Repaso Parcial 2

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$$l_x + l_y = (T_a - h_a) + (T_b - h_b)$$

III. (Examen Septiembre 2008) Considerar una economía con 1 agente y dos bienes de consumo (X e Y). La dotación inicial de capital es igual a 1, y la de trabajo es igual a L^d. Los bienes de consumo se producen mediante las funciones de producción siguientes (las empresas correspondientes son propiedad del único agente de la economía):

$$\boxed{x = (k)^{1/2},} \qquad k_x = x^2$$
$$\boxed{y = (l)^{1/2},} \qquad l_y = y^2$$

donde:

x denota la cantidad de X producida,

y denota la cantidad de Y producida,

k denota la cantidad de capital que se utiliza para producir X,

l denota la cantidad de trabajo que se utiliza para producir Y.

La función de utilidad del agente es igual a

$$U = x^{1/2} y^{1/2}.$$

Se supone que el precio por una unidad de Y es igual a 1. Se supone también que

p = precio por una unidad de X,
r = precio por una unidad de capital,
w = precio por una unidad de trabajo.

a) Calcular el valor de p en equilibrio. (Es decir: expresar p en función de L^d). Justificar la respuesta.

b) Calcular el valor de r en equilibrio. Justificar la respuesta.

c) Calcular el valor de w en equilibrio. Justificar la respuesta.

1 Firma X

$$\max_{K} \Pi_K = P_x K^{1/2} - rK$$

$$\frac{\partial \Pi_x}{\partial k} = \frac{1}{2} P_x K^{-1/2} - r = 0$$

$$P_x K^{-1/2} = 2r$$

$$\frac{P_x}{2r} = K^{1/2}$$

$$\boxed{\left(\frac{P_x}{2r}\right)^2 = K^*} \quad \text{DEMANDA } K$$

$$f_x(K) = K^{1/2} = \left[\left(\frac{P_x}{2r}\right)^2\right]^{1/2}$$

$$\boxed{= \frac{P_x}{2r}} \quad \text{OFERTA } X$$

$$\Pi_x = P_x\left(\frac{P_x}{2r}\right) - r\left(\frac{P_x}{2r}\right)^2$$

$$\Pi_x = \frac{P_x^2}{2r} - \frac{P_x^2}{4r} = \boxed{\frac{P_x^2}{4r} = \Pi_x^*}$$

$$\frac{P_x^2}{r}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{P_x^2}{r}\left(\frac{1}{4}\right)$$

Firma Y

$$\boxed{l^* = \left(\frac{P_y}{2w}\right)^2} = \left(\frac{1}{2w}\right)^2 = \boxed{\frac{1}{4w^2}}$$

$$\boxed{y^{\text{OFERTA}} = \frac{P_y}{2w}} = \boxed{\frac{1}{2w}}$$

$$\boxed{\Pi_y^* = \frac{P_y^2}{4w}} = \frac{1}{4w}$$

2 Agentes MAX

$$\max_{x,y} \; x^{1/2} y^{1/2} \quad \text{s.a} \quad P_x X + P_y y \leq 1 \cdot r + L^D \cdot w + \Pi_x^* + \Pi_y^*$$

$$\mathcal{L} = x^{1/2} y^{1/2} + \lambda\left(r + L^D w + \Pi_x^* + \Pi_y^* - P_x X - P_y y\right)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2} x^{-1/2} y^{1/2} - \lambda P_x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2} x^{1/2} y^{-1/2} - \lambda P_y = 0$$

$$\frac{y}{x} = \frac{P_x}{P_y} \Rightarrow \boxed{y = \frac{P_x \cdot X}{P_y}}$$

$$r + L^D w + \Pi_x + \Pi_y = P_x \cdot X + P_y \cdot y$$
$$= P_x \cdot X + \cancel{P_y}\left(\frac{P_x \cdot X}{\cancel{P_y}}\right)$$
$$= 2P_x \cdot X$$

