Lecture7

jueves, 17 de febrero de 2022 01:13 p. m.

Lecture7

Lecture 7: Monopoly

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- Firm is faced a problem like the following:

$$\max_{K,L} p_x f_x(L,K) - wL - rK.$$

- The firm's choice of L and K does not affect the prices p, w, r
- This is called *price-taking* behavior
- Justified if the the market is composed of many small firms

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- In many markets there is a single firm
- Since supply is completely controlled by the firm, it can use this in its favor

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- If

$$c(x) = \min_{K,L} wL + rK \text{ such that } f_x(K,L) = x$$

then the above is equivalent to:

$$\max_x px - c(x).$$

DWTP ó DAP

- When firm controls supply, then:

$$\max_x \underline{\mathbf{p(x)}}x - c(x)$$

$\llcorner$ CURVA DE DEMANDA

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- Consumers willingness to pay is given by the demand function
- $p(x)$ is the **demand** function

$$\max_P Pq - c(q(P))$$

$\llcorner \underline{q(P)}$

- We can also represent the problem as:

$$\max_p pq(p) - c(q(p))$$

- $q(p)$ is the **inverse demand function**

Lecture 7: Monopoly

Introduction

Elasticities

Monopoly

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Elasticities

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Elasticities

- Revenue: $R(q) = p(q)\,q$

$$\varepsilon_{q,P} = \frac{\%\Delta q}{\%\Delta P} = \left(\frac{\frac{\Delta q}{q}}{\frac{\Delta P}{P}}\right) = \frac{\Delta q}{\Delta P}\cdot\frac{P}{q} = \boxed{\frac{\partial q}{\partial P}\frac{P}{q}}$$

Elasticities

- Revenue: $R(q) = p(q)\,q$
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$

$$P(q)\left(1 + \underbrace{\frac{q}{P}\frac{\partial P}{\partial q}}\right) = P(q)\left(1 + \frac{1}{\varepsilon_{q,P}}\right)$$

$$\boxed{\frac{\partial R}{\partial q} > 0} \iff 1 + \frac{1}{\varepsilon_{q,P}} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,P}} \iff \boxed{\varepsilon_{q,P} < -1}$$

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- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$
- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

Elasticities

- If $\varepsilon_{q,p} \in [-1, 0)$, the demand is inelastic
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\varepsilon_{q,p} < -1$, then demand is elastic
  - An increase in price leads a big decrease in demand
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Elasticities

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- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \implies \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$

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- Suppose that the demand function is of constant elasticity κ:
- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}. \rightarrow \frac{1}{p^{k}}\, k \cdot p^{k-1} = \frac{k}{p}$$
- By the fundamental theorem of calculus:

$$\log q(p) = C + \log p^{\kappa}.$$

$$q(p) = \underbrace{\boxed{e^{c}}}_{A} \cdot p^{k} \Rightarrow \boxed{q(p) = Ap^{k}}$$

Elasticities

- What kind of demand functions have constant elasticities of demand with respect to price?
- Suppose that the demand function is of constant elasticity κ:
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- By the fundamental theorem of calculus:

$$\log q(p) = C + \log p^{\kappa}.$$

- $q(p) = e^{C}p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

Elasticities

Whenever the demand function has constant elasticity κ:

- $q(p)Ap^{\kappa}$ for some $A > 0$.
- Equivalently, $\boxed{p(q) = \left(\frac{q}{A}\right)^{1/\kappa}}$

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Elasticities

Monopoly

- We want to study the problem:

$$\max_{q} R(q) - c(q)$$

$$\frac{\partial \pi}{\partial q} = \underbrace{\frac{\partial R(q)}{\partial q}}_{Img(q)} - \underbrace{\frac{\partial c(q)}{\partial q}}_{cmg(q)} = 0$$

$$Img(q) = cmg(q)$$

$$P\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = cmg(q) \rightarrow$$

Solo se puede cumplir cuando $\varepsilon_{q,p} < -1$

- We want to study the problem:

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- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = \frac{dc}{dq} > 0.$$

$$\Rightarrow P = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}}\, cmg(q) = \frac{1}{\frac{1+\varepsilon}{\varepsilon}}\, cmg(q)$$

$$\boxed{P^{m} = \boxed{\frac{\varepsilon}{1+\varepsilon}}\, cmg(q)}$$

$$\hookrightarrow \boxed{\frac{\varepsilon}{1+\varepsilon}} > 1$$

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$$\boxed{\frac{\varepsilon}{1+\varepsilon}} > 1$$

"MARK up"

$$\lim_{\varepsilon \to -\infty} \frac{\varepsilon}{1+\varepsilon} = 1$$

$$\lim_{\varepsilon \to -1^{(-)}} \frac{\varepsilon}{1+\varepsilon} = \infty$$

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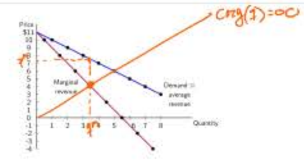
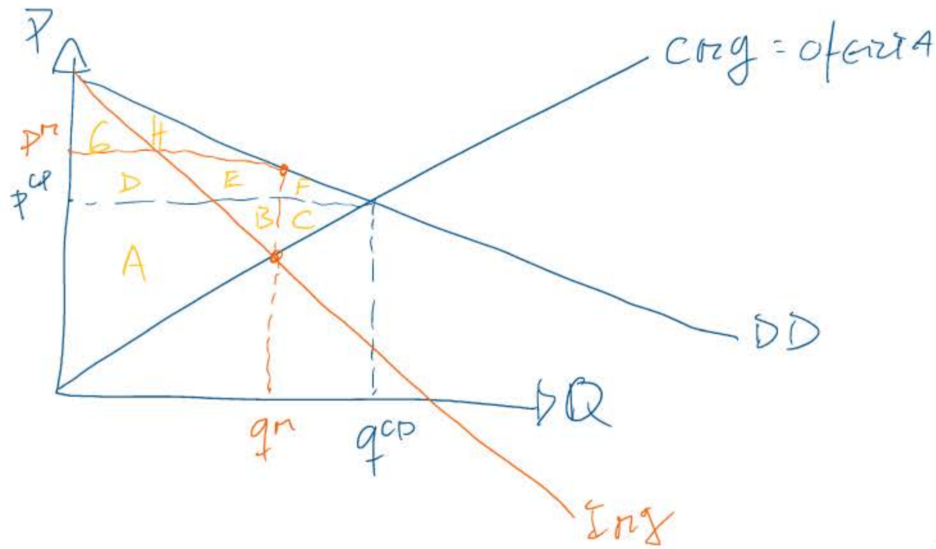
- A monopoly firm always produces at a point where demand is elastic
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits





$EC^{cp} = G + H + D + E + F$

$EP^{cp} = A + B + C$

$EC^{m} = G + H$

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$PBS = F + C$

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- The amount produced q is below the quantity where $p = MC$

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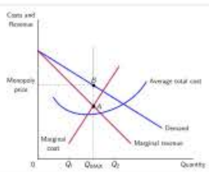
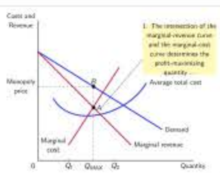
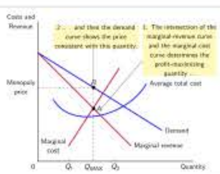
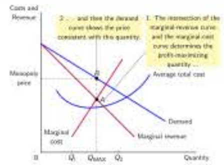
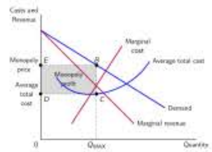
- Both consumer surplus and total surplus is less than is socially optimal
- Thus the pricing policies used by monopolies are inefficient, leading to what is called "dead-weight loss"

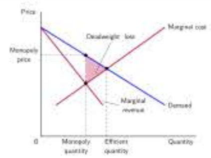












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- $$p = \frac{c}{1 + \frac{1}{\epsilon}} \Longrightarrow q(p) = A\left(\frac{c}{1 + \frac{1}{\epsilon}}\right)^{\epsilon}$$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply