

Lecture1


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Lecture1


Lecture 1: General Equilibrium

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Lecture 1: General Equilibrium

- Introduction
- Pure Exchange Economies
- Pareto efficiency
- Edgeworth Box



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Previous classes

- ▶ Consumers behavior (decision theory) was often analyzed separately from firm behavior (producer theory)
- ▶ When analyzed together, each market was viewed in isolation



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- ▶ But markets are often intertwined



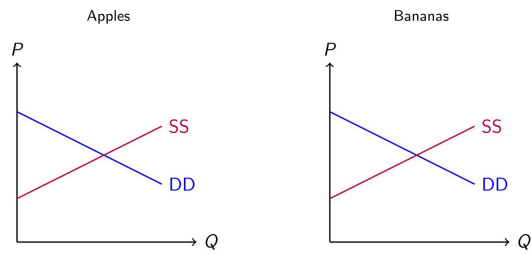
Previous classes

- ▶ Consumers behavior (decision theory) was often analyzed separately from firm behavior (producer theory)
- ▶ When analyzed together, each market was viewed in isolation
- ▶ But markets are often intertwined
 - ▶ Transportation: Uber/metro/ecobici/car
 - ▶ Wages across sectors
 - ▶ Fruits
 - ▶ Beer and tacos



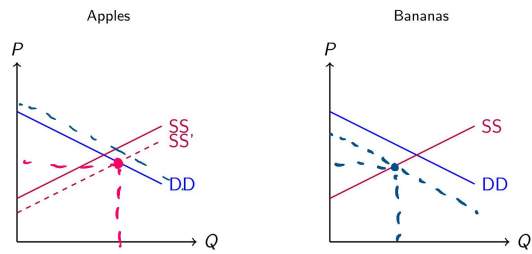
Example - Fruits

- ▶ Suppose that apple and bananas are substitutes



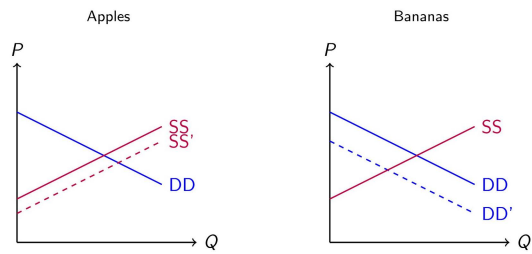
Example - Fruits

- ▶ Suppose that apple and bananas are substitutes
- ▶ Supply curve for apples shifts out



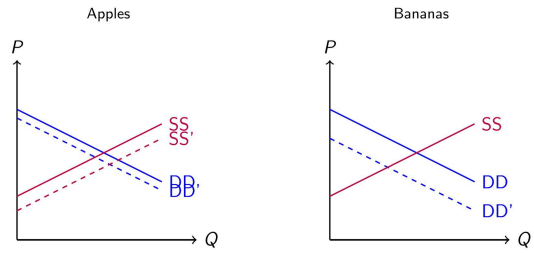
Example - Fruits

- ▶ Suppose that apple and bananas are substitutes
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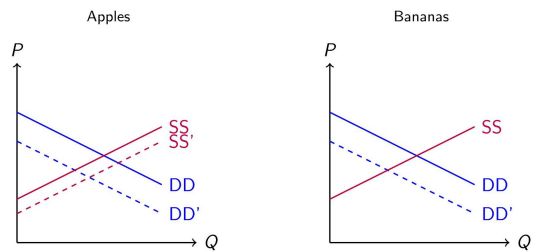
Example - Fruits

- ▶ Suppose that apple and bananas are substitutes
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Example - Fruits

- ▶ What happens if apple and bananas are complements?

A tour down memory lane

- ▶ Léon Walras started it all (1834-1910)
 - ▶ First to use mathematical tools in economics
 - ▶ Supply and demand curves as solutions to a maximization problem
 - ▶ Started the "marginal revolution"
- ▶ Walras was ultimately after normative questions (is the market economy good?)
- ▶ But first, he tackled positive questions (is there an equilibrium? is it unique?)
- ▶ Made a lot of progress. In particular came up with "Walras Law": Sum of the values of excess demands across all markets must equal zero always

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A tour down memory lane

- ▶ Vilfredo Pareto was Walras student (1848-1923)
 - ▶ Abandoned utilitarianism (i.e., utility functions)
 - ▶ Embraced "preferences"
 - ▶ Utility functions only have ordinal content
 - ▶ Comparing "utils" across individuals is meaningless
 - ▶ (Pareto) optimum/efficiency: Achieved if we can't make someone better-off without making someone worse-off