$$\boxed{X^* = \frac{r + L^D w + \Pi_x + \Pi_y}{2P_x}}$$

$$y^* = \frac{r + L^D w + \Pi_x + \Pi_y}{2P_y \cdot 1}$$

$$\boxed{y^* = \frac{r + L^D w + \Pi x + \Pi y}{2Py + 1}}$$

③ MCDOS VACIEN

$$K^* = \frac{1}{\infty} \rightarrow K^* = \boxed{1 = \left(\frac{Px}{2r}\right)^2} \rightarrow 1 = \frac{Px}{2r} \rightarrow \boxed{2r = Px}$$

$$\boxed{\ell^*} = L^D \rightarrow L^D = \left(\frac{Py}{2w}\right)^2 = \frac{1}{4w^2}$$

$$\boxed{w^* = \sqrt{\frac{1}{4L^D}}} = \boxed{\frac{1}{2L^{1/2}}}$$

$$x^* = f_x(K^*) \rightarrow \frac{r + L^D w + \Pi x + \Pi y}{2Px} = \frac{Px}{2r}$$

$$y^* = f_y(\ell^*)$$

$$\frac{r + L^D\left(\frac{1}{2L^{1/2}}\right) + \frac{Px^2}{4r} + \frac{Py}{4w}}{2Px} = \frac{Px}{2r}$$

$$\frac{r + \frac{L^{1/2}}{2} + \frac{(2r)^2}{4r} + \frac{1}{4\left(\frac{1}{2L^{1/2}}\right)}}{2(2r)} = \frac{\cancel{2r}}{\cancel{2r}}$$

$$r + \frac{L^{1/2}}{2} + r + \frac{L^{1/2}}{2} = 4r$$

$$\boxed{L_0^{1/2} = 2r}$$

$$\boxed{\frac{L_0^{1/2}}{2} = r}$$

$$\boxed{Px = 2r = L_0^{1/2}}$$

Ejercicio:

$u = x y^{1/4}$ $\quad x = \ell x^{1/4} k x^{1/4}$ $\quad y = \ell y + 10 k y$ $\quad P_Y = 1$

$\bar{L} = \bar{K} = 20$

① FPP , ② O.P.

$$\max_{\ell_x, k_x, \ell_y, k_y} \ell_x^{1/4} K_x^{1/4} \quad \text{s.a.} \quad \left.\begin{array}{l} \ell_y + 10k_y \geq \bar{y} \\ \ell_y + \ell_x \leq 20 \\ K_x + K_y \leq 20 \end{array}\right\} \rightarrow (20 - \ell_x) + 10(20 - K_x) \geq \bar{y}$$

$$\mathcal{L} = \boxed{\ell_x^{1/4} K_x^{1/4}} + \lambda\left(20 - \ell_x + 10(20 - K_x) - \bar{y}\right)$$

$$\mathcal{L} = l_x \; K_x + \lambda(20 - l_x + 10(20 \ldots) \ldots)$$

$$\frac{\partial \mathcal{L}}{\partial l_x} = \frac{1}{4} l_x^{-3/4} K_x^{1/4} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_x} = \frac{1}{4} l_x^{1/4} K_x^{-3/4} - 10\lambda = 0$$

$$\frac{K_x}{l_x} = \frac{1}{10} \rightarrow \boxed{10K_x = l_x}$$

$$20 - 10K_x + 10(20 - K_x) = \bar{y}$$

$$20 - 10K_x + 200 - 10K_x = \bar{y}$$

$$220 - \bar{y} = 20K_x$$

$$\boxed{\frac{220 - \bar{y}}{20} = K_x^*} \Longrightarrow \boxed{\frac{220 - \bar{y}}{2} = l_x^*}$$

$$f_x = l_x^{1/4} K_x^{1/4}$$

$$= \left(\frac{220 - \bar{y}}{2}\right)^{1/4} \left(\frac{220 - \bar{y}}{20}\right)^{1/4} = X$$

EQUILIBRIO

① FIRMAS MAX

MAX Πx

MAX Πy

$$\max_{l_x, K_x} \Pi x = l_x^{1/4} K_x^{1/4} - w l_x - r K_x$$

$$\frac{\partial \Pi x}{\partial l_x} = \frac{1}{4} l_x^{-3/4} K_x^{1/4} - w = 0$$

$$\frac{\partial \Pi}{\partial K_x} = \frac{1}{4} l_x^{1/4} K_x^{-3/4} - r = 0$$

② AGENTE MAM

③ MCDOS VACIAN