Lecture7

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Lecture7

Lecture 7: Monopoly

Mauricio Romero

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- Firm is faced a problem like the following:

$$\max_{K,L} p_x f_x(L,K) - wL - rK.$$

- The firm's choice of L and K does not affect the prices p, w, r
- This is called *price-taking* behavior
- Justified if the the market is composed of many small firms

- In many markets there is a single firm
- Since supply is completely controlled by the firm, it can use this in its favor

- Profit maximization condition,

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$$\max_{K,L} pf_x(K,L) - wL - rK.$$

- If

$$c(x) = \min_{K,L} wL + rK \text{ such that } f_x(K,L) = x$$

then the above is equivalent to:

$$\boxed{\max_x px - c(x).}$$

DISPONIBILIDA A PAGAR CONS. → FUNCIÓN DEMANDA

- When firm controls supply, then:

$$\max_x \mathbf{p(x)}x - c(x)$$

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- Consumers willingness to pay is given by the demand function
- $p(x)$ is the **demand** function

- We can also represent the problem as:

$$\max_p pq(p) - c(q(p))$$

- $q(p)$ is the **inverse demand function**

Lecture 7: Monopoly

Introduction

Elasticities

Monopoly

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- Revenue: R(q) = p(q) q
- $$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right)$$
- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

$$\varepsilon_{q,p} = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q/q}{\Delta p/p} = \boxed{\frac{\Delta q}{\Delta p}}\cdot\frac{p}{q}$$

$$\approx \frac{\partial q}{\partial p}\cdot\frac{p}{q}$$

Elasticities

$$p(q)\left[1 + \frac{q}{p(q)}\cdot\frac{\partial p}{\partial q}\right]$$

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$$\frac{1}{\varepsilon_{q,p}} = \frac{q}{p}\cdot\frac{\partial p}{\partial q}$$

- $$\frac{dR}{dq} > 0 \iff 1 > -\frac{1}{\varepsilon_{q,p}} \iff \varepsilon_{q,p} < -1.$$

$$1 + \frac{1}{\varepsilon} > 0$$

- $\varepsilon_{q,p}$ is the elasticity of demand with respect to price

Elasticities

- If $\varepsilon_{q,p} \in (-1, 0)$, the demand is *inelastic*
  - An increase in price leads a small decrease in demand
  - An increase in quantity leads to a big decrease in price
- If $\varepsilon_{q,p} < -1$, then demand is *elastic*
  - An increase in price leads a big decrease in demand
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Elasticities

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APAIZTE

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$$\frac{1}{p^k} k p^{k-1} = \frac{k}{p}$$

- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$

Elasticities

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- Suppose that the demand function is of constant elasticity κ
- $$\frac{dq}{dp}\frac{p}{q} = \kappa < 0.$$
- $$\frac{1}{q}\frac{dq}{dp} = \kappa\frac{1}{p} \Longrightarrow \frac{d}{dp}\log q(p) = \frac{d}{dp}\log p^{\kappa}.$$
- By the fundamental theorem of calculus:
$$\log q(p) = C + \log p^{\kappa}.$$
- $q(p) = e^{C} p^{\kappa}$ or $q(p) = Ap^{\kappa}$ for some A.

$$e^{\ln q(p)} = e^{C + \ln p^k} = e^{C} e^{\ln p^k}$$

$$q(p) = \boxed{e^{C}} p^k$$

$$q(p) = A p^{k}.$$

$$\frac{q}{A} = p^k$$

$$\left(\frac{q}{A}\right)^{\frac{1}{k}} = p$$

Elasticities

- $q(p) = e^c p^\kappa$ or $q(p) = Ap^\kappa$ for some A.

$\frac{q}{A} = p^k$

$\left(\frac{q}{A}\right)^{\frac{1}{k}} = p$

$\left(\frac{q}{10}\right) = p$

Elasticities

Whenever the demand function has constant elasticity κ

- $q(p)Ap^\kappa$ for some $A > 0$.
- Equivalently,

$$\boxed{p(q) = \left(\frac{q}{A}\right)^{1/\kappa}.}$$

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- We want to study the problem:

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$$\max_q \underbrace{R(q)}_{p\cdot q} - c(q) \longrightarrow \frac{\partial \pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial c}{\partial q} = 0$$

$$\frac{\partial R}{\partial q} = \frac{\partial c}{\partial q}$$

- The first order condition tells us:

$$\frac{dR}{dq} = \frac{dc}{dq} \Longrightarrow \boxed{p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = \frac{dc}{dq}} > 0.$$

- This implies

$$\boxed{1 + \frac{1}{\varepsilon_{q,p}} > 0 \Longleftrightarrow \varepsilon_{q,p} < -1.}$$

SOLUCION TIENE Q' ESTAR EN PARTE ELASTICA DD!

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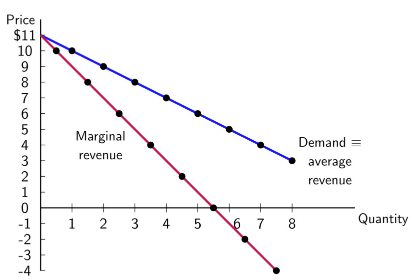
- A monopoly firm always produces at a point where demand is elastic
- If the firm produced at a point where demand was inelastic
- At such a point $\frac{dR}{dq} < 0$
- By reducing quantity (or raising the price) it could increase revenue and decrease costs simultaneously
- This strictly increases the profits



- We can simplify to:

$$p(q) = \frac{1}{1 + \frac{1}{\varepsilon_{q,p}}} \frac{dc}{dq}. \longrightarrow \left(\frac{1}{\frac{\varepsilon+1}{\varepsilon}}\right)\frac{\partial c}{\partial q} \Rightarrow \boxed{p = \frac{\varepsilon}{\varepsilon+1}\frac{\partial c}{\partial q}}$$

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- The amount produced q is below the quantity where $p = MC$.

$$p = \boxed{\frac{\varepsilon}{\varepsilon+1}}\frac{\partial c}{\partial q}$$

$$\frac{\varepsilon}{\varepsilon+1} \overset{?}{>} 1$$

$$\varepsilon < \varepsilon + 1 \checkmark$$

$$0 < 1$$

$$\lim_{\varepsilon \to -\infty} \frac{1}{1+\frac{1}{\varepsilon}} = 0$$

$$\lim_{\varepsilon \to -1^-} \frac{1}{1+\frac{1}{\varepsilon}} = \infty$$

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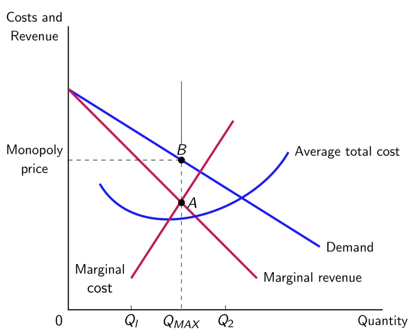
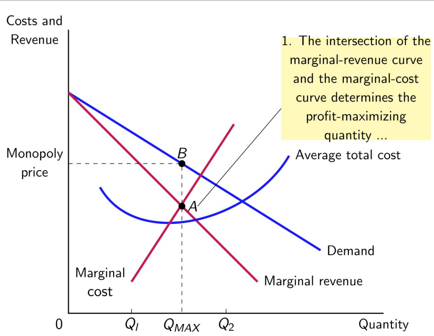
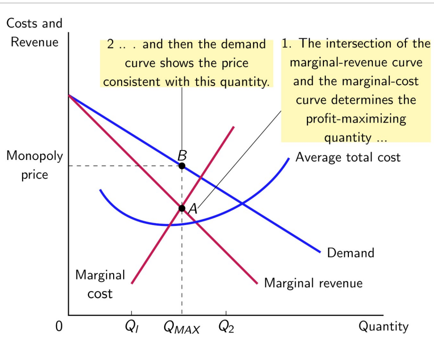
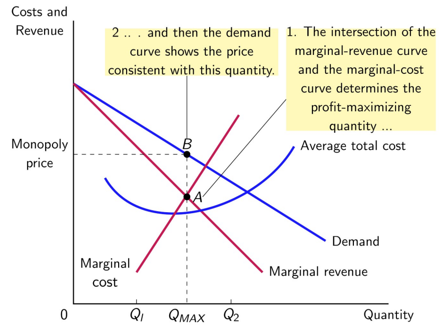
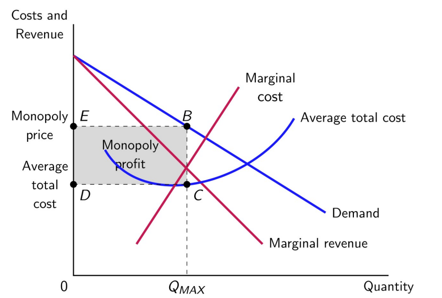
- Both consumer surplus and total surplus is less than is socially optimal
- Thus the pricing policies used by monopolies are inefficient, leading to what is called "dead-weight loss"

