
- What happened to the first welfare theorem? Is it incorrect?

Dominance (iterated)

Consider this game

	[redacted]	[redacted]	c
[redacted]	[redacted]	[redacted]	[redacted]
B	[redacted]	[redacted]	4, 5

- Player 1 has no strategy that is strictly dominated

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*

Dominance (iterated)

Consider this game

	a	b	c
A	5, 5	0, 10	3, 4
B	3, 0	2, 2	4, 5

- Player 1 has no strategy that is strictly dominated
- b dominates a for player 2, thus we can eliminate *a*
- Player 1 would foresee this...

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- *B* now dominates *A* for player 1

Dominance (iterated)

	b	c
A	0, 10	3, 4
B	2, 2	4, 5

- *B* now dominates *A* for player 1
- Player 2 would foresee this (that player 1 foresees that 2 will not play a, and thus he will not play B)

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c)

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- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)

Dominance (iterated)

	b	c
B	2, 2	4, 5

- Player 2 would play c and player 1 would play B
- We have reached a solution (B, c)
- This is known as Iterated Deletion of Strictly Dominated Strategies (IDSDS)
- The equilibrium is the set of strategies, not the payoff!

IDSDS

Definition (Solvable by IDSDS)

A game is solvable by **Iterated Deletion of Strictly Dominated Strategies** if the result of the iteration is a single strategy profile (one strategy for each player)

IDSDS

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- Is the order of elimination of the strategies important? **No**

IDSDS

- Two key assumptions:
- 1) Nobody plays a strictly dominated strategy (that is, the agents are rational)
- 2) Everyone trusts others are rational (i.e., they do not play strictly dominated strategies). That is, agents' rationality is *common information*
- Is the order of elimination of the strategies important? **No**
- Not all games are solvable by IDSDS

Battle of the sexes

S2

S1

	G	P
G	2,1	0,0
P	0,0	1,2

- No strategy is dominated for either player