# Lecture5

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# Lecture5

# Lecture 5: General Equilibrium

Mauricio Romero

### Lecture 5: General Equilibrium

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### Introducing production

General equilibrium with production

Examples

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Lecture 5: General Equilibrium

# Introducing production

Examples

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What about the producers in the economy? There are two cases. 1. There are no producers in the economy: this is what is called a pure exchange commy in which all avaible goods are those coming from endowments from consumers (up until now) 2. There are producers who can produce commodities in the economy (today)

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### Each firm j is characterized by two characteristics: 1. A production function $f_j^{\rm f}$ for producing that good l.

A production hunction f for producing that good J.

The firm j has a production function of the form:  $f_j^j(z_1^{j,l})=f_j(x_1^{j,l},x_2^{j,l},\ldots,x_L^{j,l}).$ 

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The firm j has a production function of the form:  $f_j^i(x^{j,l}) = f_j(x_1^{j,l}, x_2^{j,l}, \dots, x_L^{j,l}).$ 

*z<sup>j,j</sup>* for firm *j* describes the vector of inputs that firm *j* uses in the production of good (*l*)
 In other words, firm *j* uses *z<sup>j,j</sup>* units of commodity *i* to produce commodity *i*

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consumer i For each firm  $j, \theta_{ij} \in [0, 1]$   $\sum_{i=1}^{I} \theta_{ij} = \theta_{1i} + \theta_{2i} + \dots + \theta_{ij} = 1$ 

▶ Firms are somed by consumers in society ▶ We need to describe who ones, which firm ▶ Ownership is time as congenues. — a more relation model might involve consumers dooling which firms to one 1 by all groups the fractions of firm j that is owned by consumer i ▶ For each firm,  $k_0 \in [0, 1]$ ▶  $\sum_{i=1}^{N} k_0 = a_{ij} + b_{ij} + \cdots + b_{ij} = -1$ ▶ An implicit assumption here is that firms do not have any endowments

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# General equilibrium with production

 $\begin{array}{l} & \text{Definition} \\ ((x^*,x^*),\rho=(p_1,\ldots,p_l)) \text{ is a compatitive equilibrium if:} \\ 1. \ \text{For all produces } j=1,2,\ldots,J, \\ & x^{i^*}=((z_1^{i,1^*},\ldots,z_L^{i,1^*}),\ldots,(z_J^{i,1^*},\ldots,z_L^{i,1^*})) \text{ solves.} \end{array}$ 

# $\Pi_{j}^{*} := \max_{x^{j}} p_{t} t_{j}^{t}(x_{t}^{j}, \dots, x_{t}^{j}) - \sum_{t'=1}^{L} p_{t'} x_{t'}^{j}.$

2. For all consumers  $i=1,2,\ldots,l,\,{x^{i}}^{*}=(x_{1}^{i}^{*},\ldots,x_{k}^{i}^{*})$  solves:  $\max_{x^i} u_i(x^i)$ 

such that  $p \cdot x^{i} \leq p \cdot \omega^{i} + \sum_{i=1}^{j} \theta_{ij} \Pi_{j}^{*}$ .

3. Markets clear: For each commodity  $\ell = 1, 2, \dots, L$ :

 $\sum_{i=1}^{l} x_{\ell}^{i^*} + \sum_{j=1}^{J} \sum_{\ell'=1}^{L} x_{\ell}^{j,\ell'} = \sum_{i=1}^{l} \omega_{\ell}^{i} + \sum_{j=1}^{J} f_{j}^{\ell} \left( x_{1}^{j,\ell^*}, \dots, x_{L}^{j,\ell^*} \right).$ 

# We have exactly the same basic properties as in the case of pure exchange eccounts: 1. When utility functions are strictly momentum, and production functions are strictly increasing, prices of each commonly and prices of each input are strictly possible 2. Watra's Law, Each consumer *i* spends all of the income whenever *i* monitors utility 3. Watra's Law, IL if the mather dataling conditions hold for market *L* as well. 4. If $(e^+,e^+,r)$ is a Watrastan equilibrium, and $\alpha > 0$ , $(e^+,e^+,e^-,r)$ as a bas a Watrastan equilibrium. 5. The first and the second waffare theorems hold

