Lecture 8-9

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Lecture8-9

Lecture 8-9: Price Discrimination

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Lecture 8-9: Price Discrimination

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- In real life, firms often have different prices for different consumers/units
- We will explore some of these now
- In a competitive market such exotic pricing schemes could never arise since p = marginal cost

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- Suppose the firm can observe all characteristics of the consumer
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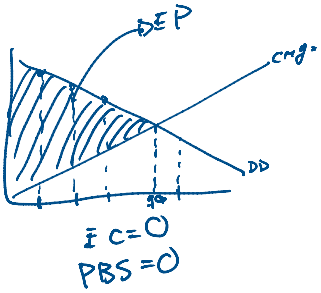
- What should the firm do?
- Demand curve illustrates the willingness to pay for the q-th unit of the product
- Firm can extract all of the surplus of the consumer. How?



- Firm will price at $p(q)$ for the q-th unit and continue to produce until $p(q) = MC(q)$

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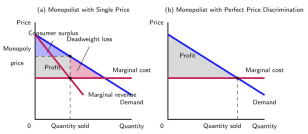
- Firm gets all of the consumer surplus as his profits:

$$\Pi = \int_0^{q^*} (p(q) - c'(q))dq = \int_0^{q^*} p(q)dq - c(q^*),$$

where q^* is the quantity at which $p(q^*) = c'(q^*)$.



- Firm can do this is because it knows the exact demand curve of each consumer
- Such activity is prohibited in many countries

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- Amazon tries to estimate everyone's demand curve

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- Market is segmented (no re-selling across markets)
- Firm knows the characteristics of each market (demand curve)
- Consider the following example: Two kinds of consumers:

$$q_A(p_A) = 24 - p_A$$
$$q_B(p_B) = 24 - 2p_B.$$

- constant marginal cost of production of 6

$Q_A = 24 - P \rightarrow \begin{cases} 0 & P \geq 24 \\ 24 - P & P \leq 24 \end{cases}$

$Q_B = 24 - 2P \rightarrow \begin{cases} 0 & P \geq 12 \\ 24 - 2P & P \leq 12 \end{cases}$

$Q_T = \begin{cases} 0 & P \geq 24 \\ 24 - P & P \in (12, 24) \\ 48 - 3P & P \leq 12 \end{cases}$

$Q_T = Q_A + Q_B = 48 - 3P$

$P_A = 24 - Q_A$

If the firm were allowed to set different prices in the different markets, then he would choose:

$$\max_{p_A}(24 - p_A)(p_A - 6) \Longrightarrow p_A^* = 15$$
$$\max_{p_B}(24 - 2p_B)(p_B - 6) \Longrightarrow p_B^* = 9.$$

$\pi = q(P)P - C(q)$

$\pi = (24 - P_A)P_A - 6(24 - P_A)$

$\pi = (24 - P_A)(P_A - 6)$

$\pi = (24 - q)q - 6q$

$\frac{\partial \pi}{\partial q} = 24 - 2q - 6 = 0$

$18 = 2q$

$q = 9$

$\frac{\partial \pi_A}{\partial P_A} = (-1)(P_A - 6) + (24 - P_A)(+1) = 0$

$= 6 - P_A + 24 - P_A = 0$

$30 = 2P_A$

$P_A^* = 15$

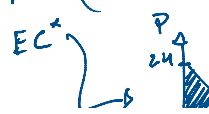
$q = 24 - P = 15$

Total consumer surplus (CS) and profits of the firm in each market:

$$\pi_A^* = 81, \pi_B^* = 18, CS_A = 40.5, CS_B = 9.$$

$\pi_A^* = (24 - P_A)(P_A - 6) = 9 \cdot 9 = 81 = EP$

EC^*



$\frac{(24 - 15)(9)}{2} = \frac{81}{2}$

$\pi_A^* = 81, \pi_B^* = 18, CS_A = 40.5, CS_B = 9.$

Firm chose to set the same price in each market. Then he would maximize the following:

$$\max\left\{\max_{p\geq 12}(24-p)(p-6), \max_{p<12}(24-p)(p-6)+(24-2p)(p-6)\right\}$$
$$= \max\{81, 75\} = 81$$

