Lecture7

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Lecture7

Lecture 7: Monopoly

Mauricio Romero

Lecture 7: Monopoly

Introduction

Elasticities

Monopoly

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Introduction

Elasticities

Monopoly

- Firm is faced a problem like the following:

$$\max_{K,L} p_x f_x(L,K) - wL - rK.$$

- The firm's choice of L and K does not affect the prices p, w, r
- This is called *price-taking* behavior
- Justified if the the market is composed of many small firms

- In many markets there is a single firm
- Since supply is completely controlled by the firm, it can use this in its favor

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- Profit maximization condition,

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$$\max_{K,L} pf_x(K,L) - wL - rK.$$

- If

$$c(x) = \min_{K,L} wL + rK \text{ such that } f_x(K,L) = x$$

then the above is equivalent to:

$$\max_{x} px - c(x).$$

- When firm controls supply, then:

$$\max_{x} \boxed{\mathbf{p(x)}} x - c(x)$$

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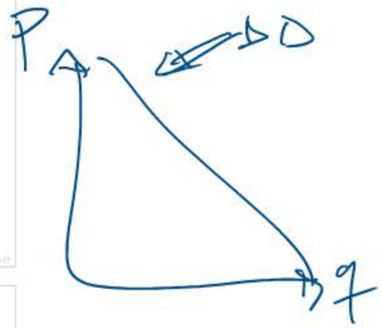
$$\boxed{\max_x \mathbf{p(x)}x - c(x)}$$

- Consumers willingness to pay is given by the demand function
- $p(x)$ is the **demand** function



- We can also represent the problem as:

$$\boxed{\max_p pq(p) - c(q(p))}$$

- $q(p)$ is the **inverse demand function**

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Elasticities

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- Revenue: R(q) = p(q) q
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

$$\varepsilon_{q,p} = \frac{\partial q}{\partial p} \cdot \frac{p}{q}$$

Elasticities

- Revenue: $\boxed{R(q) = \boxed{p(q)\ q}}$
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

$$p\left(1 + \frac{\partial p}{\partial q} \cdot \frac{q}{p}\right) = p\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

$$> 0 \rightarrow 1 + \frac{1}{\varepsilon} > 0$$

$$1 > -\frac{1}{\varepsilon}$$

$$\varepsilon < -1$$

- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

Elasticities

- If $\varepsilon_{q,p} \in (-1, 0)$, the demand is *inelastic*
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\varepsilon_{q,p} < -1$, then demand is *elastic*
  - An increase in price leads a big decrease in demand
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Elasticities

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- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$

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$$\frac{1}{p^k}\cdot k\cdot p^{\cancel{k-1}} = \frac{k}{p}$$

- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$

$$\longrightarrow q(p) = \underbrace{\boxed{e^c}}_{A}\cdot p^k.$$

$$\boxed{q(p) = A\cdot p^k.}$$

Elasticities

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- Suppose that the demand function is of constant elasticity κ
- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$
- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$
- $q(p) = e^C p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

Elasticities

Whenever the demand function has constant elasticity κ

- $q(p)Ap^{\kappa}$ for some $A > 0$.
- Equivalently,

$$p(q) = \left(\frac{q}{A}\right)^{1/\kappa}.$$

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- We want to study the problem:

$$\max_q R(q) - c(q)$$

$$\frac{\partial \pi}{\partial q} = \frac{\partial R(q)}{\partial q} - \underbrace{Cmg(q)}_{\frac{\partial c}{\partial q}} = 0$$

$$Img(q) = Cmg(q)$$

- We want to study the problem:

$$\max_q R(q) - c(q)$$

- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow \underline{p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)} = \frac{dc}{dq} > 0.$$

$\to Cmg(q)$

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- This implies

$$1+\frac{1}{\varepsilon_{q,p}} > 0 \iff \boxed{\varepsilon_{q,p} < -1.}$$

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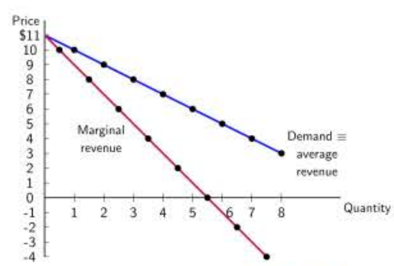
- A monopoly firm always produces at a point where demand is elastic
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits



$$P(q)\left(1+\frac{1}{\varepsilon_{q,p}}\right) = \frac{\partial c}{\partial q}$$

- We can simplify to:

$$\boxed{p(q) = \frac{1}{1+\frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq}.} = \left(\frac{1}{\frac{\varepsilon+1}{\varepsilon}}\right)\frac{\partial c}{\partial q} = \boxed{\frac{\varepsilon}{\varepsilon+1}}\frac{\partial c}{\partial q} = P(q)$$

$$p^m > cmg(q)$$

→ "MARK-UP"

$$MU(\varepsilon) = \frac{\varepsilon}{\varepsilon+1}$$

$$\lim_{\varepsilon \to -\infty} MU(\varepsilon) = 1$$

$$\lim_{\varepsilon \to -1^{(-)}} \frac{\varepsilon}{\varepsilon+1} = +\infty$$

- We can simplify to:

$$p(q) = \frac{1}{1+\frac{1}{\varepsilon_{q,p}}}\frac{dc}{dq}.$$

- Since $\varepsilon_{q,p} < -1$, then

$$p = \frac{1}{}\frac{dc}{dq} > \frac{dc}{dq}$$

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- The amount produced q is below the quantity where $p = MC$.

