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Lecture 16: Applications of Subgame Perfect Nash Equilibrium

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Ultimatum Game

Alternating offers

Stackelberg Competition

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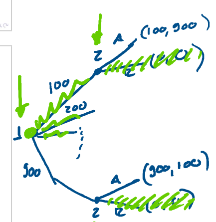
1. Player 1 makes a proposal $(x, 1000 - x)$ of how to split 1000 pesos among $(100, 900), \ldots, (800, 200), (900, 100)$
2. Player 2 accepts or rejects the proposal
3. If player 2 rejects both obtain 0. If 2 accepts, then the payoffs or the two players are determined by $(x, 1000 - x)$



EPS= (900, A)

S1 S2

- In any pure strategy SPNE, player 2 accepts all offers

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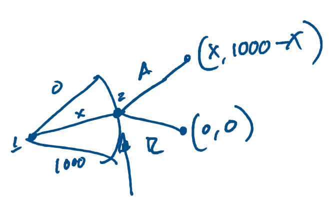
- In any SPNE, player 1 makes the proposal $(900, 100)$



S2

A si $1000 - x \geq 0$

$1000 \geq x$

SIEMPRE A

S1

$x = 1000$

EPS= (1000, A)

- This is far from what happens in reality

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SIEMPRE A

SI
$X = 1000$
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- When extreme offers like $(900, 100)$ are made, player 2 rejects in many cases

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- When extreme offers like $(900, 100)$ are made, player 2 rejects in many cases
- Player 2 may care about inequality or positive utility associated with "punishment" aversion

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- Two players are deciding how to split a pie of size 1

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- The players would rather get an agreement today than tomorrow (i.e., discount factor)

- Player 1 makes an offer θ_1

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- If player 1 accepts or rejects the proposal
- If player 1 rejects, player 1 makes an offer θ_3

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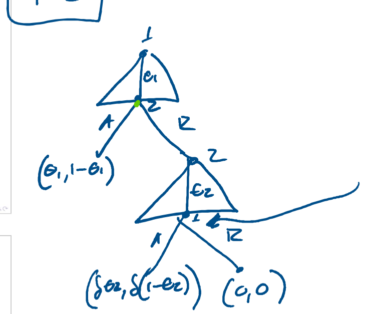
- ... and on and on for T periods

- Player 1 makes an offer θ_1
- Player 2 accepts or rejects the proposal
- If player 2 rejects, player 2 makes an offer θ_2
- If player 1 accepts or rejects the proposal
- If player 1 rejects, player 1 makes an offer θ_3
- ... and on and on for T periods
- If no offer is ever accepted, both payoffs equal zero

$\boxed{T=2}$



$\underline{\underline{S1}}$

A si $\delta\theta_2 \geq 0$

$\theta_2 \geq 0$

$\underline{\underline{S2}}$

PAGOS

The discount factor is $\delta \leq 1$.
If Player 1 offer is accepted by Player 2 in round m,

$$\pi_1 = \delta^m \theta_m,$$

$$\pi_2 = \delta^m (1-\theta_m).$$

δ = FACTOR DESCUENTO

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If Player 1 offer is accepted by Player 2 in round m,

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If Player 2 offer is accepted, reverse the subscripts

- Consider first the game without discounting

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- There is a unique SPNE: The player that makes the last offer gets the whole pie
- Last-mover advantage

- In the game with discounting, the total value of the pie is 1 in the first period, δ in the second, and so forth

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- In period T, if it is reached, Player 1 would offer 0 to Player 2

