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Lecture 3: General Equilibrium

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Lecture 3: General Equilibrium

Lecture 3: General Equilibrium

Hidden assumptions

- There is a market for each good
- Every agent can access the market without any cost
- There is a unique price for each good and all consumers know this price
- Each consumer can sell her initial endowment in the market and use the income to buy goods and services
- Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.
  - There is no centralized mechanism
  - People may not know others preferences or endowments
- There is perfect competition (i.e., everyone is a price taker)
- The only source of information agents are prices

Competitive equilibrium - Definition

Definition
A pair of an allocation and a price vector, $(x^*, p = (p_1, \dots, p_L))$ is called a competitive equilibrium if the following conditions hold:

1. For all consumers $i = 1, 2, \dots, I$, $x^{i*} = (x_1^{i*}, \dots, x_L^{i*})$ solves the following maximization problem:

$$\max_{x^i} u_i(x^i)$$

$$\text{such that } \underline{p \cdot x^i \le p \cdot \omega^i} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

2. Markets clear: For each commodity $\ell = 1, 2, \dots, L$, the following equation holds:

$$\sum_{i=1}^{I} x_\ell^{i*} = \sum_{i=1}^{I} \omega_\ell^i.$$

$\max_{x^i} U_i(x_i)$ s.t $\sum p \cdot x_i \le \sum p \cdot \omega^i$

Competitive equilibrium - Properties

Remark
Suppose that at least one consumer has **strictly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, $p_1, p_2, \dots, p_L > 0$.*

Remark
Suppose that at least one consumer has **weakly** *monotone preferences. Then if (x^*, p) is a competitive equilibrium, there for at least one ℓ, $p_\ell > 0$.*

Remark
If (x^*, p) is a competitive equilibrium, then (x^*, cp) for $c \in \mathbb{R}_+ +$ is also a competitive equilibrium.

Competitive equilibrium - Walras' Law

Theorem (Walras' Law)
Suppose that consumer i has weakly monotone preferences and that $\hat{x}^i \in x^{i*}(p)$. Then

$$p \cdot \hat{x}^i = \sum_{\ell=1}^{L} p_\ell \hat{x}_\ell^i = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i = p \cdot \omega^i.$$

Theorem (Walras' Law - II)
Suppose that utility functions are **weakly** *monotonic. Suppose that $p = (p_1, \dots, p_L)$ is such that $p_L > 0$. Take any (x^*, p) in which Condition 1 holds for each consumer $i = 1, 2, \dots, I$ and markets clear for all commodities $\ell = 1, 2, \dots, L-1$. Then the market clearing condition will hold for commodity L as well.*

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- If we sum the above across all I consumers, then we get:

$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

Walras' Law - proof

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$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- Re-arranging:

$$\sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell \omega_\ell^i.$$

$\rightarrow \sum_{\ell=1}^{L} P_\ell \sum_{i=1}^{I} x_\ell^i = \sum_{\ell=1}^{L} P_\ell \sum_{i=1}^{I} \omega_\ell^i$

$\sum_{\ell=1}^{L} P_\ell \sum_{i=1}^{I} x_\ell^i - \sum_{\ell=1}^{L} P_\ell \sum_{i=1}^{I} \omega_\ell^i = 0$

$\sum_{\ell=1}^{L} P_\ell \left(\sum_{i=1}^{I} x_\ell^i - \omega_\ell^i\right) = 0$

Walras' Law - proof

- For each consumer i, we must

$$\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- If we sum the above across all I consumers, then we get:

$$\sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell x_\ell^{i*} = \sum_{i=1}^{I}\sum_{\ell=1}^{L} p_\ell \omega_\ell^i.$$

- Re-arranging:

$$\sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell x_\ell^{i*} = \sum_{\ell=1}^{L}\sum_{i=1}^{I} p_\ell \omega_\ell^i.$$

- Re-arranging:

$$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Walras' Law - proof

- $$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

Walras' Law - proof

- $$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

- $$\boxed{p_L \sum_{i=1}^{I} \left(x_L^{i*} - \omega_L^i\right) = 0.}$$

Walras' Law - proof

- $$\sum_{\ell=1}^{L} p_\ell \sum_{i=1}^{I} \left(x_\ell^{i*} - \omega_\ell^i\right) = 0.$$

- $$p_L \sum_{i=1}^{I} \left(x_L^{i*} - \omega_L^i\right) = 0.$$

- $$\sum_{i=1}^{I} \left(x_L^{i*} - \omega_L^i\right) = 0.$$

Lecture 3: General Equilibrium

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Cobb-Douglas

$$u_A(x,y) = x^\alpha y^{1-\alpha}$$
$$u_B(x,y) = x^\beta y^{1-\beta}$$

Suppose

$$\alpha = 0.5$$
$$\beta = 0.5$$
$$\omega^A = (1.5, 0.5)$$
$$\omega^B = (0.5, 1.5)$$

$(X_A, Y_A, X_B, Y_B; P_x, P_y)$ ES EQ. Competitivo

1) $(X_i^*, Y_i^*) = \text{ARG} \max_{x_i, y_i} \; X_i^{0.5} Y_i^{0.5}$

s.t.

$\omega_x^i P_x + \omega_y^i P_y \ge X_i P_x + Y_i P_y$

2) MCDOS VACIEN:

$X_A^* + X_B^* = 2 = \omega_x^A + \omega_x^B$

$Y_A^* + Y_B^* = 2 = \omega_y^A + \omega_y^B$

$\mathcal{L} = X_i^{0.5} y_i^{0.5} + \lambda\left(\omega_x^i P_x + \omega_y^i P_y - P_x X_i - P_y y_i\right)$

$\frac{\partial \mathcal{L}}{\partial X_i} = 0.5 X_i^{-0.5} y_i^{0.5} - \lambda P_x = 0$

Cobb-Douglas

Each individual solves

$$\max_{x_i, y_i} \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \le p_x \omega_x^i + p_y \omega_y^i$$

Cobb-Douglas

Each individual solves

$$\max_{x_i, y_i} \sqrt{x^i y^i}$$

Cobb-Douglas

Each individual solves

$$\max_{x_i, y_i} \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \leq p_x w_x^i + p_y w_y^i$$

We can set up a Lagrangean:

$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

$$y = X_i^{0.5} y_i^{0.5} + \lambda\left(\underline{w_x^i P_x + w_y^i P_y - P_x X_i - P_y y_i}\right)$$

$$\frac{\partial y}{\partial X_i} = 0.5\, X_i^{-0.5} y_i^{0.5} - \lambda P_x^* = 0$$

$$\frac{\partial y}{\partial Y_i} = 0.5\, X_i^{0.5} y_i^{-0.5} - \lambda P_y^* = 0$$

Cobb-Douglas

Each individual solves

$$\max_{x_i, y_i} \sqrt{x^i y^i}$$

s.t.

$$p_x x^i + p_y y^i \leq p_x w_x^i + p_y w_y^i$$

We can set up a Lagrangean:

$$\mathcal{L} = \sqrt{x^i y^i} + \lambda\left(p_x w_x^i + p_y w_y^i - p_x x^i - p_y y^i\right)$$

The FOC are:

$$\frac{1}{2}\sqrt{\frac{y^i}{x^i}} = \lambda p_x$$
$$\frac{1}{2}\sqrt{\frac{x^i}{y^i}} = \lambda p_y$$

$$\frac{\cancel{0.5}\, X_i^{-0.5} y_i^{0.5}}{\cancel{0.5}\, X_i^{0.5} y_i^{-0.5}} = \frac{P_x^*}{P_y^*}$$

$$\boxed{\frac{y_i}{X_i} = \frac{P_x^*}{P_y^*} \Rightarrow y_i = \frac{P_x^* X}{P_y^*}}$$

$$w_x^i P_x^* + w_y^i P_y^* = X_i P_x^* + y_i P_y^*$$

$$w_x^i P_x^* + w_y^i P_y^* = X_i P_x^* + \frac{P_x^* X_i \cancel{P_y^*}}{\cancel{P_y^*}}$$

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

$$w_x^i P_x^* + w_y^i P_y^* = 2 X_i P_x^*$$

$$\boxed{\frac{w_x^i P_x^* + w_y^i P_y^*}{2 P_x^*} = X_i^*}$$ DD MARSHALLIANAS

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

$$\boxed{\frac{w_x^i P_x^* + w_y^i P_y^*}{2 P_y} = y_i^*}$$

Cobb-Douglas

Thus,

$$\frac{y^i}{x^i} = \frac{p_x}{p_y}$$
$$y^i = x^i \frac{p_x}{p_y}$$

We haven't used the budget restriction!