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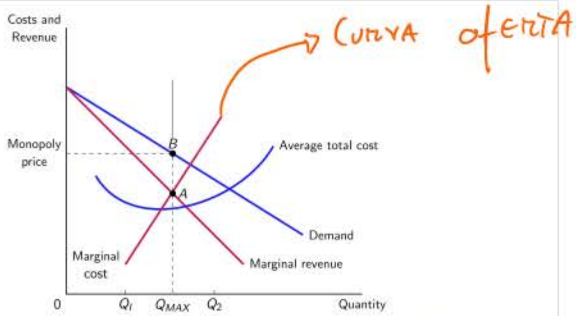
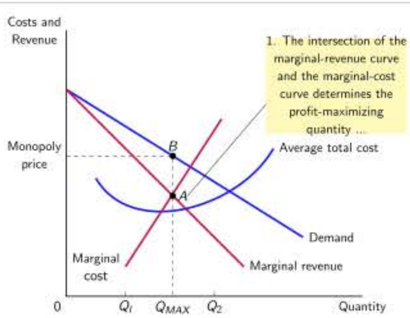
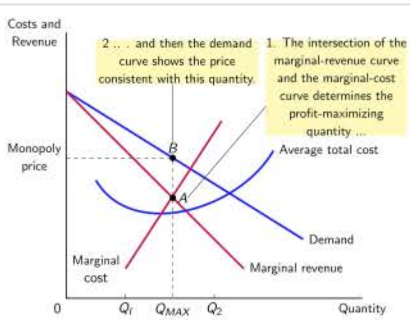
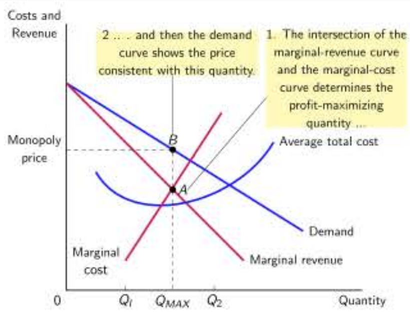
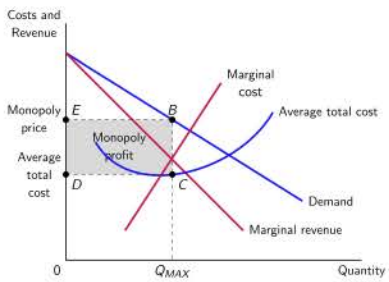
- Both consumer surplus and total surplus is less than is socially optimal
- Thus the pricing policies used by monopolies are inefficient, leading to what is called "dead-weight loss"

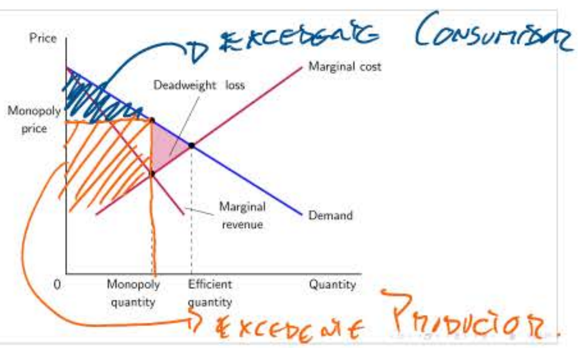


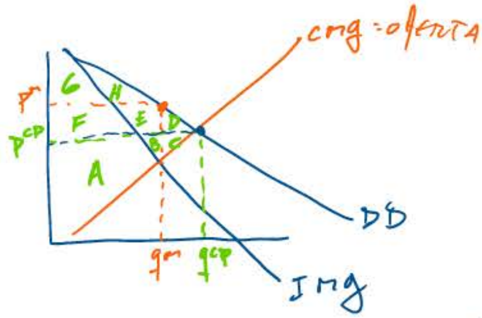












$EC^{cp} = F+E+D+G+H$

$EP^{cp} = A+B+C$

$ES: A+B+C+D+E+F+G+H$

$EC^{m} = G+H$

$EP^{m} = F+E+A+B$

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$\boxed{PBS = C+D}$

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$$\max_{p} \overbrace{pq(p) - c(q(p))}^{\pi}.$$

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- $$\boxed{p = \frac{c}{1+\frac{1}{\kappa}}} \Longrightarrow q(p) = A\left(\frac{c}{1+\frac{1}{\kappa}}\right)^{\kappa}.$$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc.
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply.