Lecture20.pdf

Thursday, May 12, 2022 3:22 PM

Lecture20....

Lecture 20: Infinitely Repeated Games

Mauricio Romero

Lecture 20: Infinitely Repeated Games

Cournot n-firms

Bertrand n-firms

Lecture 20: Infinitely Repeated Games

Cournot n-firms

Bertrand n-firms

- Single-period non-cooperative Cournot game: unique NE

- Single-period non-cooperative Cournot game: unique NE
- Firms produce higher-output, receive lower profits than if they cooperated (prisoners' dilemma)

- Single-period non-cooperative Cournot game: unique NE
- Firms produce higher-output, receive lower profits than if they cooperated (prisoners' dilemma)
- Can cooperation occur in multi-period games?

- N-firm Cournot setting game

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$

  - Firm i's best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_1 + \cdots + q_N - q_i}{2}$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Firm i's best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_1 + \cdots + q_N - q_i}{2}$

  - Symmetric NE quantities: $q^* = \frac{(a-c)}{(N+1)b}$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Firm i's best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_1 + \cdots + q_N - q_i}{2}$
  - Symmetric NE quantities: $q^* = \frac{(a-c)}{(N+1)b}$

  - Market price: $p^* = \frac{1}{(N+1)}a + \frac{N}{(N+1)}c$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Firm i's best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_1 + \cdots + q_N - q_i}{2}$
  - Symmetric NE quantities: $q^* = \frac{(a-c)}{(N+1)b}$
  - Market price: $p^* = \frac{1}{(N+1)}a + \frac{N}{(N+1)}c$

  - Per-firm profits: $\pi^* = \frac{(a-c)^2}{(N+1)^2 b}$

- N-firm Cournot setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Firm i's best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_1 + \cdots + q_N - q_i}{2}$
  - Symmetric NE quantities: $q^* = \frac{(a-c)}{(N+1)b}$
  - Market price: $p^* = \frac{1}{(N+1)}a + \frac{N}{(N+1)}c$
  - Per-firm profits: $\pi^* = \frac{(a-c)^2}{(N+1)^2 b}$

  - Note: as N grows large, $p^* \to c$ and $\pi^* \to 0$, as in PC

- If firms cooperate: $\max_q = Nq(a - b(Nq) - c) \to q^c = \frac{(a-c)}{2bN}$
- $p^c = \frac{a+c}{2}$, higher than p^*.
- $\pi^c = \frac{(a-c)^2}{4bN}$, higher than π^*.
- But why can't each firm do this? Because NE condition is not satisfied: $\max_{q_i} \pi_i = \max_{q_i} q_i \left(a - b\left((N-1)\frac{(a-c)}{2bN} + q_i\right) - c\right) \to q^d = \frac{(a-c)(n+1)}{4Nb}$
- So the profits from deviating are: $\boxed{\pi^d = \frac{(n+1)^2(a-c)^2}{16bn^2}} \to \pi(q^d, q^c)$
- What if we repeat the game?

2-period Cournot game

- Second period: unique NE in these subgames (play the NE)
- First period: Given that NE in $t = 2 \longrightarrow$ unique SPNE is to play the NE of the stage game in both periods.
- What about 3 periods?
- What about N periods?

- Infinitely repeated game

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0,1]$, which measures how "patient" a firm is

---

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0,1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0,1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$
  - Consider the following strategy:

  Firm i cooperates as long as it observes all other firms cooperating. If another firm cheats, firm i produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or "grim strategy")

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0,1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$
  - Consider the following strategy:
    1. In period t, firm i plays $q_{it} = q^c$ if $q_{-i,t-1} = q^c$.

    item Play q^* if $q_{-i,t-1} \neq q^c$

  Firm i cooperates as long as it observes all other firms cooperating. If another firm cheats, firm i produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or "grim strategy")

PERIODO T

"NADIE DESV ANTES" (JUGADO q^c $\forall i, \forall t<T$)

YA DESV ANTES (ALGUIEN JUGO $\neq q^c$)

$$\Pi(\text{DESV}) = \Pi(q^d, q^c) + \sum_{t=0}^{\infty} \pi^* \delta^t = \Pi(q^d, q^c) + \frac{\delta \Pi^*}{1-\delta}$$

$$\Pi(\text{NoDESV}) = \sum_{t=0}^{\infty} \pi^c \delta^t = \frac{\Pi^c}{1-\delta}$$

$$\Pi(\text{NO DESV}) > \Pi(\text{DESV})$$

$$\boxed{\frac{\Pi^c}{1-\delta} > \Pi^d + \frac{\delta}{1-\delta}\Pi^*}$$

$$\Pi(\text{ND}) = \sum_{t=0}^{\infty} \Pi(q^c, q^c)\delta^t$$

$$\Pi(\text{DESV}) = \Pi(q^d, q^*_{-i}) + \sum_{t=1}^{\infty} \pi^* \delta^t$$

EL q_i MAX ES q^*

$$\Pi(\text{ND}) > \Pi(\text{DESV})$$

Consider firm i (symmetric for all other firms)
There are two relevant subgames for firm i

- After a period in which cheating (either by himself or the other firm) has occurred
- Proposed strategy prescribes playing q^* forever (by all firms)
- This is NE of the subgame: playing q^* is a best-response to other firms playing q^*
- This satisfies SPE conditions.

- After a period when no cheating has occurred
- Proposed strategy prescribes cooperating and playing q^c, with discounted PV of payoffs $= \pi^c/(1-\delta)$
- The best other possible strategy is to play $BR_1(q^c_{-i}) \equiv q^d_i$ this period, but then be faced with $q_2 = q^*$ forever
- This yields discounted PV $= \pi^d + \delta(\pi^*/(1-\delta))$
- In order for q_c to be NE of this subgame, require $\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))$

- $\boxed{\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))}$
- $\frac{(a-c)^2}{4bN(1-\delta)} > \frac{(n+1)^2(a-c)^2}{16bn^2} + \delta\left(\frac{(a-c)^2}{(N+1)^2 b(1-\delta)}\right)$
- $\delta > \frac{(n+1)^2}{n^2+6b+1}$
- This value increases with n (i.e., collusion is harder to maintain as the number of firms grows)

Lecture 20: Infinitely Repeated Games

Cournot n-firms

Bertrand n-firms

Lecture 20: Infinitely Repeated Games

Cournot n-firms

Bertrand n-firms

- Single-period non-cooperative Bertrand game (with continuous prices): unique NE

- Single-period non-cooperative Bertrand game (with continuous prices): unique NE
- Firms produce higher-output, receive lower profits than if they cooperated (prisoners' dilemma)

- Single-period non-cooperative Bertrand game (with continuous prices): unique NE
- Firms produce higher-output, receive lower profits than if they cooperated (prisoners' dilemma)
- Can cooperation occur in multi-period games?

- N-firm Bertrand setting game

- N-firm Bertrand setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$

- N-firm Bertrand setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$

- N-firm Bertrand setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Symmetric NE prices: $p^* = c$

- N-firm Bertrand setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Symmetric NE prices: $p^* = c$
  - Market price: $p^* = c$

- N-firm Bertrand setting game
  - Industry inverse demand curve: $p = a - b(q_1 + q_2 + \cdots + q_N)$
  - Firm i's profit: $\pi_i = q_i(a - b(q_1 + q_2 + \cdots + q_N) - c)$
  - Symmetric NE prices: $p^* = c$
  - Market price: $p^* = c$
  - Per-firm profits: $\pi^* = 0$

- If firms cooperate: $\max_p = p\frac{a-p}{b} \rightarrow p = \frac{a+c}{2}$
- $q^c = \frac{a-c}{2bN}$
- $\pi^c = \frac{(a-c)^2}{4bN}$, higher than π^*.
- But why can't each firm do this? Because NE condition is not satisfied. If everyone else plays $p = \frac{a+c}{2}$, I charge ε less, and essentially get the monopoly earnings all for my self
- So the profits from deviating are: $\pi^d = \frac{(a-c)^2}{4b}$
- What if we repeat the game?

2-period Cournot game

- Second period: unique NE in these subgames (play the NE)
- First period: Given that NE in $t = 2 \longrightarrow$ unique SPNE is to play the NE of the stage game in both periods.
- What about 3 periods?
- What about N periods?

- Infinitely repeated game

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is

---

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$
  - Consider the following strategy:

    Firm i cooperates as long as it observes all other firms cooperating. If another firm cheats, firm i produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or "grim strategy")

- Infinitely repeated game
- Are there SPNE of this game in which both firms play q^c (cooperate) each period?
  - Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is
  - Property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$
  - Consider the following strategy:
    1. In period t, firm i plays $p_{i,t} = p^c$ if $p_{-i,t-1} = p^c$.

       item Play p^* if $p_{-i,t-1} \neq p^c$

    Firm i cooperates as long as it observes all other firms cooperating. If another firm cheats, firm i produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or "grim strategy")

$\boxed{T}$

"NO DESV ANTES"

$\Pi(\text{NO DESV}) = \sum_{t=0}^{\infty} \Pi^c \delta^t = \frac{\Pi^c}{1-\delta}$

$\Pi(\text{DESV}) = \Pi^d + \sum_{t=1}^{\infty} \Pi^* \delta^t = \Pi^d + \frac{\Pi^* \delta}{1-\delta}$

$\Pi(\text{ND}) > \Pi(\text{DESV})$

$\frac{\Pi^c}{\ldots} > \Pi^d + \frac{\Pi^* \delta}{1-\delta}$

"SI HUBO DESV ANTES"

$U(\text{ND}) = \sum_{t=0}^{\infty} \Pi^* \delta^t$

$U(\text{DESV}) = \Pi(P^d, P^*) + \sum_{t=1}^{\infty} \Pi^* \delta^t$

$U(\text{ND}) \geq U(\text{DESV})$ ✓

1. In period t, firm i plays $q_{it} = q^c$ if $q_{-i,t-1} = q^c$.

   item Play q^* if $q_{-i,t-1} \neq q^c$

Firm i cooperates as long as it observes all other firms cooperating. If another firm cheats, firm i produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or "grim strategy")

$\pi(ND) > \pi(DESV)$

$$\boxed{\frac{\pi^c}{1-\delta} > \pi^d + \pi^e \frac{\delta}{1-\delta}}$$

$U(DESV) = \pi(P, P) + \sum_{t=1} \ldots \delta$

$U(ND) \geq U(DESV)$ ✓

Consider firm i (symmetric for all other firms)
There are two relevant subgames for firm i

- After a period in which cheating (either by himself or the other firm) has occurred
- Proposed strategy prescribes playing q^* forever (by all firms)
- This is NE of the subgame: playing q^* is a best-response to other firms playing q^*
- This satisfies SPE conditions.

- After a period when no cheating has occurred
- Proposed strategy prescribes cooperating and playing q^c, with discounted PV of payoffs $= \pi^c/(1-\delta)$
- The best other possible strategy is to play $BR_1(q^c_{-i}) \equiv q^d_i$ this period, but then be faced with $q_2 = q^*$ forever
- This yields discounted PV $= \pi^d + \delta(\pi^*/(1-\delta))$
- In order for q_c to be NE of this subgame, require $\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))$

- $\boxed{\pi^c/(1-\delta) > \pi^d + \delta(\pi^*/(1-\delta))}$
- $\frac{(a-c)^2}{4bN(1-\delta)} > \frac{(a-c)^2}{4b} + \delta 0$
- $\boxed{\delta > \frac{n-1}{n}}$
- This value increases with n (i.e., collusion is harder to maintain as the number of firms grows)