Lecture14

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Lecture 14: Game Theory // Nash equilibrium

Mauricio Romero

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Mixed strategies

Examples

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Mixed strategies

Consider rock/paper/scissors

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

- This game is entirely stochastic (ability has nothing to do with your chances of winning)

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- The probability of winning with every strategy is the same
- Thus, people *tend* choose randomly which of the three options to play
- We would like the concept of Nash equilibrium to reflect this

---

Mixed strategies

Definition
A mixed strategy σ_i is a function $\sigma_i : S_i \to [0,1]$ such that

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1.$$

- $\sigma_i(s_i)$ represents the probability with which player i plays s_i

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- We will denote the set of all mixed strategies of player i by Σ_i

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- $$u_1(\sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_{s \in S} u_1(s_1, s_2, \ldots, s_n)\sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n).$$

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$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

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- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)
- The expected utility of playing "rock" is

$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

$+0 \cdot 0$

- If I'm randomizing over rock and scissors (i.e., $\sigma_i = (\frac{1}{2}, 0, \frac{1}{2})$) then

$$E(U_i(\sigma, \sigma_{-i})) = \underbrace{-1\frac{1}{4}}_{\text{rock vs paper}} + \underbrace{1\frac{1}{4}}_{\text{rock vs scissors}} + \underbrace{1\frac{1}{4}}_{\text{scissors vs paper}} + \underbrace{0\frac{1}{4}}_{\text{scissors vs scissors}} = \frac{1}{4}$$

Mixed strategies

Definition
A (possibly mixed) strategy profile $(\sigma_1^*, \sigma_2^*, \ldots, \sigma_n)^*$ is a Nash equilibrium if and only if for every i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

$\Rightarrow \sigma_i^* \in MR_i(\sigma_{-i}^*) \ \forall i$

for all $\sigma_i \in \Sigma_i$.

$$\Rightarrow \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) U_i(\sigma_i, s_{-i}) > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) U_i(\sigma_i', s_{-i})$$

Mixed strategies

Definition (Mixed Strategy Dominance Definition A)
Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all mixed strategies of the opponents, σ_{-i},

$$\boxed{u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma_i', \sigma_{-i}).}$$

Mixed strategies

If σ_i is better than σ_i' no matter what **pure strategy** opponents play, then σ_i is also strictly better than σ_i' no matter what **mixed strategies** opponents play

~~**Theorem**~~
~~*Let σ_i and σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if and only if for all $s_{-i} \in S_{-i}$,*~~

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof- Part 1

- Since $S_{-i} \subseteq \Sigma_{-i}$, if σ_i strictly dominates σ_i'

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- Then for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof - Part 2

- To prove the other direction, suppose that for all $s_{-i} \in S_{-i}$,

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- For any σ_{-i},

$$\begin{aligned} u_i(\sigma_i, \sigma_{-i}) &= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) \end{aligned}$$

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- So

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i', s_{-i}) = u_i(\sigma_i', \sigma_{-i})$$

Mixed strategies

Definition (Mixed Strategy Dominance Definition B)
Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all pure strategies of the opponents, $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

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Battle of the sexes

S_2

S_1	G	P
G	2,1	0,0
P	0,0	1,2

$EN = \{(G,G); (P,P)\}$

Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

- There are two pure strategy equilibria (G, G) and (P, P)

Battle of the sexes

	G	P
G	2,1	0,0
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- There are two pure strategy equilibria (G, G) and (P, P)
- We now look for Nash equilibria that involve randomizationby the players

Battle of the sexes

PASO 1

$\boxed{MR_1(\sigma_2 = (\underbrace{\lambda}_{Prob_2(G)}, \overbrace{1-\lambda}^{Prob_2(P)}))}$

$\mathbb{E}(U_1(G, \sigma_2)) = 2 \cdot \lambda + 0 \cdot (1-\lambda) = 2\lambda$

$\mathbb{E}(U_1(P, \sigma_2)) = 0 \cdot \lambda + 1 \cdot (1-\lambda) = 1-\lambda$

$G \succ_1 P$ $\quad$ $P \succ_1 G$ $\quad$ $P \sim_1 G$

$2\lambda > 1-\lambda$ $\quad$ $1-\lambda > 2\lambda$ $\quad$ $1-\lambda = 2\lambda$

$3\lambda > 1$ $\quad$ $\lambda < 1/3$ $\quad$ $\lambda = 1/3$

$\lambda > 1/3$

$$MR_1(\sigma_2 = (\lambda, 1-\lambda)) = \begin{cases} G & \lambda > 1/3 \\ \sigma_1 = (\alpha, 1-\alpha),\ \alpha \in [0,1] & \lambda = 1/3 \\ P & \lambda < 1/3 \end{cases}$$

$MR_2(\sigma_1 = (\alpha, 1-\alpha))$

$\mathbb{E}(U_2(\sigma_1, G)) = 1 \cdot \alpha + 0(1-\alpha) = \alpha$

$2(1-\alpha) = 2 - 2\alpha$

- We now look for the mixed strategy Nash equilibria of the game.

