



Lecture 8-9: Price Discrimination
Mauricio Romero

Lecture 8-9: Price Discrimination

- Introduction
- First Degree Price Discrimination
- Third Degree Price Discrimination
- Monopsony
- Double Marginalization Problem
- Profit Sharing and Double Marginalization

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- ▶ In real life, firms often have different prices for different consumers/units
- ▶ We will explore some of these now
- ▶ In a competitive market such exotic pricing schemes could never arise since $p = \text{marginal cost}$

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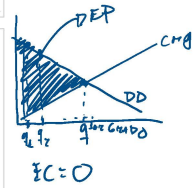
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- First Degree Price Discrimination
- Third Degree Price Discrimination
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- ▶ Suppose the firm can observe all characteristics of the consumer
- ▶ What should the firm do?

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- ▶ Demand curve illustrates the willingness to pay for the q -th unit of the product

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- Suppose the firm can observe all characteristics of the consumer
- What should the firm do?
- Demand curve illustrates the willingness to pay for the q-th unit of the product
- Firm can extract all of the surplus of the consumer. How?

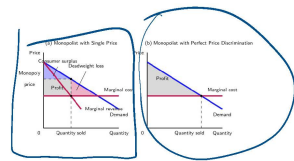
Firm will price at $p(q)$ for the q-th unit and continue to produce until $p(q) = MC(q)$

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Firm gets all of the consumer surplus as his profits:

$$\Pi = \int_0^{q^*} (p(q) - c(q))dq = \int_0^{q^*} p(q)dq - c(q^*)$$

where q^* is the quantity at which $p(q^*) = c(q^*)$.



- Firm can do this because it knows the exact demand curve of each consumer
- Such activity is prohibited in many countries

- Firm can do this because it knows the exact demand curve of each consumer
- Such activity is prohibited in many countries
- Amazon tries to estimate everyone's demand curve

Lecture 9-9: Price Discrimination

- Introduction
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- Third Degree Price Discrimination
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- Double Marginalization Problem
- Profit Shaving and Double Marginalization

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- Introduction
- First Degree Price Discrimination
- Third Degree Price Discrimination
- Monopsony
- Double Marginalization Problem
- Profit Shaving and Double Marginalization

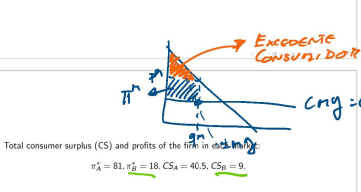
- Market is segmented (no re-selling across markets)
- Firm knows the characteristics of each market (demand curve)
- Consider the following example: Two kinds of consumers:

$$q_A(p_A) = 24 - p_A$$

$$q_B(p_B) = 24 - 2p_B$$
- constant marginal cost of production of 6

If the firm were allowed to set different prices in the different markets, then he would choose:

$$\max_{p_A} (24 - p_A)(p_A - 6) \Rightarrow p_A^* = 15$$

$$\max_{p_B} (24 - 2p_B)(p_B - 6) \Rightarrow p_B^* = 9$$


Firm chose to set the same price in each market. Then he would maximize the following:

$$\max_{p} \{ \max_{p_A} (24 - p_A)(p_A - 6), \max_{p_B} (24 - 2p_B)(p_B - 6) \}$$

The firm chooses $p = 11$.

- Price of $p = 11$ in both markets, which leads to only consumers in market A buying
- To summarize, the consumer surplus and profits in each market are:
 $\pi_A^* = 81, \pi_B^* = 0, CS_A = 40.5, CS_B = 0$
- Prohibiting third degree price discrimination can exclude a whole market altogether
- Highly inefficient compared to the social welfare outcome given third degree price discrimination

- Suppose that the constant marginal cost of production is now 4 instead of 6
- With third degree price discrimination, the firm sets the following prices:
 $\max_{p_A} (24 - p_A)(p_A - 4) \Rightarrow p_A^* = 14$
 $\max_{p_B} (24 - 2p_B)(p_B - 4) \Rightarrow p_B^* = 8$
- In this case, the profits and consumer surplus in each market is given by:
 $\pi_A^* = 100, \pi_B^* = 32, CS_A = 50, CS_B = 16, TS = 198$

If the firm were prohibited from using third degree price discrimination, then:

$$\max_{p} \{ \max_{p_A} (24 - p_A)(p_A - 4), \max_{p_B} (24 - 2p_B)(p_B - 4) \}$$

The firm chooses $p = 10$.

profits in both markets and the consumer surplus in both markets:
 $\pi_A^* = 84, \pi_B^* = 24, CS_A = 98, CS_B = 4, TS = 210$

- Consumers in region B are hurt but consumers in region A gain significantly leading to an increase in consumer surplus
- The firm's joint profits are hurt but the total surplus actually increases
- Total surplus decreases

- Third degree price discrimination is considered illegal in many countries and the European Union

$$\pi_A = (24 - p)(p - 6)$$

$$\frac{\partial \pi}{\partial p} = (-1)(p - 6) + (24 - p)(1)$$

$$= 6 - p + 24 - p = 0$$

$$= 30 - 2p = 0$$

$$\Rightarrow 15 = p^* \rightarrow q_A^* = 24 - p = 9$$

$$q_B^* = 9$$

$$\pi^* = (24 - 15)(15 - 6) = 9 \cdot 9 = 81$$

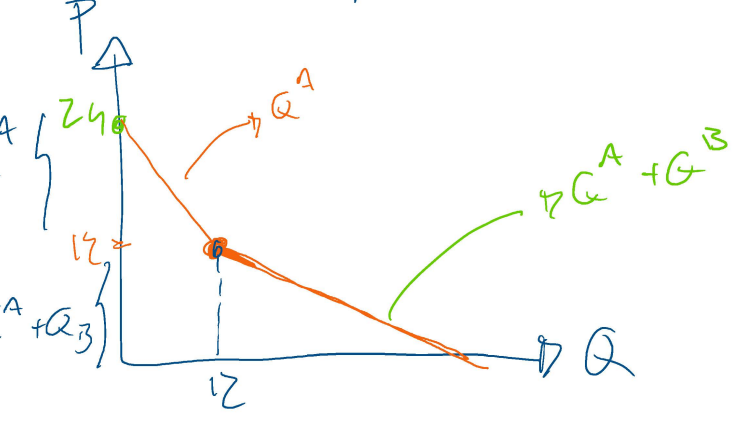
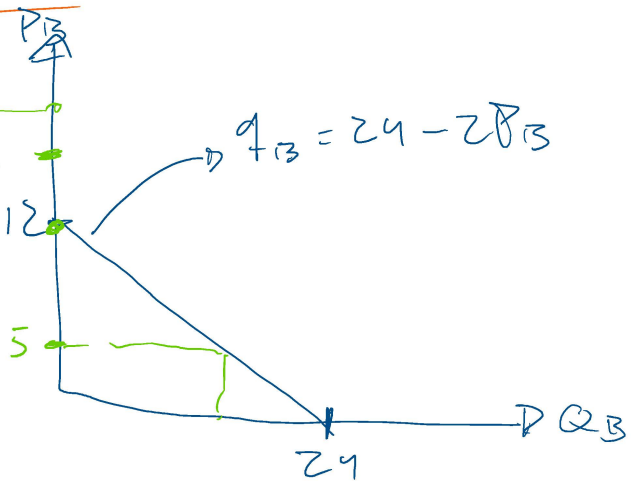
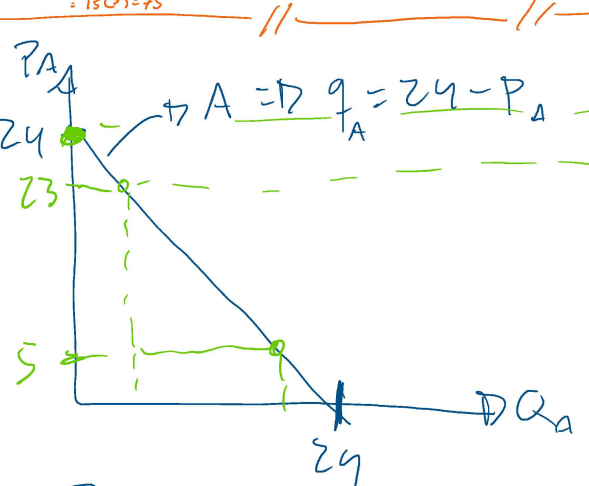
$$\pi = (24 - p)(p - 6)$$

$$\frac{\partial \pi}{\partial p} = (-1)(p - 6) + (24 - p)(1)$$

$$= 6 - p + 24 - p = 0$$

$$= 30 - 2p = 0$$

$$\Rightarrow p = 11 \rightarrow \pi = (24 - 11)(11 - 6) = 13 \cdot 5 = 65$$



$$Q_A = 24 - p$$

$$Q_B = 24 - 2p$$

$$Q_A = \begin{cases} 24 - p & \text{si } p \leq 24 \\ 0 & \text{si } p > 24 \end{cases}$$

$$Q_B = \begin{cases} 24 - 2p & \text{si } p \leq 12 \\ 0 & \text{si } p > 12 \end{cases}$$

$$Q_{total} = Q_A(p=13) + Q_B(p=13)$$

Third degree price discrimination is considered illegal in many countries and the European Union

It is possible to get around such allegations by claiming that the differential pricing comes from cost reasons

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When someone or some firm is the sole buyer (monopsony is the sole seller)

Often arises in the context of firms being the sole buyers of labor

Let us study the profit maximization problem of a firm

$\max_{K, L} P(K, L) - rK - wL$

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w is now a function of the amount of labor demanded (reflecting the power of the firm in the labor market)

The first order condition yields:

$\frac{\partial P}{\partial L}(K^*, L^*) = w'(L^*)L^* + w(L^*) \Rightarrow pMPL = L^*w' + w$

In a competitive market $w' = 0$ and so $pMPL = w$

Wages and labor below the competitive level (an argument for minimum wages and union)

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Third Degree Price Discrimination

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12

$$\pi = P(Q) \cdot Q - C \cdot Q$$

~~$Q_T = Q_A + Q_B = 48 - 3P$~~

~~$= 48 - 3(13)$~~

~~$= 48 - 39$~~

~~$= 9$~~

$$Q_T(P=13) = Q_A(P=13) + Q_B(P=13)$$

$$= 11 + 0$$

$$Q_T = Q_A + Q_B = \begin{cases} 48 - 3P & P \leq 12 \\ 24 - P & P > 12 \end{cases}$$

$P \leq 12$

$P > 24$

$PE(12, 24)$

$\pi = P(f(L)) - rK - w(L)L$

$\frac{\partial \pi}{\partial L} = P \frac{\partial f(L)}{\partial L} - w'(L)L - w(L) = 0$

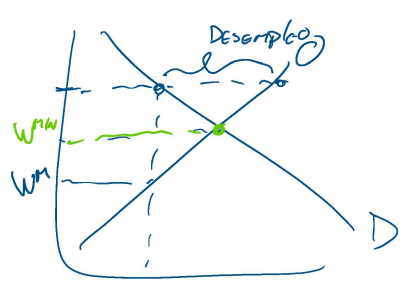
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$\Rightarrow w(L) = 0 \Rightarrow \frac{\partial f(L)}{\partial L} = \frac{w}{P}$

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$\frac{\partial f(L)}{\partial L} = \frac{w'(L)L + w(L)}{P}$

$\left(\frac{w}{P}\right)^M < \left(\frac{w}{P}\right)^{CP}$



► What happens when there are multiple monopolies involved in the market?

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$$C(q_b) = (p_a + c)q_b$$

► What happens when there are multiple monopolies involved in the market?

► Firm A produces factor a at no cost

► Firm b in order to supply q_b units of b must buy q_a units of a → $q_a = q_b$

► Firm B produces according to a cost function:

$$C(q_b) = (p_a + c)q_b$$

► Demand equation for good b is linear:

$$q_b(p_b) = 100 - p_b$$

► Firm B 's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - p_a q_b - c q_b$$

$$\rightarrow \Pi_b = p_b q_b - (p_a + c)q_b$$

$$\Pi_b = (100 - q_b)q_b - (p_a + c)q_b$$

$$\frac{\partial \Pi_b}{\partial q_b} = 100 - 2q_b - p_a - c = 0$$

► Firm B 's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - p_a q_b - c q_b$$

► The first order condition tells us:

$$100 - 2q_b - p_a - c = 0 \Rightarrow p_a = 100 - 2q_b - c$$

► Firm B 's optimization problem becomes:

$$\max_{q_b} (100 - q_b)q_b - p_a q_b - c q_b$$

► The first order condition tells us:

$$100 - 2q_b - p_a - c = 0 \Rightarrow p_a = 100 - 2q_b - c$$

$$\frac{100 - p_a - c}{2} = q_b$$

$$\frac{100 - p_a - c}{2} = q_b^{\text{DEMAND}}$$

$$\Pi_a = p_a q_a - \text{Costos} \rightarrow 0$$

$$\Pi_a = \left(\frac{100 - p_a - c}{2} \right) (p_a)$$

$$\frac{\partial \Pi_a}{\partial p_a} = \frac{100 - 2p_a - c}{2} = 0$$

Firm B's optimization problem becomes:
 $\max_{q_b} (100 - q_b)q_b - p_a q_b - c q_b$

The first order condition tells us:
 $100 - 2q_b - p_a - c = 0 \Rightarrow p_a = 100 - 2q_b - c$

Since firm B is the only demander of commodity a, we have:
 $p_a = 100 - 2q_b - c = 100 - 2q_a - c$

If the price is p_a then the q_a that solves the above equation would be the amount demanded of good a

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Thus firm B's maximization problem has given us an inverse demand function for commodity a

Since firm A is also a monopolist in producing good a, we can solve firm A's maximization problem in the following way:
 $\max_{q_a} q_a(100 - 2q_a - c)$

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As a result, we get:
 $100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, p_a^* = 50 - \frac{c}{2}$

Since firm A is also a monopolist in producing good a, we can solve firm A's maximization problem in the following way:
 $\max_{q_a} q_a(100 - 2q_a - c)$

As a result, we get:
 $100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, p_a^* = 50 - \frac{c}{2}$

Firm a decides to supply the above units of a at a price $50 - c/2$

Firm B will produce $q_b^* = q_a^* = \frac{100 - c}{4}$

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Then the price is given by:
 $p_b^* = 100 - \frac{100 - c}{4} = 75 + \frac{c}{4}$

Firm B will produce $q_b^* = q_a^* = \frac{100 - c}{4}$

Then the price is given by:
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To summarize, we have:

$p_a^* = 50 - \frac{c}{2}$	(1)
$q_a^* = \frac{100 - c}{4}$	(2)
$p_b^* = 75 + \frac{c}{4}$	(3)

Handwritten notes:

$$\frac{\partial \Pi_a}{\partial p_a} = \frac{100 - 2p_a - c}{2} = 0$$

$$\frac{100 - c}{2} = p_a$$

$$q_a = 100 - \frac{(100 - c) - c}{2}$$

$$q_a = \frac{100 - c}{4}$$

$$q_b = \frac{100 - c}{4}$$

$$p_b = 100 - \left(\frac{100 - c}{4}\right)$$

$$p_b = 75 + \frac{c}{4}$$

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$$\Pi_T = \Pi_A + \Pi_B$$

$$= p_a q_a + p_b q_b - (p_a + c) q_b$$

Handwritten notes:

$$p_b = 100 - q_b$$

$P_b = 100 - \frac{1}{4}Q_b = 100 - \frac{1}{4}Q_b$

To summarize, we have:

$$P_a^c = 50 - \frac{c}{2} \quad (1)$$

$$Q_a^c = \frac{100-c}{2} \quad (2)$$

$$P_b^c = 75 + \frac{c}{4} \quad (3)$$

$$Q_b^c = \frac{100-c}{4} \quad (4)$$

► **Case 1: $c = 0$**

$P_a^c = 50, Q_a^c = 25, P_b^c = 75, Q_b^c = 25.$

► If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?

► The monopolists problem becomes:

$$\max_q q(100 - q).$$

► The first order condition states that:

$$100 - 2q^* = 0 \implies q^* = 50, p^* = 50.$$

► Price of good b comes down from 75 to 50

► Production of good b goes up from 25 to 50

► This increases both the profits of the firm and the consumer surplus!

► **Case 1: $c = 10$**

$P_a^c = 45, Q_a^c = 22.5, P_b^c = 77.5, Q_b^c = 22.5.$

► If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?

► The monopolists problem becomes:

$$\max_q q(100 - q) - 10q$$

► The first order condition states that:

$$100 - 2q = 10 \implies q^* = 55, p^* = 45.$$

► This increases both the profits of the firm and the consumer surplus!

► What is going on in the above examples?

► Because the first firm is a monopolist, it charges a mark up above marginal cost for its intermediate good.

► This then distorts the marginal cost of firm B up additionally.

► This then leads an even larger mark up on top of this additional marginal cost that affects the price of good b .

► Essentially a markup on product a indirectly leads to an even larger markup on the final product b .

► This is called the **double marginalization problem**.

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► Double marginalization can lead to inefficiently high prices and inefficiently low levels of production.

► By merging, both profits of the firm and consumer surplus may simultaneously go up.

► Difficult to tell if two firms are merging to solve a double marginalization problem or if they are simply merging to create a monopoly.

► What are some potential ways to solve this problem without mergers?

► One possible way might be to engage in profit sharing.

► Firms agree to share profits according to the following rule

► Prices charged for good a are zero

► In exchange, the profits of firm B are shared via a split of α going to firm A and $(1 - \alpha)$ going to firm B .

$$\Pi_T = \Pi_A + \Pi_B$$

$$= P_a Q_a + P_b Q_b - (P_a + c) Q_b$$

$$Q_a = Q_b$$

$$= P_a Q_b + P_b Q_b - P_a Q_b - c Q_b$$

$$= P_b Q_b - c Q_b$$

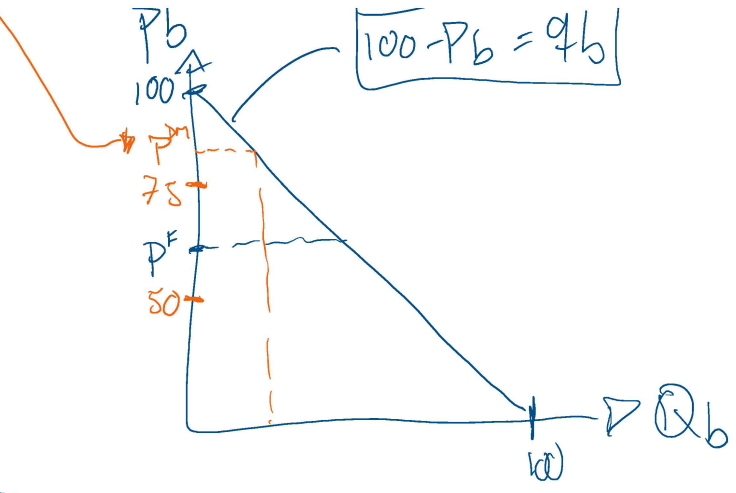
$$\Pi^F = (100 - Q_b) Q_b - c Q_b$$

$$\frac{\partial \Pi^F}{\partial Q_b} = 100 - 2Q_b - c = 0$$

$$\frac{100 - c}{2} = Q_b = Q_a$$

$$P_b = 100 - \left(\frac{100 - c}{2}\right)$$

$$P_b = 50 + \frac{c}{2}$$



- Firms agree to share profits according to the following rule
- Prices charged for good a are zero
- In exchange, the profits of firm B are shared via a split of π going to firm A and $(1-\alpha)$ going to firm B .
- Firm A 's decision is trivial. He simply produces $q_a = q_b$
- Firm B chooses to maximize:

$$\max_q (1-\alpha)((100-q)q - cq) = (1-\alpha) (\max_q (100-q)q - cq)$$
- Term inside the parentheses is just the monopoly profits if the two firms merged:

$$(1-\alpha) \max_q \pi^m(q)$$

$$\pi_b = (1-\alpha) [q_b \bar{p} - (2c)q_b]$$

$$(1-\alpha) [q_b(100-q_b) - cq_b]$$

$$\frac{\partial \pi_b}{\partial q_b} = (1-\alpha) [100 - 2q_b - c] = 0$$

$$\frac{100-c}{2} = q_b$$

The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement

The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement

The price will be the monopoly price

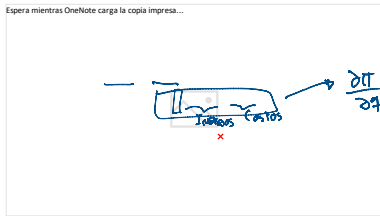


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$$\alpha \pi_b > \pi_a$$

$$(1-\alpha) \pi_b > \pi_b$$





$$50 - q - c = 0$$
$$\boxed{50 - c = q}$$



$$\boxed{B}$$
$$\alpha P_b q_b - c q_b \geq \pi_b^{\text{aprox}}$$

$$\boxed{A}$$
$$(1 - \alpha) P_b q_b \geq \pi_b^{\text{aprox}}$$