Solución Taller 7

① Colusión tácita: Pañales

- Competencia en cantidades
- $P = 10 - q_1 - q_2$
- $c_1 = c_2 = 5$

ⓐ Fase cooperativa

$$\max \Pi^m = (10 - Q)Q - 5Q \qquad \Pi^m = \left(10 - \frac{5}{2} - 5\right)\frac{5}{2} = \frac{25}{4}$$

FOC: $10 - 2Q - 5 = 0$

$$Q^m = \frac{5}{2}$$

$$q^{coop} = \frac{5}{4} \qquad \Pi^{coop} = \left(10 - \frac{5}{2} - 5\right)\frac{5}{4} = \frac{5}{2}\cdot\frac{5}{4} = \frac{25}{8}$$

Fase desvio

$$\max \Pi^d = \left(10 - z - \frac{5}{4}\right)z - 5z$$

FOC: $10 - 2z - \frac{5}{4} - 5 = 0$

$$\frac{15}{4} = 2z$$

$$z = \frac{15}{8}$$

$$\Pi^{des} = \left(10 - \frac{15}{8} - \frac{5}{4} - 5\right)\frac{15}{8} = \left(\frac{5\cdot 8 - 15 - 10}{8}\right)\frac{15}{8} = \frac{225}{64}$$

Fase castigo

Ⓕ1 $\max \Pi_1 = (10 - q_1 - q_2 - 5)q_1$ $\qquad \Pi_1 = \left(5 - 2\left(\frac{5}{3}\right)\right)\left(\frac{5}{3}\right) = \left(\frac{15-10}{3}\right)\left(\frac{5}{3}\right)$

FOC: $5 - 2q_1 - q_2 = 0$

$$q_1(q_2) = \frac{5 - q_2}{2}$$

Ⓕ2 $\max \Pi_2 = (10 - q_1 - q_2 - 5)q_2$ $\qquad \Pi_1^{cour} = \Pi_2^{cour} = \frac{25}{9}$

FOC: $5 - q_1 - 2q_2 = 0$

$$q_2(q_1) = \frac{5 - q_1}{2}$$

$$2q_1 = 5 - \left(\frac{5 - q_1}{2}\right) \qquad q_1 = q_2 = \frac{5}{3}$$

$$4q_1 = 10 - 5 + q_1$$

$$3q_1 = 5$$

$$\delta^{Gatillo} \geq \frac{\pi^{desv} - \pi^{coop}}{\pi^{desv} - \pi^{cast}}$$

$$\delta^{Gatillo} \geq \frac{\left(\frac{225}{64}\right) - \left(\frac{25}{8}\right)}{\left(\frac{225}{64}\right) - \left(\frac{25}{9}\right)} = \frac{\frac{225 - 200}{64}}{\frac{2025 - 1600}{576}} = \frac{14\,400}{27\,200} = \frac{144}{272}$$

$$\delta^{Gatillo} \geq \frac{9}{17}$$

(b)

$$\delta^{Gatillo} \geq \frac{\pi^{desv} - \pi^{coop}}{\pi^{desv} - \pi^{cast}} = \frac{\pi^{m} - \pi^{m}/2}{\pi^{m} - 0} = 1 - \frac{1}{2} = \frac{1}{2}$$

(c) $\frac{9}{17} > \frac{1}{2}$

- Es mayor el factor de descuento de la Competencia en cantidades
- Cuando las firmas compiten en precios, el beneficio del castigo es 0 (paradoja de Bertrand) por lo que desviarse tiene mayores consecuencias

(2) Colusión Tácita: Cemento

- $P = 130 - q_1 - q_2$
- $C_1 = C_2 = 10$
- $\delta = \frac{3}{4}$

Fase cooperativa

$\max \pi^m = (130 - Q - 10)Q$

FOC: $120 - 2Q = 0$

$Q = \frac{120}{2} = 60$

$q_1 = q_2 = 30$

$\pi_1 = (120 - 60)30 = 1800$

$\pi_1(q^*) = \pi_2(q^*) = 1800$

Fase desvío de la cooperación

$\max \pi^d = (130 - 30 - z - 10)z$

FOC: $90 - 2z = 0$

$45 = z$

$\pi^d = (130 - 30 - 45 - 10)45 = 2025$

$\pi_1^d(q^*) = \pi_2^d(q^*) = 2025$

Fase de castigo

$$\pi_1(\hat{q}) = \pi_2(\hat{q}) = (130 - 2\hat{q} - 10)\hat{q} = (120 - 2\hat{q})\hat{q}$$

Fase desvío del castigo

$$\text{Max } \pi^d = (130 - \hat{q} - z - 10)z$$

FOC: $120 - \hat{q} - 2z = 0$

$$\frac{120 - \hat{q}}{2} = z$$

$$\pi^d = \left(130 - \hat{q} - \left(\frac{120 - \hat{q}}{2}\right) - 10\right)\left(\frac{120 - \hat{q}}{2}\right)$$

$$\pi^d = \left(\frac{240 - 2\hat{q} - 120 + \hat{q}}{2}\right)\left(\frac{120 - \hat{q}}{2}\right) = \left(\frac{120 - \hat{q}}{2}\right)^2$$

$$\pi_1^d(\hat{q}) = \pi_2^d(\hat{q}) = \frac{(120 - \hat{q})^2}{4}$$

CONDICIÓN DE CREDIBILIDAD

$$\delta[\pi(q^*) - \pi(\hat{q})] \geq \pi^d(\hat{q}) - \pi(\hat{q})$$

reemplazando.

$$\frac{3}{4}[1800 - (120 - 2\hat{q})\hat{q}] \geq \frac{(120 - \hat{q})^2}{4} - (120 - 2\hat{q})\hat{q}$$

CONDICIÓN DE SOSTENIBILIDAD

$$\delta^{GYZ} \geq \frac{\pi^d(q^*) - \pi(q^*)}{\pi(q^*) - \pi(\hat{q})}$$

$$\delta^{GYZ} \geq \frac{2025 - 1800}{1800 - [(120 - 2\hat{q})\hat{q}]}$$