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G.
- $$u(\lambda, q) = 2\lambda q + (1-\lambda)(1-q)$$
- Case 1: If $q > 1/3$, then $2q > 2/3 > 1 - q$ and therefore, the best response is $\lambda = 1$.
- Case 2: If $q = 1/3$, then $2q = 2/3 = 1 - q$ and therefore, the best response is $\lambda \in [0, 1]$.
- Case 3: If $q < 1/3$, then $2q < 2/3 < 1 - q$ and therefore, the best response is $\lambda = 0$.
- Thus, the best response function is given by:

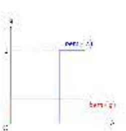
$$BR_1(q) = \begin{cases} 1 & \text{if } q > 1/3 \\ [0,1] & \text{if } q = 1/3 \\ 0 & \text{if } q < 1/3 \end{cases}$$

Battle of the sexes

Similarly we can calculate the best response function for player 2 and we get:

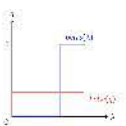
$$BR_2(\lambda) = \begin{cases} 1 & \text{if } \lambda > 2/3 \\ [0,1] & \text{if } \lambda = 2/3 \\ 0 & \text{if } \lambda < 2/3 \end{cases}$$

Battle of the sexes



- There are three points where the best response curves cross.

Battle of the sexes



- There are three points where the best response curves cross.
- First two are the pure strategy NE we had found before.

$$\mathbb{E}(U_2(\sigma_1, G)) = 1\cdot\alpha + 0(1-\alpha) = \alpha$$

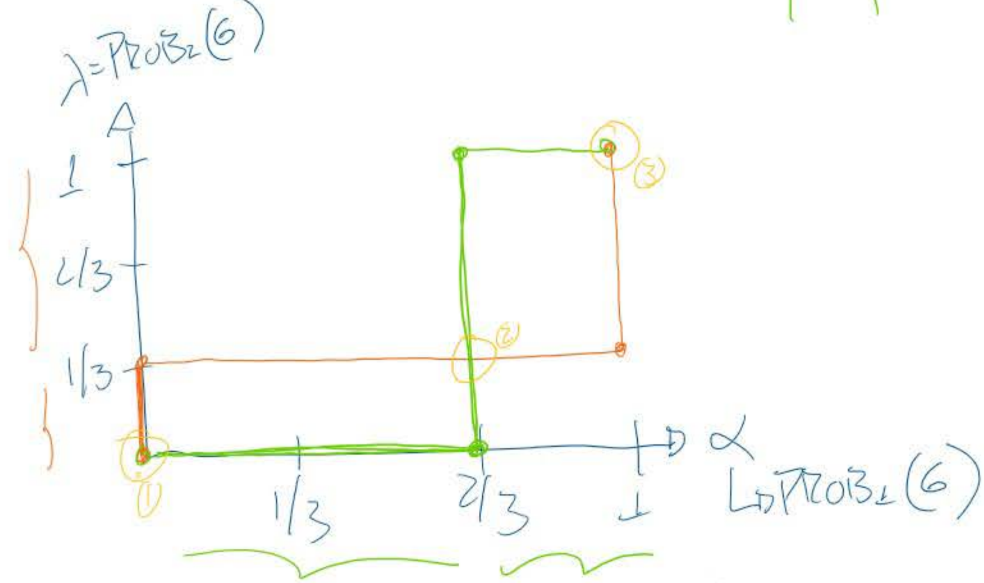
$$\mathbb{E}(U_2(\sigma_1, P)) = 0\cdot\alpha + 2(1-\alpha) = 2 - 2\alpha$$

$G \succ_2 P$: $\alpha > 2 - 2\alpha$, $3\alpha > 2$, $\alpha > 2/3$

$P \succ_2 G$: $2 - 2\alpha > \alpha$, $\alpha < 2/3$

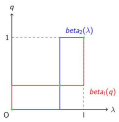
$P \sim_2 G$: $2 - 2\alpha = \alpha$, $\alpha = 2/3$

$$MR_2(\sigma_1 = (\alpha, 1-\alpha)) = \begin{cases} G & \alpha > 2/3 \\ \sigma_2 = (\lambda, 1-\lambda),\ \lambda \in [0,1] & \alpha = 2/3 \\ P & \alpha < 2/3 \end{cases}$$



$$EN = \left\{ \begin{array}{l} (\sigma_1 = (0,1),\ \sigma_2 = (0,1)) \\ (\sigma_1 = (2/3, 1/3),\ \sigma_2 = (1/3, 2/3)) \\ (\sigma_1 = (1,0),\ \sigma_2 = (1,0)) \end{array} \right\}$$

Battle of the sexes



- There are three points where the best response curves cross: $(1,1), (0,0,), (\frac{2}{3}, \frac{1}{3})$
- First two are the pure strategy NE we had found before
- Last is a strictly mixed NE: both players randomize