- Price of $p^* = 15$ in both markets, which leads to only consumers in market A buying
- To summarize, the consumer surplus and profits in each market are:

$$\pi_A^* = 81, \pi_B^* = 0, CS_A = 40.5, CS_B = 0.$$

- Prohibiting third degree price discrimination can exclude a whole market altogether
- Highly inefficient compared to the social welfare outcome given third degree price discrimination

- Suppose that the constant marginal cost of production is now 4 instead of 6
- With third degree price discrimination, the firm sets the following prices:

$$\max_{p_A}(24-p_A)(p_A-4) \Longrightarrow p_A^* = 14,$$
$$\max_{p_B}(24-2p_B)(p_B-4) \Longrightarrow p_B^* = 8.$$

- In this case, the profits and consumer surplus in each market is given by:

$$\pi_A^* = 100, \pi_B^* = 32, CS_A = 50, CS_B = 16, TS = 198.$$

- If the firm were prohibited from using third degree price discrimination, then:

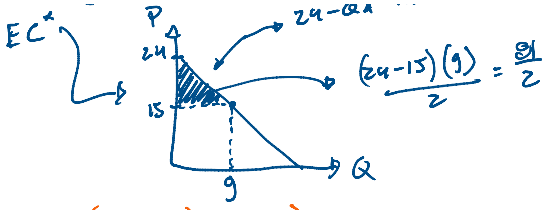
$$\max\left\{\max_{p\geq 12}(24-p)(p-4), \max_{p<12}(48-3p)(p-4)\right\}$$
$$= \max\{100, 108\} = 108.$$

- $p = 10$
- profits in both markets and the consumer surplus in both markets:

$$\pi_A^* = 84, \pi_B^* = 24, CS_A = 98, CS_B = 4, TS = 210.$$

- Consumers in region B are hurt but consumers in region A gain significantly leading to an increase in consumer surplus
- The firm's joint profits are hurt but the total surplus actually increases
- Total surplus decreases

- Third degree price discrimination is considered illegal in many countries and the European union
- It is possible to get around such allegations by claiming that the differential pricing comes from cost reasons



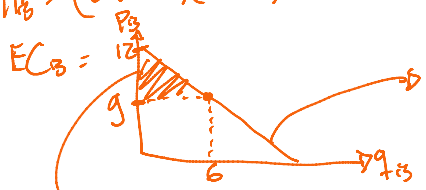
$\Pi_B = (24-2P_B)(P_B-6)$

$\Pi_B^* = (24-18)(9-6) = 6\cdot 3 = 18$

$q_B = 24-2P_B$

$P_B = \frac{24-q_B}{2}$

$= 12-\frac{1}{2}q_B$



$\frac{(12-9)(6)}{2} = \frac{3\cdot 6}{2} = \frac{18}{2}$

s.t. $P^* \geq 12$

$\Pi = (24-P)(P-6)$

$\frac{\partial \Pi}{\partial P} = (-1)(P-6)+(24-P)1 = 0$

$6-P+24-P = 0$

$30-2P = 0$

$P^* = 15$

$\Pi = (24-15)(15-6) = 81$

s.t. $P \leq 12$

$\Pi = (48-3P)(P-6)$

$\frac{\partial \Pi}{\partial P} = (48-3P)+(-3)(P-6) = 0$

$48-3P-3P+18 = 0$

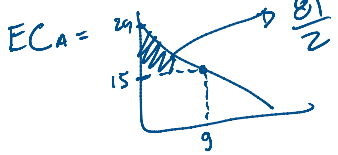
$66 = 6P$

$P = 11$

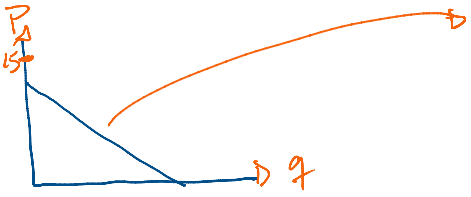
$\Pi^* = (48-33)(11-6)$

$= (15)(5) = 75$

$\Pi^* = 81 < EP$



$EC_B = 0$



$q_b = 24-2P_b$

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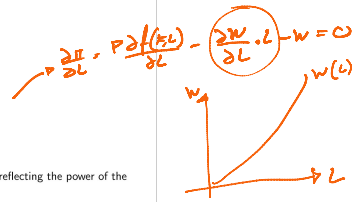
- When someone or some firm is the sole buyer (monopoly is the sole seller)
- Often arises in the context of firms being the sole buyers of labor