# Lecture 5: General Equilibrium

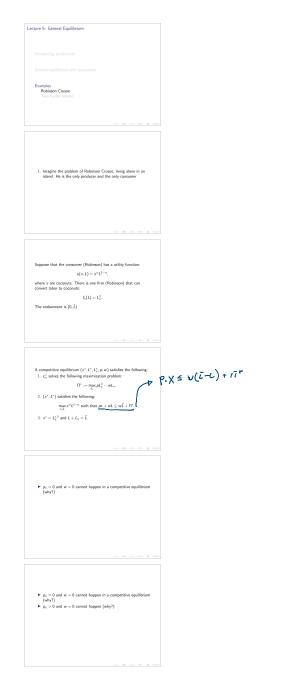
Introducing production

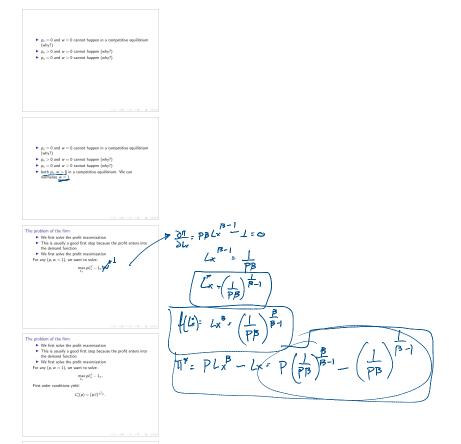
General equilibrium with production

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# Lecture 5: General Equilibrium

Examples





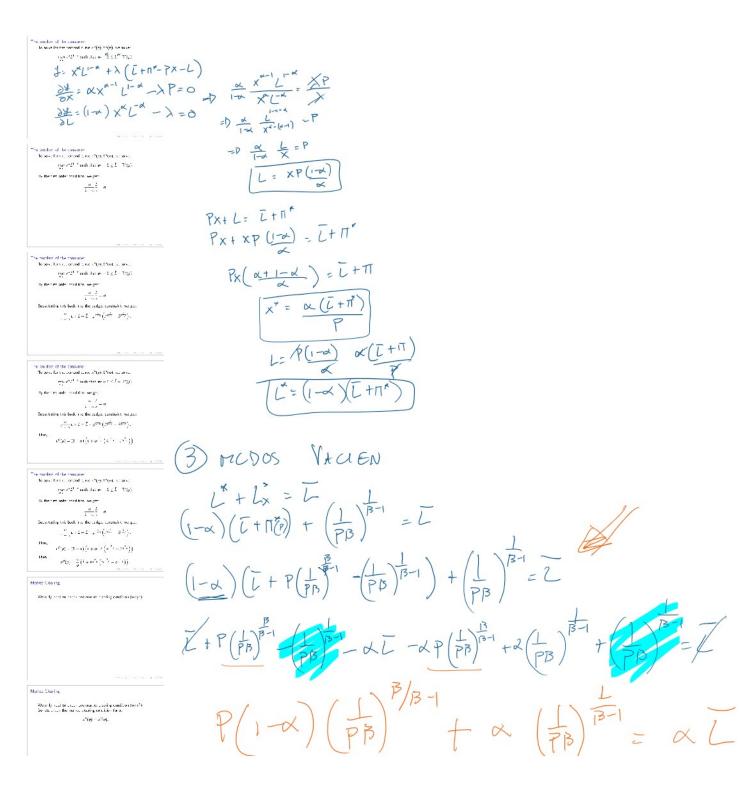
The problem of the firm
 We first solve the profit maximization
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 the demand function
 We first solve the profit maximization
 For any (p, w = 1), we want to solve:  $\max_{\mathbf{L}_{\mathbf{x}}} p L_{\mathbf{x}}^{\beta} - L_{\mathbf{x}}.$ 

First order conditions yield:  $L^*_x(\rho) = (\rho\beta)^{\frac{1}{1-\beta}}$ . Therefore,