Consider the following game

$G \succ_2 F$
$D \succ_2 B$

		E	[illegible]	G
P_1	A	5, 10	[illegible]	3, 4
P_2	[illegible]	[illegible]	[illegible]	[illegible]
P_3	C	4, 2	[illegible]	3, 8
$1-P_1-P_2-P_3$	D	2, 4	[illegible]	8, 4

- Consider $\sigma_1 = (\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6})$

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- $\mathbb{E}U(E, \sigma_1) = 10\frac{1}{3} + 4\frac{1}{4} + 2\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(F, \sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$

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- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(G, \sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$
- Then $BR_2(\sigma_1) = \{(p, 0, 1-p), p \in [0,1]\}$

- G dominates F (player 2)

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- D dominates B (player 1)

Reduced game

		E	G
P_A	A	5, 10	3, 4
P_C	[illegible]	[illegible]	[illegible]
$P_D = 1-P_A-P_C$	D	2, 4	8, 4

OBJETIVO:
BUSCAR $\sigma_1 = (P_A, 0, 1-P_A)$
G' DOMINE A C!
$\rightarrow$ 2 CONDICIONES:

$U_1(\sigma_1, E) > U_1(C, E) \rightarrow 5 \cdot P_A + 2 \cdot (1-P_A) > 4$

$U_1(\sigma_1, G) > U_1(C, G) \rightarrow 3 \cdot P_A + 8(1-P_A) > 3$

$5P_A + 2 - 2P_A > 4 \Rightarrow 3P_A > 2 \rightarrow P_A > 2/3$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C

Las condiciones: $U_1(\sigma_1, G) > U_1(C, G) \to$

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\begin{aligned} \mathbb{E}U(\sigma_1, E) &> U(C, E) \\ 3p + 2 &> 4 \\ p &> \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbb{E}U(\sigma_1, G) &> \mathbb{E}U(C, G) \\ 8 - 5p &> 3 \\ p &< \frac{5}{5} = 1 \end{aligned}$$

$\Rightarrow$ $5P_A + 2 - 2P_A > 4 \Rightarrow 3P_A > 2 \to P_A > 2/3$

$3P_A + 8 - 8P_A > 3 \Rightarrow -5P_A > -5 \to P_A < 1$

$\boxed{P_A \in (2/3, 1)}$

Reduced game

		P_E	$1-P_E$
		E	G
σ_1: P_A	A	5, 10	3, 4
$1-P_A$	D	2, 4	8, 4

$\to MR_1(\sigma_2 = (P_E, 1-P_E))$

$\mathbb{E}(U_1(A, \sigma_2)) = 5P_E + 3(1-P_E) = 2P_E + 3$

$\mathbb{E}(U_1(D, \sigma_2)) = 2P_E + 8(1-P_E) = 8 - 6P_E$

$A \succ_1 D$

$2P_E + 3 > 8 - 6P_E$

$8P_E > 5$

$P_E > 5/8$

$D \succ_1 A$

$P_E < 5/8$

$D \sim_1 A$

$P_E = 5/8$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$

$MR_2(\sigma_1 = (P_A, 1-P_A))$

$\mathbb{E}(U_2(\sigma_1, E)) = 10P_A + 4(1-P_A) = 6P_A + 4$

$\mathbb{E}(U_2(\sigma_1, G)) = 4P_A + 4(1-P_A) = 4$

$E \succ_2 G$

$6P_A + 4 > 4$

$P_A > 0$

$G \succ_2 E$

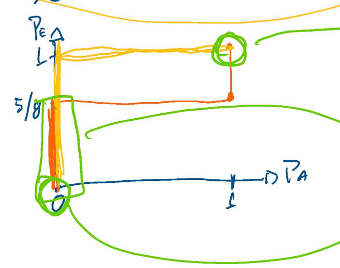
$P_A < 0$

(NUNCA)

$G \sim_2 E$

$P_A = 0$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$



EN = {(A, E), (D, G), $(D, \sigma_2 = (P_E, 1-P_E);\ P_E \in (0, 5/8))$}

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
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- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$

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- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
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- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$
- Thus

$$BR_1(q, 1-q) = \begin{cases} \sigma_1 = (0,1) & \text{if } 0 \le q < \frac{5}{8} \\ \sigma_1 = (1,0) & \text{if } \frac{5}{8} < q \le 1 \\ \sigma_1 = (p, 1-p) & \text{if } \frac{5}{8} = q \end{cases}$$

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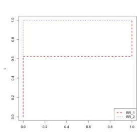
- $\mathbb{E}U(\sigma_1, E) = 10p + 4(1-p) = 6p + 4$
- $\mathbb{E}U(\sigma_1, G) = 4p + 4(1-p) = 4$
- $6p + 4 > 4$ if $p > 0$
- $6p + 4 < 4$ if $p < 0$.
- Thus

$$BR_2(p, 1-p) = \begin{cases} \sigma_2 = (1,0) & \text{if } p > 0 \\ \sigma_2 = (q, 1-q) & \text{if } p = 0 \end{cases}$$

Best responses



$NE = \{(A, E), (D, \sigma_2^q)\}$ where $\sigma_2^q = (q, 1-q)$ and $0 \leq q \leq \frac{5}{8}$