- Let us study the profit maximization problem of a firm:

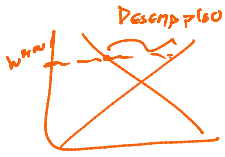
$$\max_{K,L} pf(K,L) - rK - w(L)L.$$



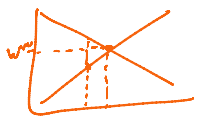
- w is now a function of the amount of labor demanded (reflecting the power of the firm in the labor market)

- The first order condition yields:

$$p\frac{\partial f}{\partial L}(K^*,L^*) = w'(L^*)L^* + w(L^*) \Longrightarrow pMPL = L^*w' + w$$



- In a competitive market $w' = 0$ and so $pMPL = w$
- Wages and labor below the competitive level (an argument for minimum wages and union)



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- Demand equation for good b is linear:

$$q_b(p_b) = 100 - p_b.$$

- Firm B's optimization problem becomes:

$$\max_{q_b}(100 - q_b)q_b - p_a q_b - cq_b.$$

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- The first order condition tells us:

$$100 - 2q_b = p_a + c \Longrightarrow p_a = 100 - 2q_b - c.$$

- Firm B's optimization problem becomes:

$$\max_{q_b}(100 - q_b)q_b - p_a q_b - cq_b.$$

- The first order condition tells us:

$$100 - 2q_b = p_a + c \Longrightarrow p_a = 100 - 2q_b - c.$$

- Since firm b is the only demander of commodity a, we have:

$$p_a = 100 - 2q_b - c = 100 - 2q_a - c.$$

- If the price is p_a then the q_a that solves the above equation would be the amount demanded of good a

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- Thus firm B's maximization problem has given us an inverse demand function for commodity a

- Since firm A is also a monopolist in producing good a, we can solve firm A's maximization problem in the following way:

$$\max_{q_a} q_a (100 - 2q_a - c).$$

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- As a result, we get:

$$100 - 4q_a - c = 0 \Rightarrow q_a^* = \frac{100 - c}{4}, p_a^* = 50 - \frac{c}{2}.$$

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- Firm a decides to supply the above units of a at a price $50 - c/2$

- Firm B will produce $q_b^* = q_a^* = \frac{100-c}{4}$

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- Then the price is given by:

$$p_b^* = 100 - \frac{100 - c}{4} = 75 + \frac{c}{4}.$$

- Firm B will produce $q_b^* = q_a^* = \frac{100-c}{4}$
- Then the price is given by:

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- To summarize, we have:

$$p_a^* = 50 - \frac{c}{2} \quad (1)$$
$$q_a^* = \frac{100 - c}{4} \quad (2)$$
$$p_b^* = 75 + \frac{c}{4} \quad (3)$$
$$q_b^* = \frac{100 - c}{4} \quad (4)$$

- **Case 1:** $c = 0$
-

$$p_a^* = 50, q_a^* = 25, p_b^* = 75, q_b^* = 25.$$

- If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?
- The monopolists problem becomes:

$$\max_{q} q(100 - q).$$

- The first order condition states that:

$$100 - 2q^* = 0 \longrightarrow q^* = 50, p^* = 50$$

$\pi_A = p_A q_A$

$\pi_B = p_B q_B - q_A p_A - \cancel{T_B C}^{0}$

$\pi_T = p_B q_B$

- **Case 1:** $c = 0$
- $$p_a^* = 50, q_a^* = 25, p_b^* = 75, q_b^* = 25.$$
- If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?
- The monopolists problem becomes:
$$\max_q q(100 - q).$$
- The first order condition states that:
$$100 - 2q^* = 0 \implies q^* = 50, p^* = 50.$$
- Price of good b comes down from 75 to 50
- Production of good b goes up from 25 to 50
- This increases both the profits of the firm *and* the consumer surplus!