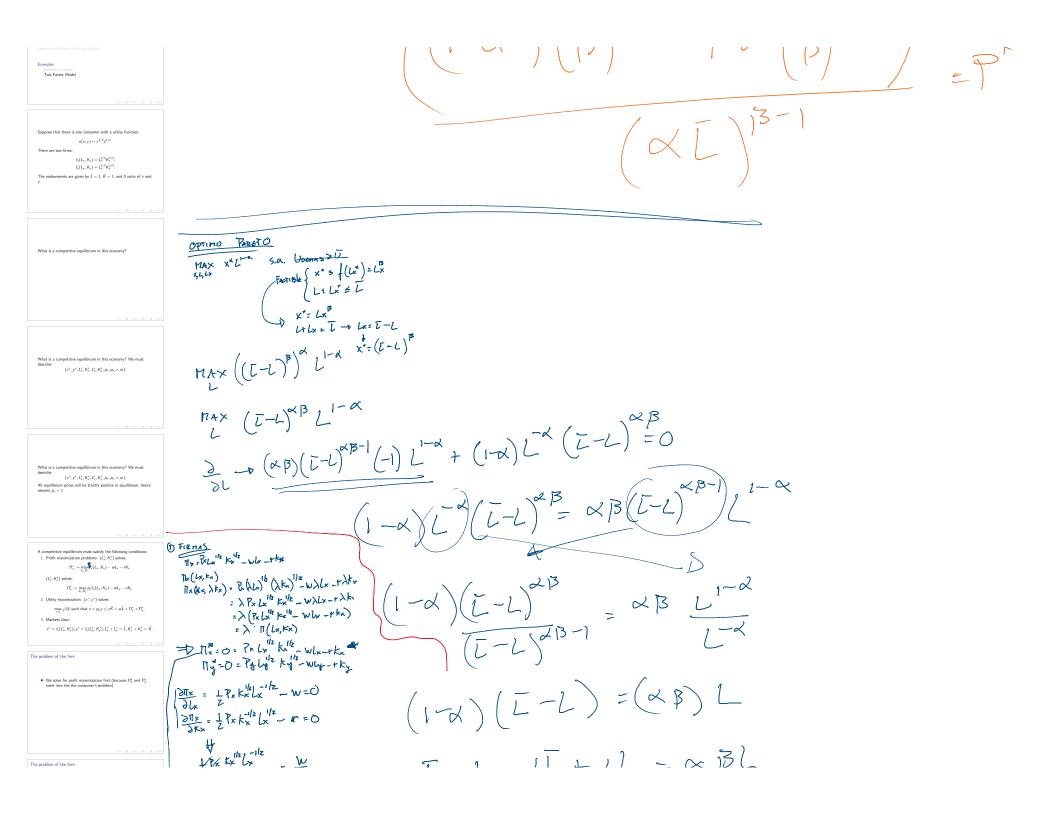
 $\Pi^*(p) = \rho \left( p\beta \right)^{\frac{\beta}{1-\beta}} - \left( p\beta \right)^{\frac{1}{1-\beta}} = p^{\frac{1}{1-\beta}} \left( \beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right).$ 

The problem of the firm The problem of the firm P We first solve the profit maximization P This is usually a proof first tep because the profit enters into the demand function P We first solve the profit maximization For any (p, w = 1), we want to solve:  $\max_{L_{\mathcal{X}}} p L_{\mathcal{X}}^{\beta} - L_{\mathcal{X}}.$ First order conditions yield:  $L_{\chi}^{*}(\rho) = (\rho\beta)^{\frac{1}{1-\beta}}$ . Therefore,  $\Pi^{+}(p) = \rho \left( p\beta \right)^{\frac{\beta}{1-\beta}} - \left( p\beta \right)^{\frac{1}{1-\beta}} = p^{\frac{1}{1-\beta}} \left( \beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right).$ 

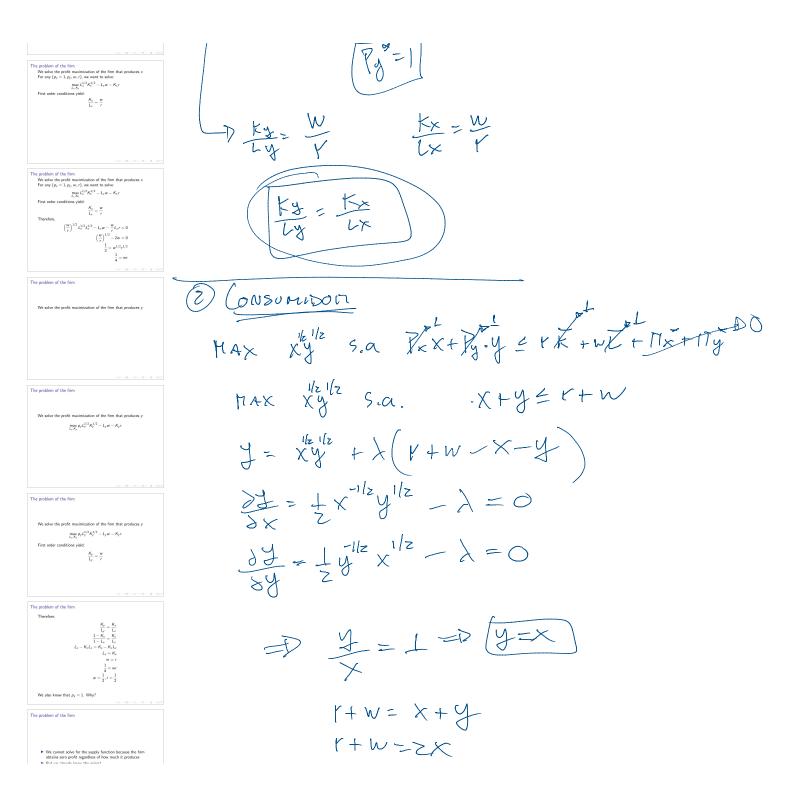
The supply of x is then given by:  $x^{\delta}(p)=L^*_x(p)^{\beta}=(p\beta)^{\frac{\beta}{1-\beta}}$ 

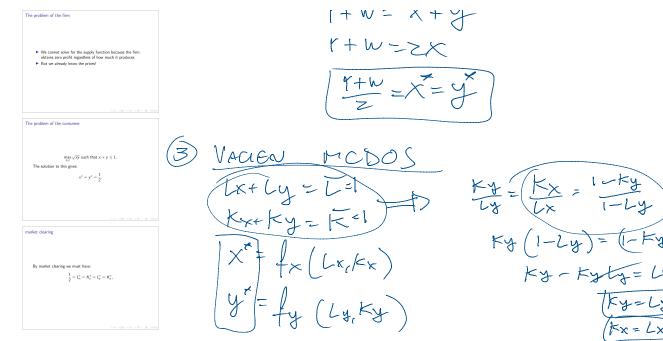


So lets check the market clearing condition for x: $x^{d}(\rho) = x^{s}(\rho).$	$P(1-\alpha)(\overline{p}\overline{p})$ $t \propto (\overline{p}\overline{p})^{\overline{p}-1} = \alpha L$
Market Clearing. We only need to check one market clearing condition (why?) So lass check the market clearing condition for <i>x</i> : $x^{(4)}(p) = x^{4}(p).$ As a result, $\frac{\alpha}{p} \left( I + p^{\frac{1}{10}} \left( p^{\frac{1}{10}} - g^{\frac{1}{10}} p \right) \right) = p^{\frac{1}{10}} p^{\frac{1}{10}},$	$P\left(\frac{1}{P}\right)^{B/B-1}\left(1-\alpha\right)\left(\frac{1}{P}\right)^{B/B-1} + \propto \left(\frac{1}{P}\right)^{B-1}\left(\frac{1}{P}\right)^{1-1} = \propto \overline{L}$
$\label{eq:Market Clearing} \end{tabular}$ We only need to check one market clearing condition (why?) So lets check the market clearing condition for <i>x</i> : $\kappa^{(d)}(p) = \kappa^{(d)}(p) = \kappa^{(d)}(p) = r^{(d)}(p) = r^{($	$\begin{pmatrix} 1 \\ P \end{pmatrix}^{B}/B-1^{-1} \begin{pmatrix} 1 - \alpha \end{pmatrix} \begin{pmatrix} 1 \\ B \end{pmatrix}^{B}/B-1 + \alpha \begin{pmatrix} 1 \\ P \end{pmatrix}^{B-1} \begin{pmatrix} 1 \\ B \end{pmatrix}^{B-1} = \alpha Z$
Solving this, we obtain $p^* = \left(\frac{\alpha I}{\alpha \beta^{1/2} + (1-\alpha)\beta^{1/2}}\right)^{1/\beta}.$ To solve for $s^*, L^1, L^2_0$ , we plug the pice back into the demand and supply function: $\left(\frac{\beta T^{1/2} \alpha I}{s}, \frac{\beta T^{1/2} \alpha I}{s}\right)^{\beta}$ $\left(\frac{s}{s} = I - L^2_0 + \alpha \beta^{1/2}\right)^{\beta}$	$ \left( \begin{array}{c} 1 \\ p \end{array} \right) \xrightarrow{F-1} \left( (-x) \left( \begin{array}{c} 1 \\ p \end{array} \right) \xrightarrow{F-1} + x \left( \begin{array}{c} 1 \\ p \end{array} \right) \xrightarrow{F-1} = x \\ \end{array} \right) \xrightarrow{F-1} = x \\ \end{array} $
$\begin{aligned} \mathcal{L}_{s}^{*} = \mathcal{L}_{s}^{*}(\mathbf{p}^{*}) = \frac{\alpha \sigma}{1 - \alpha + \alpha \sigma^{2}} \mathbf{I}. \end{aligned}$ To solve for $s^{*}, \mathcal{L}^{*}, \mathcal{L}^{*}_{s}$ , we plug the price back into the demand and supply functions: $\mathbf{x}^{*} = \mathbf{x}^{*}(\mathbf{p}^{*}) = \left(\frac{\beta \tau^{*} \gamma \alpha \mathbf{I}}{\beta \tau^{*} (1 - \alpha) + \alpha \beta \tau^{*} \gamma}\right)^{\beta}$ $\mathcal{L}^{*}_{s} = \mathbf{I} - \mathcal{L}_{s} = \frac{1 - \alpha}{2\beta} \mathbf{I}.$ $\mathcal{L}^{*}_{s} = \mathcal{L}_{s} - \mathbf{I}_{s} = \frac{\alpha}{2\beta} \mathbf{I}.$	$ \begin{pmatrix} \bot \\ P \end{pmatrix}^{\frac{1}{p}-1} \begin{pmatrix} (-\chi) \begin{pmatrix} \bot \\ P \end{pmatrix}^{\frac{p}{p}} \begin{pmatrix} B \\ P \end{pmatrix}^{\frac{p}{p}-1} + \chi \begin{pmatrix} \bot \\ P \end{pmatrix}^{\frac{1}{p}-1} \end{pmatrix} = \propto L $
We can also solve for the profiles of the firm in equilibrium: $\begin{split} & \Pi^*(p^*) = pr^{\frac{1}{2}}\left(gr^{\frac{1}{2}}-gr^{\frac{1}{2}}gr^{\frac{1}{2}}\right) \\ & = \frac{oI}{\alpha\beta^{\frac{1}{2}}+(1-\alpha)\beta^{\frac{1}{2}}g}\left(\beta r^{\frac{1}{2}}-\beta r^{\frac{1}{2}}g\right). \end{split}$	$\frac{1}{2}\left(\left(-\alpha\right)\left(\frac{1}{2}\right)^{\beta/\beta-1}+\tau\left(\frac{1}{2}\right)^{\beta-1}\right)=62$
Lecture 5: General Equilibrium Introducing production General equilibrium with production Examples Two Factor Medel	$\left(\left(1-\alpha\right)\left(\frac{1}{5}\right)^{\beta}\left(\beta-1\right) + \alpha\left(\frac{1}{5}\right)^{\beta-1}\right) = p^{\alpha}$



$$\frac{1}{2} \sum_{\substack{w \in w \\ w & w \\$$





$$F = \frac{1}{1 - Ly}$$

$$\frac{1}{2y} = (-Fy) Ly$$

$$-Fy Ly = Ly - Fy Ly$$

$$\frac{Fy = Ly}{Fx = Ly}$$

$$\frac{Fy = Ly}{Fx = Lx}$$

$$\frac{Fy = V}{F} = L$$

$$\frac{W = F^{2}}{L}$$

$$\frac{1}{2} = W, F$$

$$\frac{L}{2} = W = F^{2}$$

$$\frac{V_{GSUMGN}}{P_{x}=1, P_{y}=1, w=1/2, y=1/2}$$

$$\chi^{*} = l|z, q^{*} = l|z$$

$$\int_{z}^{1/z} L_{x} = L_{x} = L_{x} = L_{x} = L_{y} = L_{y}$$

Q.F.  
MAX 
$$\chi^{lk} y^{ll2}$$
. S.a.  $V_{Demas \neq k}$   
Facture  $\chi^* = f_{\chi}(L_{\chi}, L_{\chi})$   
 $\Gamma_{\Lambda} + Ly = L$   
 $K_{\chi} + Ky = K$