$(\delta\theta_2, \delta(1-\theta_2))$ $(0,0)$

δ = FACTOR DESCUENTO

$\delta \in [0,1]$

S2

$\theta_2 = 0$ → PAGOS $(0, \delta)$

S2

A si $1-\theta_1 \geq \delta$

$1-\delta \geq \theta_1$

$\theta_1 \leq 1-\delta$

S1

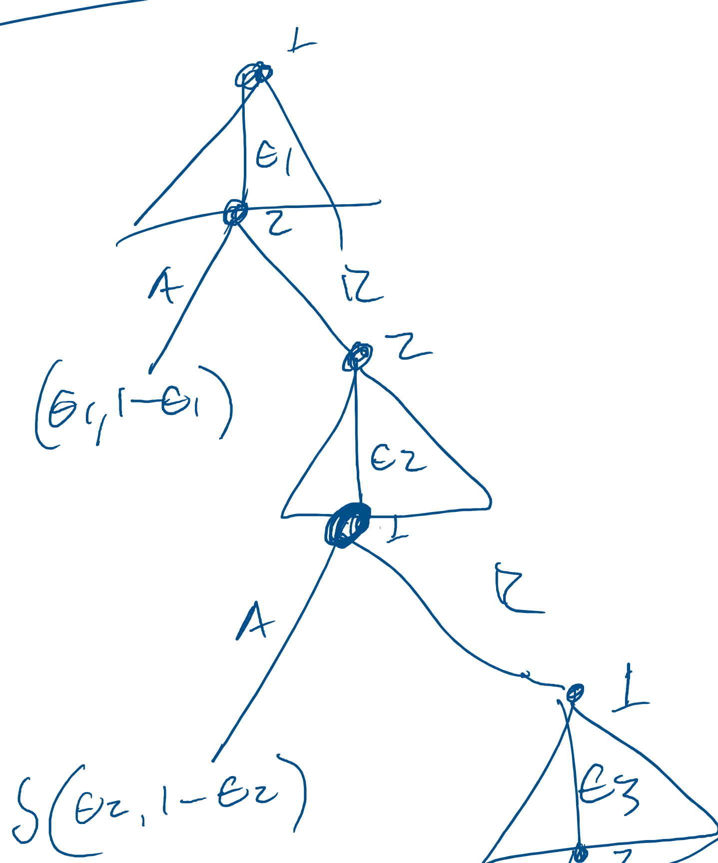
$\theta_1 = 1-\delta$

⇒ LLEGAMOS A UN TRATO $T = 1$

$\theta_1 = 1-\delta$

PAGOS $(1-\delta, \delta)$

$\boxed{T = 3}$



S2

A si $\delta^2 \theta_3 \geq 0$

$\theta_3 \geq 0$

S1

$\theta_3 = 1$

PAGOS $(\delta^2, 0)$

- In period T, if it is reached, Player 1 would offer 0 to Player 2

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- In period $(T-1)$, Player 2 could offer Smith δ, keeping $(1-\delta)$ for himself
- Player 1 would accept (indifferent between accepting and rejecting) since the **whole pie** in the next period is worth δ

- In period $(T-2)$, Player 1 would offer Player 2 $\delta(1-\delta)$, keeping $(1-\delta(1-\delta))$ for himself

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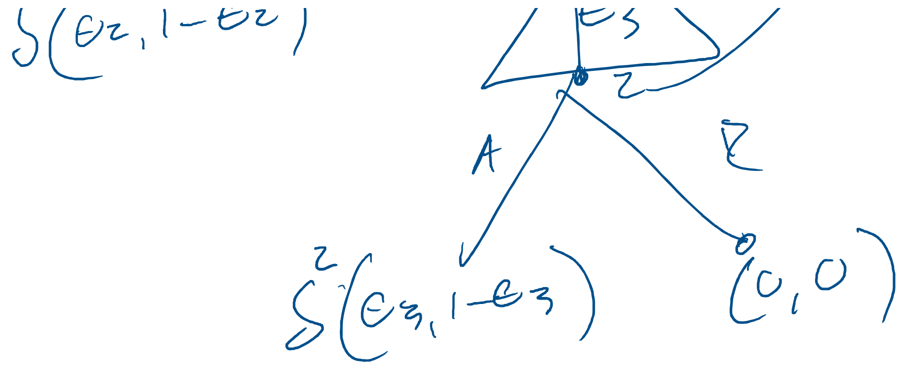

$\underline{S_1}$ A o R

A si $\delta\theta_2 \geq \delta^2$

$\theta_2 \geq \delta$

$\underline{S_2}$ $\underline{\theta_2 = \delta} \rightarrow$ PAGOS $(\delta^2, (1-\delta)\delta)$

$\underline{S_2}$ A o R

A $1-\theta_1 \geq (1-\delta)\delta$

$1-(1-\delta)\delta \geq \theta_1$

$\underline{S_1}$ $\theta_1 = 1-(1-\delta)\delta$

$\underline{\text{PAGOS}}$ $\left(1-(1-\delta)\delta, (1-\delta)\delta\right)$

- In period $(T-2)$, Player 1 would offer Player 2 $\delta(1-\delta)$, keeping $(1-\delta(1-\delta))$ for himself
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- In period $(T-3)$, Player 2 would offer Player 1 $\delta[1-\delta(1-\delta)]$, keeping $(1-\delta[1-\delta(1-\delta)])$ for himself
- Player 1 would accept...
- ...
- In equilibrium, the very first offer would be accepted, since it is chosen precisely so that the other player can do no better by waiting

Table 1 shows the progression of Player 1's shares when $\delta = 0.9$.

Table 1: Alternating Offers over Finite Time

Round	1's share	2's share	Total value	Who offers?
$T-3$	$\delta(1-\delta(1-\delta))$	$1-\delta(1-\delta(1-\delta))$	δ^{T-4}	2
$T-2$	$1-\delta(1-\delta)$	$\delta(1-\delta)$	δ^{T-3}	1
$T-1$	δ	$1-\delta$	δ^{T-2}	2
T	1	0	δ^{T-1}	1

- If $T=3$ (i.e, 1 offers, 2 offers, 1 offers)

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- One offers $\delta(1-\delta)$, 2 accepts in period 1

- Player 1 always does a little better when he makes the offer than when Player 2 does

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- If we consider just the class of periods in which Player 1 makes the offer, Player 1's share falls

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- Suppose instead that the firms move in sequence which is called a **Stackelberg competition game**
- Suppose that the inverse demand function is given by:

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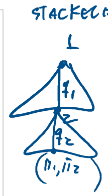
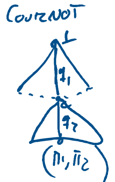
- Firms have the cost functions $c_i(q_i)$.

STACKELBERG



COURNOT



Te timing of the game is given by:

1. First Firm 1 chooses $q_1 \geq 0$
2. Second Firm 2 observes the chosen q_1 and then chooses q_2

- The game tree in this game is then depicted by an infinite tree

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- A pure strategy for firm 1 is just a choice of $q_1 \geq 0$
- A strategy for firm 2 specifies what it does after every choice of q_1
- Firm 2's strategy is a function $q_2(q_1)$ which specifies exactly what firm 2 does if q_1 is the chosen strategy of player 1

ESTRATEGIAS $\left\{ \begin{array}{l} S_1 = q_1 \in \mathbb{R}^+ \\ S_2 = \{ \text{FUNCIONES: } q_1 \to q_2 \} \\ \qquad\qquad\quad \mathbb{R}^+ \to \mathbb{R}^+ \end{array} \right.$

$q_1(q_2)$

The utility functions for firm i when firm 1 chooses q_1 and firm 2 chooses the strategy (or function) $q_2(\cdot)$ is given by:

$$\pi_1(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_1 - c_1(q_1)$$
$$\pi_2(q_1, q_2(\cdot)) = P(q_1 + q_2(q_1))q_2(q_1) - c_2(q_2(q_1))$$

$\boxed{\text{FUNCIONES PAGO}}$

$\pi_1 = P(q_1 + q_2(q_1))q_1 - C_1(q_1)$

$\pi_2 = P(q_1 + q_2(q_1))q_2(q_1) - C_2(q_2(q_1))$

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- Consider the following specific game with demand function given by:

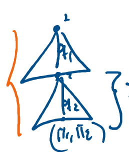
$$\boxed{P(q_1 + q_2) = A - q_1 - q_2.}$$

- Let the marginal costs of both firms be zero
- Then the normal form simplifies:

$$u_1(q_1, q_2(\cdot)) = (A - q_1 - q_2(q_1))q_1,$$
$$u_2(q_1, q_2(\cdot)) = (A - q_1 - q_2(q_1))q_2(q_1).$$

EPS

$\boxed{T=1}$



$T=2$

$$\Pi_2 = (A - q_1 - q_2)q_2$$

$$\frac{\partial \Pi_2}{\partial q_2} = A - q_1 - 2q_2 = 0$$

$$\boxed{q_2(q_1) = \frac{A - q_1}{2}}$$

① $\Pi_1 = (A - q_1 - q_2)q_1$

$$\Pi_1 = \left(A - q_1 - \left(\frac{A - q_1}{2}\right)\right) q_1$$

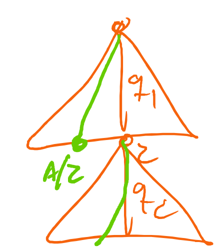
$$\Pi_1 = \left(\frac{A - q_1}{2}\right) q_1 = \left(\frac{Aq_1 - q_1^2}{2}\right)$$

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{A}{2} - q_1 = 0$$

$$\boxed{q_1^* = \frac{A}{2}}$$

$$EPS = \left(q_1^* = \frac{A}{2},\ q_2(q_1) = \frac{A - q_1}{2}\right)$$

"SENDA DEL EQ"



$$q_1 = A/2$$
$$q_2 = A/4$$
$$\Pi_1 = (A - A/2 - A/4)A/2$$
$$\Pi_2 = (A - A/2 - A/4)A/4$$

- What is an example of a Nash equilibrium of this game?

X $$q_1 = \frac{A}{2},\ q_2 = \frac{A - \frac{A}{2}}{2} = \frac{A}{4}$$

- What is an example of a Nash equilibrium of this game?
- Let $\alpha \in [0, A)$ and consider the following strategy profile:

$$q_1^* = \alpha,\ q_2^*(q_1) = \begin{cases} A & \text{if } q_1 \neq \alpha, \\ \frac{A-\alpha}{2} & \text{if } q_1 = \alpha. \end{cases}$$

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- Let us check that indeed this constitutes a Nash equilibrium

$$\Pi_1 = (A - q_1 - q_2)q_1$$

$$\left(A - q_1 - \frac{A}{4}\right) q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = A - 2q_1 - \frac{A}{4} = 0$$

$$\frac{3A}{4} = 2q_1$$

- First we check the best response of player 1

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- If player 2 plays q_2^*, then player 1's utility function is given by:

$$u_1(q_1, q_2^*(\cdot)) = \begin{cases} \left(A - \alpha - \left(\frac{A-\alpha}{2}\right)\right)\alpha > 0 & \text{if } q_1 = \alpha \\ -q_1^2 \leq 0 & \text{if } q_1 \neq \alpha. \end{cases}$$

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- Thus,

$$\max_{q_1 \geq 0} u_1(q_1, q_2^*(\cdot))$$

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- Firm 1 is best responding to player 2's strategy.

- Suppose that firm 1 plays the strategy q_1^*. Is firm 2 best responding?

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- Firm 2's utility function is given by:

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- Thus, firm 2 wants to choose the optimal strategy $q_2(\cdot)$ that maximizes the following utility:

$$\max_{q_2(\cdot)} (A - \alpha - q_2(\alpha))q_2(\alpha)$$

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$$q_2(\alpha) = \frac{A - \alpha}{2}.$$

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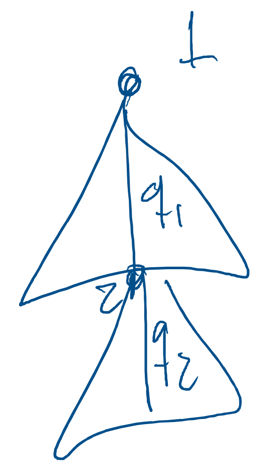
$$\frac{3A}{4} = 2q_1$$

$$\boxed{\frac{3A}{8} = q_1}$$

¿HAY EN G' NO SON EPS?



$$P = A - q_1 - q_2$$

$$\boxed{S_2(q_1) = q_2(q_1) = \begin{cases} A & \text{si } q_1 > 0 \\ \frac{A}{2} = q^m & \text{si } q_1 = 0 \end{cases}}$$

$$\pi = (A - Q)Q$$

$$\frac{\partial \pi}{\partial Q} = A - 2Q = 0 \Rightarrow Q = \frac{A}{2}$$

$$¿EN? \left(q_1 = 0,\ q_2(q_1) = \begin{cases} A & \text{si } q_1 > 0 \\ \frac{A}{2} & \text{si } q_1 = 0 \end{cases}\right) \checkmark$$

$\frac{A}{2}$ si $q_1 = 0$

following utility:

$$\max_{q_2(\cdot)}(A - \alpha - q_2(\alpha))q_2(\alpha)$$

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- Suppose that firm 1 plays the strategy q_1^*. Is firm 2 best responding?
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- The utility function of firm 2 does not depend at all on what it chooses for $q_2^*(q_1)$ when $q_1 \neq \alpha$
- In particular, q_2^* is a best response for firm 2

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- The equilibrium outcome of the above Nash equilibrium above is that firm 1 sets the price α and firm 2 sets the price $(A-\alpha)/2$.
- In particular, in the Nash equilibrium corresponding to $\alpha = 0$, the equilibrium outcome is for firm 1 to choose a quantity of 0 and firm 2 setting a price of $A/2$
- This would be the same outcome if firm 2 were the monopolist in this market

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- As a result, the best that firm 1 can do is to produce nothing
- If firm 1 were to hypothetically choose $q_1 > 0$, then firm 2 would obtain negative profits if it indeed follows through with $q_2^*(q_1)$.

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- Lets continue with the setting in which marginal costs are zero and the demand function is given by $A - q_1 - q_2$

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- So, player 2 solves:
$$\max_{q_2(\cdot)} (A - q_1 - q_2(q_1))q_2(q_1).$$

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- In this case, the first order condition implies:

$$q_2^*(q_1) = \frac{A - q_1}{2}.$$

- Thus, in any SPNE, player 2 must play the following strategy:

$$q_2^*(q_1) = \begin{cases} \frac{A-q_1}{2} & \text{if } q_1 \leq A \\ 0 & \text{if } q_1 > A. \end{cases}$$

- Then player 1's utility function given that player 2 plays q_2^* is given by:

$$u_1(q_1, q_2^*(\cdot)) = q_1(A - q_1 - q_2^*(q_1)) = \begin{cases} q_1(A - q_1) & \text{if } q_1 > A, \\ q_1 \frac{A-q_1}{2} & \text{if } q_1 \leq A. \end{cases}$$

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- Firm 1 will never choose $q_1 > A$ since then it obtains negative profits
- Thus, firm 1 maximizes:

$$\max_{q_1 \in [0, A]} q_1 \frac{A - q_1}{2}.$$

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- The **equilibrium outcome** is for firm 1 to choose $A/2$ and firm 2 to choose $A/4$

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- The Cournot game was one in which all firms chose quantities simultaneously
- In that game, since there is only one subgame, SPNE was the same as the set of NE
- Lets solve for the set of SPNE (which is the same as NE) in the Cournot game with the same demand function and same costs
- In this case, (q_1^*, q_2^*) is a NE if and only if

$$q_1^* \in BR_1(q_2^*), q_2^* \in BR_2(q_1^*).$$

- For $q_1^* \in BR_1(q_2^*)$, we need q_1^* to solve the following maximization problem:

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- As a result, solving these two equations, we get:

$$q_1^* = q_2^* = \frac{A}{3}.$$

In the Cournot game, note that firms' payoffs are:

$$\pi_1^c = \frac{A^2}{9}, \pi_2^c = \frac{A^2}{9}.$$

As we already saw, this was not Pareto efficient since each firm is getting a payoff that is strictly less than 1/2 of the monopoly profits.

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- This is intuitive since firm 1 always has the option of choosing the Cournot quantity $q_1 = A/3$, in which case firm 2 will indeed choose $q_2^*(q_1) = A/3$ giving a payoff of $A^2/9$
- But by choosing something optimal, firm 1 will be able to do even better