$\pi_A = P_A q_A$

$\pi_B = P_B q_B - q_A P_A - \cancel{\pi_B C}^{0}$

$\pi_T = P_B q_B$

- **Case 1:** $c = 10$
- $$p_a^* = 45, q_a^* = 22.5, p_b^* = 77.5, q_b^* = 22.5.$$ 45 45
- If the firms were to merge so that whatever is produced by one of the firms can be used freely by that firm?
- The monopolists problem becomes:
$$\max_q q(100 - q) - 10q$$
- The first order condition states that:
$$100 - 2q = 10 \implies p^* = 55, q^* = 45.$$
- This increases both the profits of the firm *and* the consumer surplus!

- What is going on in the above examples?
- because the first firm is a monopolist, it charges a mark up above marginal cost for its intermediate good
- This then distorts the marginal cost of firm B up additionally
- This then leads an even larger mark up on top of this additional marginal cost that affects the price of good b
- Essentially a markup on product a indirectly leads to an even larger markup on the final product b
- This is called the **double marginalization problem**

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Introduction

First Degree Price Discrimination

Third Degree Price Discrimination

Monopsony

Double Marginalization Problem

Profit Sharing and Double Marginalization

- Double marginalization can lead to inefficiently high prices and inefficiently low levels of production
- By merging, both profits of the firm and consumer surplus may simultaneously go up
- Difficult to tell if two firms are merging to solve a double marginalization problem or if they are simply merging to create a monopoly
- What are some potential ways to solve this problem without mergers?
- One possible way might be to engage in profit sharing

- Firms agree to share profits according to the following rule
- Prices charged for good a are zero
- In exchange, the profits of firm B are shared via a split of α going to firm A and $(1-\alpha)$ going to firm B

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- Prices charged for good a are zero
- In exchange, the profits of firm B are shared via a split of α going to firm A and $(1-\alpha)$ going to firm B
- Firm A's decision is trivial. He simply produces $q_a = q_b$
- Firm B chooses to maximize:

$$\max_q (1-\alpha)\left((100-q)q - cq\right) = (1-\alpha)\left(\max_q (100-q)q - cq\right).$$

- Term inside the parentheses is just the monopoly profits if the two firms merged:

$$(1-\alpha)\max_q \Pi^m(q).$$

- The firms will produce at the monopoly quantities which we were found were strictly greater than if the two firms produced completely separately without any such agreement

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- For any $\alpha \in (0,1)$, we get an increase in consumer surplus and total profits
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- Such arrangements can break down easily. Profits are hard to verify.

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- Firms enter into an arrangement where the revenues are shared according to α (firm A) and $(1-\alpha)$ (firm B) split
- Suppose that $\alpha = 1/2$ and $c = 10$. Then firm 2 maximizes:

$$\max_q \frac{1}{2} q(100 - q) - 10q.$$

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$$\frac{1}{2} MR(q) = MC = 10 \Longrightarrow MR(q) = 2MC = 20.$$

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$\frac{\partial \Pi}{\partial q} = \frac{1}{2}(100 - 2q) - 10 = 0$

$100 - 2q = 20$

$80 = 2q$

$q = 40$

$P = 100 - 80$

$P = 20$

- The first order condition gives:

$$\frac{1}{2} MR(q) = MC = 10 \Longrightarrow MR(q) = 2MC = 20.$$

- Firm will produce below monopoly profits since it will produce at a point where $MR = 2MC$ instead of $MR = MC$

- Solving, we get:

$$100 - 2q = 20 \Longrightarrow p^* = 60 > p^m = 55, q^* = 40 < q^m = 45.$$

- Solving, we get:

$$100 - 2q = 20 \Longrightarrow p^* = 60 > p^m = 55, q^* = 40 < q^m = 45.$$

- This does solve the double marginalization problem slightly:

$$p_b^* = 77.5 > p^* = 60, q_b^* = 22.5 < q^* = 40.$$