Lecture14

Tuesday, April 11, 2023 11:24 AM

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Lecture 14: Game Theory // Nash equilibrium

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Lecture 14: Game Theory // Nash equilibrium

Mixed strategies

Examples

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Mixed strategies

Consider rock/paper/scissors

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

- This game is entirely stochastic (ability has nothing to do with your chances of winning)

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- The probability of winning with every strategy is the same
- Thus, people *tend* choose randomly which of the three options to play
- We would like the concept of Nash equilibrium to reflect this

Mixed strategies

Definition
A mixed strategy σ_i is a function $\sigma_i : S_i \to [0,1]$ such that

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1.$$

- $\sigma_i(s_i)$ represents the probability with which player i plays s_i

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- A **pure strategy** is simply a mixed strategy σ_i that plays some strategy $s_i \in S_i$ with probability one
- We will denote the set of all mixed strategies of player i by Σ_i

$S_i = \text{PAPEL}$

$\sigma_i(\text{PAPEL}) = 1$

Mixed strategies

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- The expected utility of playing "rock" is

$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

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- For instance, assume my opponent is playing randomizing over paper and scissors with probability $\frac{1}{2}$ (i.e., $\sigma_{-i} = (0, \frac{1}{2}, \frac{1}{2})$)
- The expected utility of playing "rock" is

$$E(U_i(rock, \sigma_{-i})) = -1\frac{1}{2} + 1\frac{1}{2} = 0$$

- If I'm randomizing over rock and scissors (i.e., $\sigma_i = (\frac{1}{2}, 0, \frac{1}{2})$) then

$$E(U_i(\sigma, \sigma_{-i})) = \underbrace{-1\frac{1}{4}}_{\text{rock vs paper}} + \underbrace{1\frac{1}{4}}_{\text{rock vs scissors}} + \underbrace{1\frac{1}{4}}_{\text{scissors vs paper}} + \underbrace{0\frac{1}{4}}_{\text{scissors vs scissors}} = \frac{1}{4}$$

Mixed strategies

Definition
A (possibly mixed) strategy profile $(\sigma_1^*, \sigma_2^*, \ldots, \sigma_n)^*$ is a Nash equilibrium if and only if for every i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

for all $\sigma_i \in \Sigma_i$.

S_i

Σ_i

Mixed strategies

Definition (Mixed Strategy Dominance Definition A)
Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all mixed strategies of the opponents, σ_{-i},

$$u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma_i', \sigma_{-i}).$$

Mixed strategies

If σ_i is better than σ_i' no matter what **pure strategy** opponents play, then σ_i is also strictly better than σ_i' no matter what **mixed strategies** opponents play

Theorem
Let σ_i and σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if and only if for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof- Part 1

- Since $S_{-i} \subseteq \Sigma_{-i}$, if σ_i strictly dominates σ_i'

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- Then for all $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

Proof - Part 2

- To prove the other direction, suppose that for all $s_{-i} \in S_{-i}$,

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- For any σ_{-i},

$$\begin{aligned} u_i(\sigma_i, \sigma_{-i}) &= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \sigma_i(s_i)\sigma_{-i}(s_{-i})u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) \sum_{s_i \in S_i} \sigma_i(s_i)u_i(s_i, s_{-i}) \\ &= \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) \end{aligned}$$

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$\boxed{u_i(\sigma_i', \sigma_{-i})}$

$> \sum_{s_{-i}} \sigma_{-i}(s_{-i}) \, u_i(\sigma_i', s_{-i})$

- So

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i, s_{-i}) > \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i})u_i(\sigma_i', s_{-i}) = u_i(\sigma_i', \sigma_{-i})$$

Mixed strategies

Definition (Mixed Strategy Dominance Definition B)
Let σ_i, σ_i' be two mixed strategies of player i. Then σ_i strictly dominates σ_i' if for all pure strategies of the opponents, $s_{-i} \in S_{-i}$,

$$u_i(\sigma_i, s_{-i}) > u_i(\sigma_i', s_{-i}).$$

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Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

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Battle of the sexes

	G	P
G	2,1	0,0
P	0,0	1,2

$\sigma_2 = (q, 1-q)$
$\hookrightarrow q = \sigma_i(G)$
$1-q = \sigma_i(P)$

- There are two pure strategy equilibria (G,G) and (P,P) $\boxed{\sigma_1 = (\lambda, 1-\lambda)}$
- We now look for Nash equilibria that involve randomizationby the players

Battle of the sexes

- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G

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- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$

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- **Case 1:** If $q > 1/3$, then $2q > 2/3 > 1-q$ and therefore, the best response is $\lambda = 1$

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- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$

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- Let λ be the probability with which player 1 chooses G and q be the probability with which player 2 plays G
- $$u_1(\lambda, q) = 2\lambda q + (1-\lambda)(1-q).$$
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- **Case 2:** if $q = 1/3$, then $2q = 2/3 = 1-q$ and therefore, the best response is $\lambda \in [0,1]$
- **Case 3:** If $q < 1/3$, then $2q < 2/3 < 1-q$ and therefore the best response is $\lambda = 0$
- Thus, the best response function is given by:

$$BR_1(q) = \begin{cases} 1 & \text{if } q > 1/3 \\ [0,1] & \text{if } q = 1/3 \\ 0 & \text{if } q < 1/3. \end{cases}$$

Battle of the sexes

Similarly we can calculate the best response function for player 2 and we get:

$$BR_2(\lambda) = \begin{cases} 1 & \text{if } \lambda > 2/3 \\ [0,1] & \text{if } \lambda = 2/3 \\ 0 & \text{if } \lambda < 2/3. \end{cases}$$

MR S1

$\sigma_2 = (q, 1-q)$

$\mathbb{E}(U_1(G, \sigma_2)) = q \cdot 2 + (1-q) \cdot 0 = 2q$

$\mathbb{E}(U_1(P, \sigma_2)) = q \cdot 0 + (1-q) \cdot 1 = 1-q$

$G \succ P$	$P \succ G$	$P \sim G$
$2q > 1-q$	$1-q > 2q$	$1-q = 2q$
$3q > 1$	$1 > 3q$	$q = 1/3$
$\boxed{q > 1/3}$	$q < 1/3$	

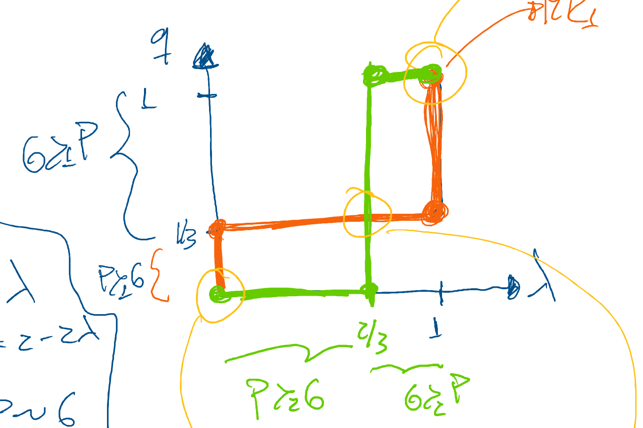
MR S2

$MR_2(\sigma_1 = (\lambda, 1-\lambda))$

$\mathbb{E}(U_2(\sigma_1, G)) = 1 \lambda + 0(1-\lambda) = \lambda$

$\mathbb{E}(U_2(\sigma_1, P)) = 0 \cdot \lambda + 2(1-\lambda) = 2-2\lambda$

$G \succ P$	$P \succ G$	$P \sim G$
$\lambda > 2-2\lambda$	$2-2\lambda > \lambda$	$2-2\lambda = \lambda$
$3\lambda > 2$	$\lambda < 2/3$	$\lambda = 2/3$
$\lambda > 2/3$		

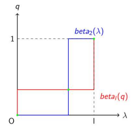


$EN = \left(\sigma_1 = (1,0), \sigma_2 = (1,0) \right)$
$(S_1 = G, S_2 = G)$

$\{ \sigma_1 = (0,1), \sigma_2 = (0,1)$
$S_1 = P, S_2 = P$

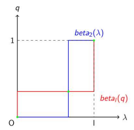
$\{ \sigma_1 = (\frac{2}{3}, \frac{1}{3}), \sigma_2 = (\frac{1}{3}, \frac{2}{3}) \}$

Battle of the sexes



- There are three points where the best response curves cross: $(1,1),(0,0,),(\frac{2}{3},\frac{1}{3})$

Battle of the sexes



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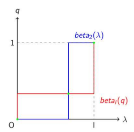

- There are three points where the best response curves cross: $(1,1),(0,0,),(\frac{2}{3},\frac{1}{3})$
- First two are the pure strategy NE we had found before
- Last is a strictly mixed NE: both players randomize

Consider the following game

	E	[illegible]	G
A	5, 10	[illegible]	3, 4
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
D	2, 4	[illegible]	8, 4

$G \succsim F$

$D \succ_1 B$

Buscar $\sigma_1 = (P, \underset{B}{0}, \underset{C}{0}, 1-P) \succ C$

$E(U_1(\sigma_1, E)) > U_1(C, E) \rightarrow 5P + 2(1-P) > 4 \rightarrow 5P + 2 - 2P > 4 \rightarrow 3P > 2 \rightarrow P > 2/3$

$E(U_1(\sigma_1, G)) > U_1(C, G) \rightarrow 3P + 8(1-P) > 3 \rightarrow 3P + 8 - 8P > 3 \rightarrow 5 > 5P \rightarrow P < 1$

$\boxed{P = 0.9}$

$\Rightarrow$ BUSQUEMOS E.N.

$\rightarrow MR_1(\sigma_2 = (q, \overset{F}{0}, 1-q))$

$E(U_1(A, \sigma_2)) = 5 \cdot q + 3(1-q) = 5q + 3 - 3q = 2q + 3$

$E(U_1(D, \sigma_2)) = 2 \cdot q + 8(1-q) = 2q + 8 - 8q = 8 - 6q$

$A \succ D$ | $D \succ A$ | $D \sim A$

$2q + 3 > 8 - 6q$ | $q < 5/8$ | $q = 5/8$

$8q > 5$

$q > 5/8$

$MR_2(\sigma_1 = (P, \underset{B}{0}, \underset{C}{0}, 1-P))$

$E(U_2(\sigma_1, E)) = 10 \cdot P + 4(1-P) = 10P + 4 - 4P = 6P + 4$

$E(U_2(\sigma_1, G)) = 4 \cdot P + 4(1-P) = 4$

$E \succ G$ ✓ | $G \succ E$ | $G \sim E$

MR_2

- Consider $\sigma_1 = (\frac{1}{3},\frac{1}{4},\frac{1}{4},\frac{1}{6})$

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- $\mathbb{E}U(F,\sigma_1) = 3\frac{1}{3} + 2\frac{1}{4} + 4\frac{1}{4} + 3\frac{1}{6} = 3$

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- $\mathbb{E}U(G,\sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$

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- $\mathbb{E}U(G,\sigma_1) = 4\frac{1}{3} + 6\frac{1}{4} + 8\frac{1}{4} + 4\frac{1}{6} = 5.5$
- Then $BR_2(\sigma_1) = \{(p, 0, 1-p), p \in [0,1]\}$

- G dominates F (player 2)

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- D dominates B (player 1)

Reduced game

	E	G
A	5, 10	3, 4
C	4, 2	3, 8
D	2, 4	8, 4

- Note that $\sigma_1 = (p, 0, 1-p)$ with $p > \frac{2}{3}$ dominates C
- $\mathbb{E}U(\sigma_1, E) = 5p + 2(1-p) = 3p + 2$
- $\mathbb{E}U(\sigma_1, G) = 3p + 8(1-p) = 8 - 5p$
-

$$\begin{aligned} \mathbb{E}U(\sigma_1, E) &> U(C, E) \\ 3p + 2 &> 4 \\ p &> \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbb{E}U(\sigma_1, G) &> \mathbb{E}U(C, G) \\ 8 - 5p &> 3 \\ p &< \frac{5}{5} = 1 \end{aligned}$$

Reduced game

	E	G
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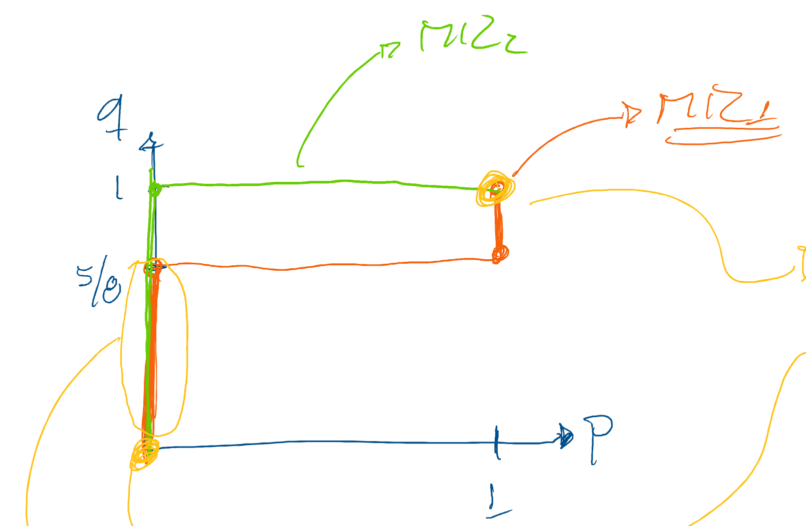
- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$

$E \succ G$ ✓
$6p + 4 > 4$
$6p > 0$
$p > 0$

$G \succ E$
$\boxed{p < 0}$
NUNCA

$G \sim E$
$\underline{p = 0}$



$\quad\ (A \qquad\qquad\qquad\qquad E)$
$EN = \left(\sigma_1 = (1,0,0,0),\ \sigma_2 = (1,0,0)\right)$

$\left(\sigma_1 = (0,0,0,1),\ \sigma_2 = (0,0,1)\right)$
$\quad\ (D \qquad\qquad\qquad G)$

$\sigma_1 = (0,0,0,1),\ \sigma_2 = (q, 0, 1-q)$
con $q \in [0, 5/8]$

- Lets find $BR_1(\sigma_2 = (q, 1-q))$
- $\mathbb{E}U(A, \sigma_2) = 5q + 3(1-q) = 2q + 3$
- $\mathbb{E}U(D, \sigma_2) = 2q + 8(1-q) = 8 - 6q$
- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$

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- $8 - 6q > 2q + 3$ if $\frac{5}{8} > q$
- $8 - 6q < 2q + 3$ if $\frac{5}{8} < q$
- Thus

$$BR_1(q, 1-q) = \begin{cases} \sigma_1 = (0,1) & \text{if } 0 \leq q < \frac{5}{8} \\ \sigma_1 = (1,0) & \text{if } \frac{5}{8} < q \leq 1 \\ \sigma_1 = (p, 1-p) & \text{if } \frac{5}{8} = q \end{cases}$$

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- $6p + 4 > 4$ if $p > 0$

- Lets find $BR_2(\sigma_1 = (p, 1-p))$
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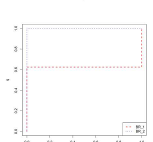
- Thus

$$BR_2(p, 1-p) = \begin{cases} \sigma_2 = (1,0) & \text{if } p > 0 \\ \sigma_2 = (q, 1-q) & \text{if } p = 0 \end{cases}$$

Best responses



$NE = \{(A, E), (D, \sigma_2^q)\}$ where $\sigma_2^q = (q, 1-q)$ and $0 \leq q \leq \frac{5}{8}$