$$p_x x^i + p_y y^i = p_x w_x^i + p_y w_y^i$$
$$p_x x^i + p_y x^i \frac{p_x}{p_y} = p_x w_x^i + p_y w_y^i$$
$$x^i = \frac{w_x^i p_x + w_y^i p_y}{2p_x}$$
$$y^i = \frac{w_x^i p_x + w_y^i p_y}{2p_y}$$

$$\boxed{\frac{1.5P_x + 0.5P_y}{2P_x} = X_A^*} \qquad \boxed{\frac{0.5P_x + 1.5P_y}{2P_x} = X_B^*}$$

$$\boxed{\frac{1.5P_x + 0.5P_y}{2P_y} = y_A^*} \qquad \boxed{\frac{0.5P_x + 1.5P_y}{2P_y} = y_B^*}$$

Cobb-Douglas

$$x^A = \frac{1.5p_x + 0.5p_y}{2p_x}$$
$$y^A = \frac{1.5p_x + 0.5p_y}{2p_y}$$
$$x^B = \frac{0.5p_x + 1.5p_y}{2p_x}$$
$$y^B = \frac{0.5p_x + 1.5p_y}{2p_y}$$

Now we can use condition 2 (market clear)

Cobb-Douglas

$$x^A = \frac{1.5p_x + 0.5p_y}{2p_x}$$
$$y^A = \frac{1.5p_x + 0.5p_y}{2p_y}$$
$$x^B = \frac{0.5p_x + 1.5p_y}{2p_x}$$
$$y^B = \frac{0.5p_x + 1.5p_y}{2p_y}$$

Now we can use condition 2 (market clear)

$$x^A + x^B = 2$$
$$y^A + y^B = 2$$

$$\underbrace{\frac{1.5P_x + 0.5P_y}{2P_x}}_{X_A^*} + \underbrace{\frac{0.5P_x + 1.5P_y}{2P_x}}_{X_B^*} = 2$$

Cobb-Douglas

$$\frac{1.5p_x + 0.5p_y}{2p_x} + \frac{0.5p_x + 1.5p_y}{2p_x} = 2$$
$$\frac{p_x}{p_y} = 1$$

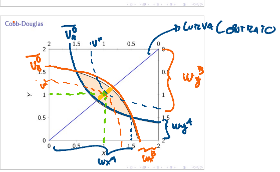
$$\frac{2P_x + 2P_y}{2P_x} = 2$$

Cobb-Douglas

$$\frac{1.5p_x + 0.5p_y}{2p_x} + \frac{0.5p_x + 1.5p_y}{2p_x} = 2$$
$$\frac{p_x}{p_y} = 1$$
$$x^A = x^B = y^A = y^B = 1$$

$$1 + \frac{P_y}{P_x} = 2$$

Cobb-Douglas



$$\boxed{\frac{P_y}{P_x} = 1}$$

$(1, 1)$

$(2, 2)$

$(1/2, 1/2)$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

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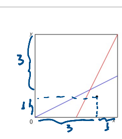
EN EL EQ

$$\boxed{\begin{aligned} X_A^* &= X_B^* = 1 \\ y_A^* &= y_B^* = 1 \end{aligned}}$$

Perfect complements

Suppose that

$$u_A(x^A, y^A) = \min(x^A, 2y^A) = \text{MIN}(X^A, 2y^A)$$
$$u_B(x^B, y^B) = \min(2x^B, y^B) = \text{MIN}(2X^B, Y^B)$$
$$\omega^A = (3, 1)$$
$$\omega^B = (1, 3)$$



A PUEDE COMPRAR (X_A, Y_A)

$$P_x X_A + P_y y_A \leq w_x^A P_x + w_y^A P_y$$

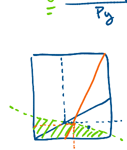
$$y_A \leq \frac{w_x^A P_x + w_y^A P_y - P_x X_A}{P_y}$$

$$y_A \leq \frac{w_x^A P_x + w_y P_y}{P_y} - \frac{P_x}{P_y} X_A$$

Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$



Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$

Perfect complements

At a given price vector, consumer A can buy any combination (x^A, y^A) such that:

$$p_x w_x^A + p_y w_y^A \geq p_x x^A + p_y y^A$$

or equivalently

$$y^A \leq \frac{p_x w_x^A + p_y w_y^A}{p_y} - \frac{p_x}{p_y} x^A$$

How does this looks in the Edgeworth box?