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A tour down memory lane

- ▶ Francis Edgeworth (1845 – 1926)
 - ▶ Introduced indifference curves
 - ▶ Was the first to ask: Where will voluntary exchange lead to?
 - ▶ He conjecture his result was aligned with Walras' result

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A tour down memory lane

- ▶ No more advances for a while (until 1950's) then
 - ▶ Kenneth Arrow
 - ▶ Gerard Debreu
 - ▶ Lionel McKenzie
- ▶ Existence
- ▶ Showed it was Pareto efficient
- ▶ Two Nobel prizes (Arrow — 1972 and Debreu — 1974)



Lecture 1: General Equilibrium

Introduction

Pure Exchange Economies

Pareto efficiency

Edgeworth Box



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Pure Exchange Economies

- ▶ How are goods distributed among consumers?
- ▶ What incentives are there to exchange goods? What institutions mediate the exchange?
- ▶ Is there a distribution of goods that leaves everyone satisfied and there aren't any incentives to deviate?

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Pure Exchange Economies

- ▶ What are the properties of such an equilibrium?
 - ▶ Is it unique?
 - ▶ Is it stable?
 - ▶ Is it efficient?

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Pure Exchange Economies

- ▶ Assume there are
 - ▶ I consumers, $\mathcal{I} = \{1, \dots, I\}$
 - ▶ L goods, $\mathcal{L} = \{1, \dots, L\}$
 - ▶ Each consumer i is characterized by a utility function u^i .
 - ▶ Each consumer can consume goods in $x_i \in \mathbb{R}_+^L \rightarrow (x_i^1, x_i^2, \dots, x_i^L)$
 - ▶ Each consumer has an initial endowment of $w^i \in \mathbb{R}_+^L$.
 - ▶ Each consumer is characterized by the pair: (u^i, w^i) .
 - ▶ Assume the utility functions represent neoclassic preferences

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Utility functions and neoclassic preferences

- ▶ A brief reminder



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Utility functions and neoclassic preferences

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- ▶ Utility functions are ordinal not cardinal
- ▶ They are used to represent preferences
 - ▶ If $x \succ_i y$ then $u^i(x) > u^i(y)$
 - ▶ If f is any increasing function then $f(u^i(x)) > f(u^i(y))$
 - ▶ Hence $f(u^i(\cdot))$ also represents \succ_i
 - ▶ $u^i(x) > u^i(y)$ means something, but $u^i(x) - u^i(y)$ does not
- ▶ Neoclassic preferences are well behaved

Utility functions and neoclassic preferences

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 - ▶ Hence $f(u^i(\cdot))$ also represents \succ_i
 - ▶ $u^i(x) > u^i(y)$ means something, but $u^i(x) - u^i(y)$ does not
- ▶ Neoclassic preferences are well behaved
 - ▶ They can be represented by a utility function
 - ▶ They are weakly monotonic
 - ▶ They are quasi-concave

Handwritten notes:

$$e^{x^1 y^2} = x^1 y^2$$

$$\sqrt{x^1 y^2} = x^1 y^2$$

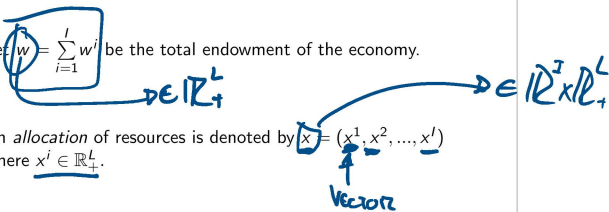
Pure Exchange Economies

Definition (Exchange economy)

A pure exchange economy is $\mathcal{E} = \langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \rangle$ where \mathcal{I} is the set of agents, u^i is a representation of consumer i 's preferences and w^i is consumer i 's initial endowment.

- ▶ Let $w = \sum_{i=1}^I w^i$ be the total endowment of the economy.

- ▶ An allocation of resources is denoted by $x = (x^1, x^2, \dots, x^I)$ where $x^i \in \mathbb{R}_+^L$.



Pure Exchange Economies

Definition (Feasible allocation)

The set of *feasible* allocation F of an economy $\mathcal{E} = \langle \mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}} \rangle$ is defined by:

$$F = \left\{ x = (x^1, x^2, \dots, x^I) : x^i \in \mathbb{R}_+^L, \sum_{i=1}^I x^i = \sum_{i=1}^I w^i \right\}$$

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Pareto efficiency

Definition (Pareto efficiency)

Let \mathcal{E} be an economy. A feasible allocation of resources $x = (x^1, x^2, \dots, x^I)$ is Pareto efficient if there isn't another feasible allocation $\hat{x} = (\hat{x}^1, \hat{x}^2, \dots, \hat{x}^I)$ such that for every agent i , $u^i(\hat{x}^i) \geq u^i(x^i)$ and for at least one agent i^* , $u^{i^*}(\hat{x}^{i^*}) > u^{i^*}(x^{i^*})$.

X ES O.P.
SI NO EXISTE
UN \hat{x} C'LO
PARZETO DOMINE.

Pareto efficiency

Definition (Pareto domination)

Take two feasible allocations x and \hat{x} . We say that \hat{x} Pareto dominates x if for all $i = 1, \dots, I$,

$$u_i(\hat{x}_i^1, \dots, \hat{x}_i^I) \geq u_i(x_i^1, \dots, x_i^I)$$

and there is at least one consumer j for which

$$u_j(\hat{x}_j^1, \dots, \hat{x}_j^I) > u_j(x_j^1, \dots, x_j^I).$$

Thinking about Pareto efficiency

- ▶ If x is a Pareto efficient feasible allocation, does it mean that x Pareto dominates all other feasible allocations?
- ▶ If there are two allocations (x and y) is it always the case that one Pareto dominates the other?
- ▶ For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy
- ▶ Social planner should strive to achieve Pareto efficiency at the very least!

Thinking about Pareto efficiency

- ▶ If x is a Pareto efficient feasible allocation, does it mean that x Pareto dominates all other feasible allocations?
- ▶ If there are two allocations (x and y) is it always the case that one Pareto dominates the other?
- ▶ For Pareto efficiency, the initial endowments only matter in the sense that they determined the total endowment of the economy
- ▶ Social planner should strive to achieve Pareto efficiency at the very least! However, she may have other concerns such as fairness

Thinking about Pareto efficiency

- ▶ If utility is strictly increasing, then can a Pareto efficient allocation be such that $\sum_{i=1}^I x_i^j > \sum_{i=1}^I w_i^j$?

$$\sum_{i=1}^I x_i^j > \sum_{i=1}^I w_i^j$$

- ▶ The set of all Pareto allocations is known as the **contract curve**

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→ \sum INDIVIDUOS
 \sum BIENES

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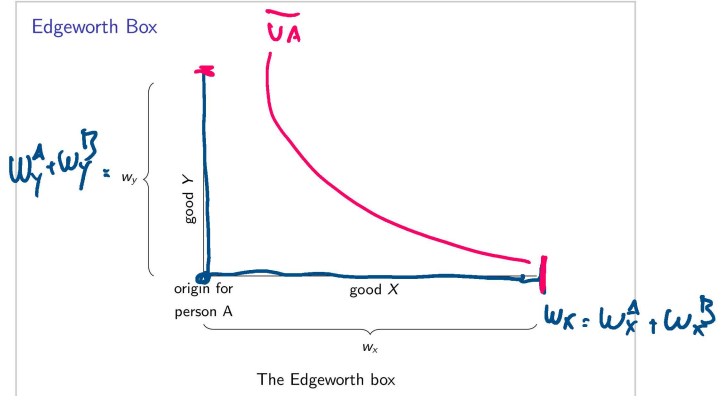
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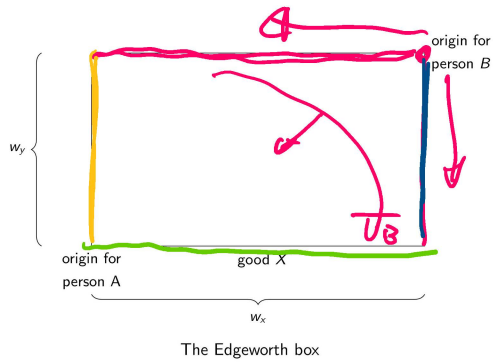
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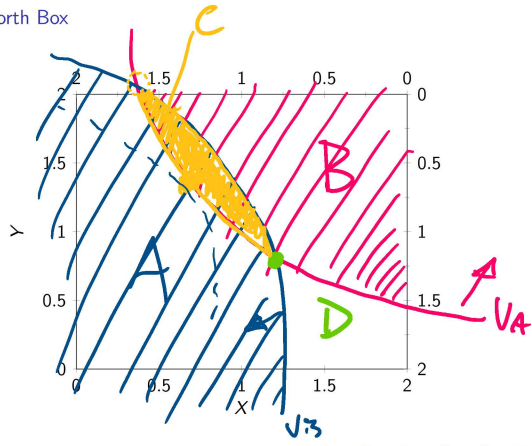


Edgeworth Box

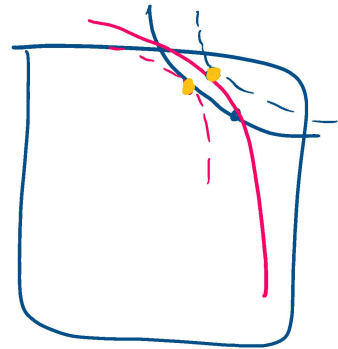
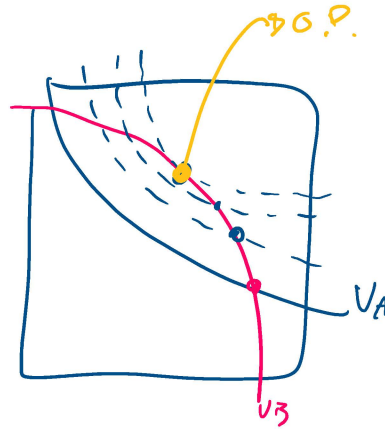
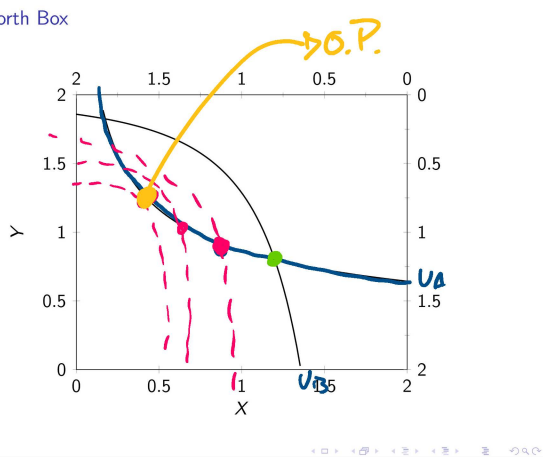


The Edgeworth box

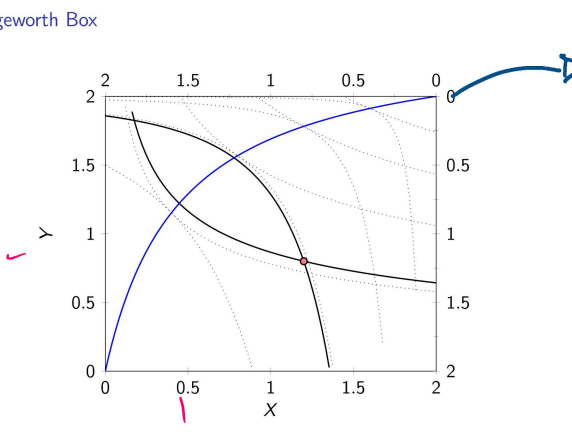
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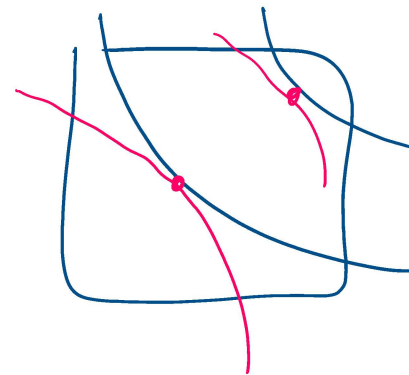
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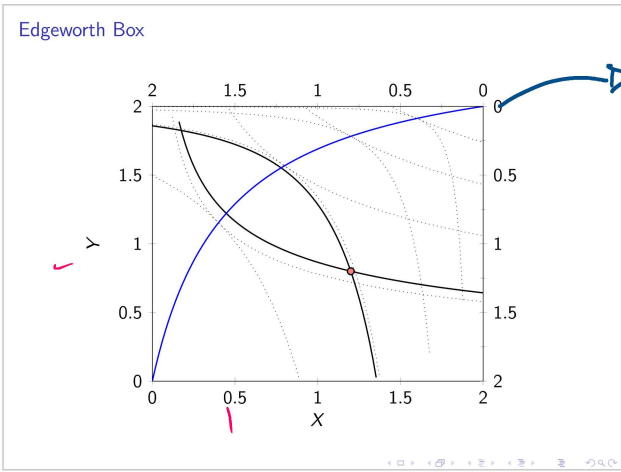


Edgeworth Box



CURVA DE CONTRATO





→ CURVA DE CONÍZATO