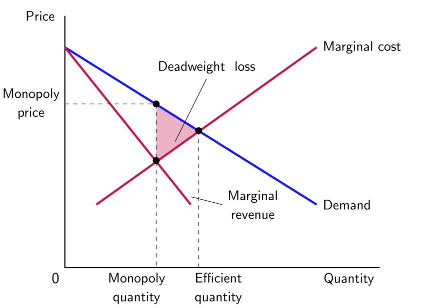


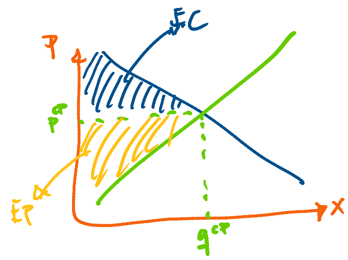


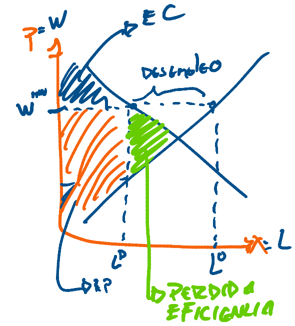


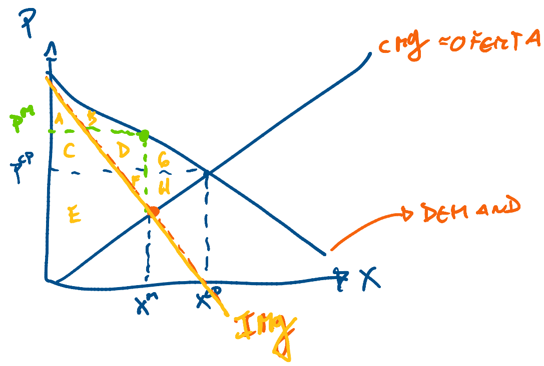












Comp. Perf

EC = A+B+C+D+G

EP = E+F+H

MONOP

EC = A+B

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PBS = G+H

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- $$p = \frac{c}{1 + \frac{1}{\kappa}} \Longrightarrow q(p) = A \left(\frac{c}{1 + \frac{1}{\kappa}} \right)^{\kappa}.$$

If profits are positive, why aren't more firms entering the market?

- Natural monopoly (Microsoft)
- Patents
- Political Lobbying: Televisa, Azteca, etc.
- Regulation (Moody and S & P's)
- Demand externalities
  - Classic network externalities (Microsoft): Microsoft Word and Windows are only valuable if a lot of consumers use it.
  - Two-sided markets (Ticketmaster or Uber): consumers value these markets only if there is enough supply of tickets. Similarly suppliers only value these markets if there is demand to meet the supply.