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

If $\frac{p_x}{p_y} \neq 1$ Then, we will have the following restriction:

$$y^A \leq \frac{p_x}{p_y}\left(w_x^A - x^A\right) + w_y^A$$

Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

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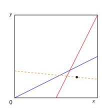
Thus, replacing the values of w_x^A and w_y^A, we have:

$$y^A \leq \frac{p_x}{p_y}\left(3 - x^A\right) + 1$$

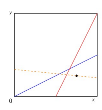
Note that, for the case $\frac{p_x}{p_y} = 1$, we have the following restriction:

$$y^A \leq 4 - x^A$$

$\frac{p_x}{p_y} < 1$

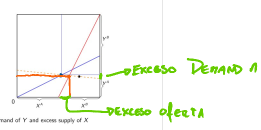


$\frac{p_x}{p_y} < 1$



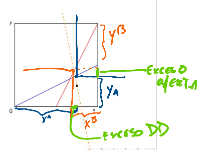
A can buy whats below the orange line, B what is above

$\frac{p_x}{p_y} < 1$

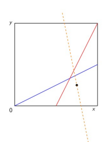


Excess demand of Y and excess supply of X

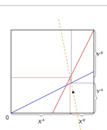
$\frac{p_x}{p_y} > 1$



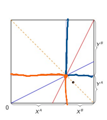
$\frac{p_x}{p_y} > 1$



$\frac{p_x}{p_y} > 1$



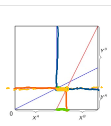
$\boxed{\frac{p_x}{p_y} = 1}$



No excess demand or supply

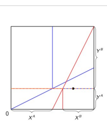
What about zero prices?

$p_x = 0$



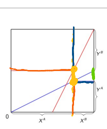
Excess supply of X? (and Y balanced?)

$p_x = 0$



Excess supply of X? (and Y balanced?) Not really since both A and B are indifferent over a wide range that would make the market clear

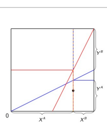
$p_y = 0$



$\frac{P_x}{P_y}$

Excess supply of Y? (and X balanced?)

$p_y = 0$



Excess supply of Y? (and X balanced?) Not really since both A and B are indifferent over a wide range that would make the market clear

To sum up...

- There are multiple equilibria
- There are three price vectors associated with these equilibria
- One price vector has a unique resource allocation associated with it
- Two price vectors ($p_x = 0$ and $p_y = 0$) have *infinity* resource allocations associated with them

Perfect complements

Try at home:

$$u_A(x^A, y^A) = \min(x^A, y^A)$$
$$u_B(x^B, y^B) = \min(x^B, y^B)$$
$$\omega^A = (1, 1)$$
$$\omega^B = (3, 1)$$

Lecture 3: General Equilibrium

Competitive equilibrium

Examples: Cobb-Douglas

Examples: Perfect Complements

Examples: Perfect Substitutes

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Competitive equilibrium

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Perfect Substitutes

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$\boxed{u_B(x^B, y^B) = x^B + y^B}$$
$$\omega^A = (1, 1)$$
$$\omega^B = (1, 1)$$

MAX U s.t R.P.

$\cancel{y = 2X^A + Y^A + \lambda(P_x + P_y - xP_x - yP_y)}$

$\cancel{\frac{\partial y}{\partial X} = 2 - \lambda P_x}$

$\cancel{\frac{\partial y}{\partial Y} = 1 - \lambda P_y}$

$\boxed{P_x < P_y}$ $\boxed{P_x = P_y}$ $\boxed{P_x > P_y}$

P_Y

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$\boxed{u_B(x^B, y^B) = x^B + y^B}$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

$\frac{\partial y}{\partial x} = 2 - \lambda P_x$

$\frac{\partial y}{\partial y} = 1 - \lambda P_y$

Perfect Substitutes

$$\boxed{u_B(x^B, y^B) = x^B + y^B}$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

$p_x > 0$ and $p_y > 0$, why?

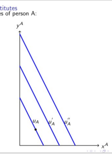
Perfect Substitutes

$$u_A(x^A, y^A) = 2x^A + y^A$$
$$u_B(x^B, y^B) = x^B + y^B$$
$$\omega^A = (1,1)$$
$$\omega^B = (1,1)$$

$p_x > 0$ and $p_y > 0$, why? hence, normalize $p_x = 1$

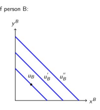
Perfect Substitutes

Peferences of person A:



Perfect Substitutes

Peferences of person B:



Perfect Substitutes

Algebraic solution

$$\max_{x^A, y^A} 2x^A + y^A$$

subject to:

$$I = x^A + p_y y^A$$
$$y^A \geq 0$$
$$x^A \geq 0$$

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subject to:

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$$y^A \geq 0$$
$$x^A \geq 0$$

From the budget constraint we can obtain $y^A = \frac{I - x^A}{p_y}$, and adding the condition $y^A \geq 0$, we can conclude that $x^A \in [0, I]$.

Perfect Substitutes

Introducing y^A into the original maximization problem:

$$\max(2 - \frac{1}{p_y})x^A + \frac{I}{p_y} \quad s.t.\ x^A \in [0, I]$$

Which is a maximization of a straight line with slope $\left(2 - \frac{1}{p_y}\right)$ over an interval.

Perfect Substitutes

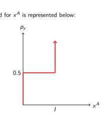
The demand for goods of individual A is

$$X^A = \begin{cases} 0 & \text{if } p_y < 0.5 \\ [0, I] & \text{if } p_y = 0.5 \\ I & \text{if } p_y > 0.5 \end{cases}$$

$$Y^A = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 0.5 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 0.5 \\ 0 & \text{if } p_y > 0.5 \end{cases}$$

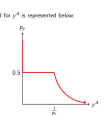
Perfect Substitutes

The demand for x^A is represented below:



Perfect Substitutes

The demand for y^A is represented below:



Perfect Substitutes

Algebraic solution
For person B the solution is analogous, but we have the following maximization problem: Introducing y^A into the original maximization problem:

$$\max(1 - \frac{1}{p_y})x^B + \frac{I}{p_y} \quad s.t.\ x^B \in [0, I]$$

Which is a maximization of a straight line with slope $\left(1 - \frac{1}{p_y}\right)$ over an interval.

Perfect Substitutes

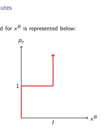
The demand for goods of individual B is

$$X^B = \begin{cases} 0 & \text{if } p_y < 1 \\ [0, I] & \text{if } p_y = 1 \\ I & \text{if } p_y > 1 \end{cases}$$

$$Y^B = \begin{cases} \frac{I}{p_y} & \text{if } p_y < 1 \\ [0, \frac{I}{p_y}] & \text{if } p_y = 1 \\ 0 & \text{if } p_y > 1 \end{cases}$$

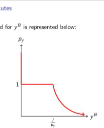
Perfect Substitutes

The demand for x^B is represented below:



Perfect Substitutes

The demand for y^B is represented below:



Perfect Substitutes

When is the market for good X balanced (how about good y?)

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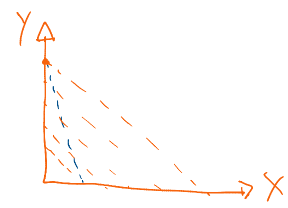
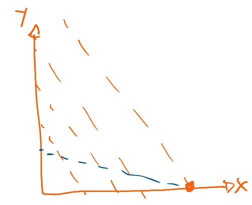
When is the market for good X balanced (how about good y?)
- Try $p_y < 0.5$

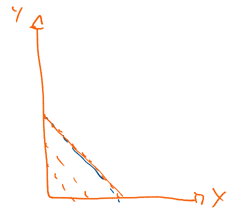
$\boxed{P_x < P_y}$ TODO EN Y $\boxed{P_x = P_y}$ CUALQUIER COSA $\boxed{P_x > P_y}$ TODO EN X

$$X^B(P_x, P_y) = \begin{cases} \frac{I}{P_x} & P_x < P_y \\ [0, \frac{I}{P_x}] & P_x = P_y \\ 0 & P_x > P_y \end{cases} \qquad Y^B = \begin{cases} 0 & P_x < P_y \\ \frac{I - X \cdot P_x}{P_y} & P_x = P_y \\ \frac{I}{P_y} & P_x > P_y \end{cases}$$

$$\rightarrow X^B = \begin{cases} I/P_x & \frac{P_x}{P_y} < 1 \\ 0, I/P_x & \frac{P_x}{P_y} = 1 \\ 0 & \frac{P_x}{P_y} > 1 \end{cases} \qquad Y^B = \begin{cases} 0 & \frac{P_x}{P_y} < 1 \\ \frac{I - X \cdot P_y}{P_y} & \frac{P_x}{P_y} = 1 \\ I/P_y & \frac{P_x}{P_y} > 1 \end{cases}$$







$$X^A = \begin{cases} \frac{I}{P_y} & \frac{P_x}{P_y} < 2 \\ [0, \frac{I}{P_x}] & \frac{P_x}{P_y} = 2 \\ 0 & \frac{P_x}{P_y} > 2 \end{cases} \qquad Y^A = \begin{cases} 0 & \frac{P_x}{P_y} < 2 \\ \frac{I - X^A \cdot P_x}{P_y} & \frac{P_x}{P_y} = 2 \\ \frac{I}{P_y} & \frac{P_x}{P_y} > 2 \end{cases}$$

CASO 1 $\quad \frac{P_x}{P_y} < 1$

DD AG   OO AG

$0 + 0 \neq 2$

NO EQ.

CASO 2

$$\frac{P_x}{P_y} = 1$$

OO AG

$$0 + Y^B \overset{?}{=} 2$$

Y^A

$\left[0, \frac{I^B}{P_y}\right]$

$$\frac{P_x + P_y}{P_y}$$

$$\frac{P_x}{P_y} + 1$$

$$\boxed{2} \overset{?}{=} 2 \checkmark$$

EQUILIBRIO!

$\frac{P_x}{P_y} = 1 \qquad Y^A = 0 \qquad X^A = \frac{P_x + P_y}{P_y} = 2$

$\qquad\qquad\quad Y^B = 2 \qquad X^B = 0$

CASO 3 $\quad \frac{P_x}{P_y} > 2$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium ($X^A = 2, X^B = 0, Y^A = 0, Y^B = 2$)

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium ($X^A = 2, X^B = 0, Y^A = 0, Y^B = 2$)
- Try $p_y > 1$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium ($X^A = 2, X^B = 0, Y^A = 0, Y^B = 2$)
- Try $p_y > 1$
- $X^A = I$ and $X^B = I$

Perfect Substitutes

When is the market for good X balanced (how about good y?)

- Try $p_y < 0.5$
- $X^A = 0$ and $X^B = 0$
- Try $p_y = 0.5$
- $X^A = [0, I]$ and $X^B = 0$
- Can't be an equilibrium since $I = 1.5$ when $p_y = 0.5$, thus $X^A + X^B < 2$
- Try $0.5 < p_y < 1$
- $X^A = I$ and $X^B = 0$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B < 2$
- Try $p_y = 1$
- $X^A = I = 2$ and $X^B = [0, 2]$
- One possible equilibrium ($X^A = 2, X^B = 0, Y^A = 0, Y^B = 2$)
- Try $p_y > 1$
- $X^A = I$ and $X^B = I$
- $X^A = I = 1 + p_y$ and $X^B = I = 1 + p_y$
- Can't be an equilibrium since $I = 1 + p_y$, thus $X^A + X^B = 2 + 2p_y > 2$

$\boxed{\text{CASO 3}}$ $1 < \frac{P_x}{P_y} < 2$

$$\underset{Y^A}{\underset{\uparrow}{0}} + \underset{Y^B}{\underset{\uparrow}{\frac{I^B}{P_y}}} \overset{?}{=} 2$$

$$\frac{P_x + P_y}{P_y}$$

$$\underbrace{\frac{P_x}{P_y} + 1}_{>2} \overset{?}{=} 2$$

$$>2 \neq 2$$

NO ES EQ!

$\boxed{\text{CASO 4}}$ $\frac{P_x}{P_y} = 2$

$$Y^A + \frac{I^A}{P_y} \overset{?}{=} 2$$

$$Y^A + \frac{P_x + P_y}{P_y} \overset{?}{=} 2$$

$$Y^A + \underbrace{\frac{P_x}{P_y} + 1}_{3} \overset{?}{=} 2$$

$\boxed{\text{NO ES EQ}}$

$\boxed{\text{CASO 5}}$ $\frac{P_x}{P_y} > 2$

$$\underbrace{\frac{P_x + P_y}{P_y}}_{Y^A} + \underbrace{\frac{P_x + P_y}{P_y}}_{Y^B} \overset{?}{=} 2$$

$$\underbrace{\frac{2P_x}{P_y} + 2}_{>2} \overset{?}{=} 2$$

NO ES EQ.